Comparison of bifurcation analysis and maximum force criteria in the prediction of necking in stretched metal sheets

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ABSTRACT

In the literature dealing with plastic instabilities, in general, and especially those related to material (local) instabilities in relation to sheet metal forming analysis, a large number of necking and formability criteria have been proposed. However, a thorough and rigorous comparison of their theoretical foundations and underlying assumptions is still lacking.

In the present work, diffuse necking is investigated in the context of stretched metal sheets using two different approaches, namely bifurcation theory and maximum force principle. The contribution includes a critical analysis and a systematic comparison of their respective ability to predict necking. In particular it is shown that, in contrast to bifurcation theory, which is of quite general applicability, some restrictions are associated with the application of maximum force conditions. Also, the analysis emphasizes that, despite their similar predictions for some particular loading paths and constitutive models, the two approaches are quite different in general, which confirms the distinct nature of their theoretical foundations.

Analytical developments corresponding to the application of bifurcation theory and maximum force criteria allows us to derive closed-form expressions for the critical states (in terms of limit stresses and strains) in the context of elasto-plasticity and rigid-plasticity for a variety of hardening models. It is also shown that the resulting useful formulas, obtained within the rigid-plastic constitutive framework, represent accurate approximations to those found in elasto-plasticity, under the condition of small hardening modulus to Young's modulus ratios. It is noteworthy that in the context of rigid-plasticity, the well-known Swift's diffuse necking criterion is recovered through bifurcation analysis. Recall that Swift's criterion has long been attributed in the literature to the maximum force principle, while it is shown here to rather originate from the bifurcation analysis, which provides it with a sound theoretical justification.

REFERENCES

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