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PROPER ORTHOGONAL DECOMPOSITION IN DIRECT AND INVERSE ELASTIC-PLASTIC ANALYSIS

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Key words: inverse problems, parameter identification in metal plasticity, simulation of indentation test, Proper Orthogonal Decomposition, Radial Basis Function.

Summary: With reference to indentation tests on metal structures a material characterization procedure is proposed based on experimental date from both indentation curves and imprint profile and on inverse analyses centered on Proper Orthogonal Decomposition, Radial Basis Functions and Trust Region algorithm.

1 INTRODUCTION

Deterioration of structural materials due to chemical and physical processes implies reduction of mechanical properties such as elastic stiffness, yield limit and hardening parameters. In order to assess the level of accumulated damage diagnostic analysis of industrial components may be required. Frequently in the real-life engineering practice, possibly deteriorated components are still in service and, hence, require non-destructive tests and diagnoses to be performed in situ. In order to perform the whole parameter identification process routinely in situ, it is necessary to employ a fast numerical procedure for solving the inverse problem fed by data collected from experiments, like indentation tests.

In this communication it is shown that the computational burden associated with the repetitive simulations of such tests required by the algorithms for the minimization of the discrepancy function can be significantly reduced by an interpolation procedure based on Proper Orthogonal Decomposition (POD) combined with Radial Basis Function (RBF). It is evidenced how preliminarily performed simulations can be exploited to construct low-order approximations that can eventually be employed to compute the structural responses practically without any loss of accuracy with significant reduction of computing time. Such approach is adopted herein for elasto-plastic inverse analysis where repetitive analyses of the same system (namely the same geometry, same boundary conditions etc.) need to be performed for numerous diverse sets of material parameters.

2 DIRECT INELASTIC ANALYSIS BASED ON PROPER ORTHOGONAL DECOMPOSITION

For simulations of typical indentation tests on a metal specimen by a conical indenter a commercial Finite Element code (ABAQUS) using the model shown in Figure 1 have been adopted. The model exhibits the following main features: axiall-symmetry; 1600 quadrilateral elements (each with 4 nodes); regime of large-strain deformations; Huber-Mises elasto-plasticity without hardening (with parameters: Young's modulus E=202GPa, Poisson's ratio υ=0.3; yield limit σ_Y=300MPa,); elastically deformable indenter with E=1170GPa and υ=0.1.

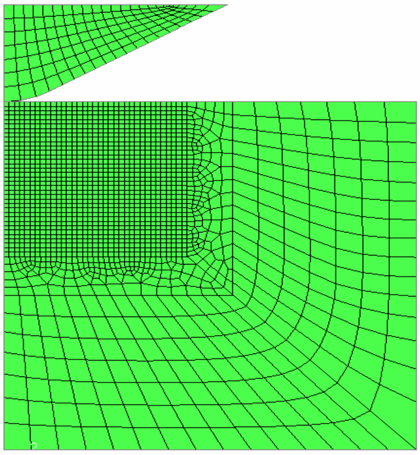

Figure 1: Finite Element model of the conical indentation test

Traditional direct analyses have been performed by attributing to parameters the following set of values: Young's modulus from 150GPa to 250Gpa with the step of 20GPa; yield limit from 150MPa to 400MPa with the step 25MPa. The results of these 66 direct analyses of indentations (the so-called "snapshots") are correlated as obviously expected, namely they are almost parallel if represented as vectors. In what follows, the results in focus are von Mises equivalent stresses at Gauss integration points, therefore 6400 components in each snapshot.

The expected correlation suggests to apply the mathematical procedure usually called Proper Orthogonal Decomposition (e.g.[4]). The procedure adopted can be outlined as follows: in the snapshot space (of dimensionality 6400) a new orthogonal system is determined such that with respect to the first direction the sum of projections of all (66) snapshots attains its maximum; a second direction is characterized by the same maximum property among the directions orthogonal to the first one, and so on. Such procedure implies the computation of eigenvalues and eigenvectors of a matrix (symmetric and positive-semi-definite here with dimensions 66×66) generated from the matrix which gathers all snapshots as columns pre-multiplying by its transpose. In the new reference system many components of the correlated snapshots are negligible. The computed eigenvalues can be used to select those which can be neglected (for details see [1,6]). In order to computationally exploit for a new analysis the compressed information extracted by the above POD procedure from the original snapshots, interpolation is now performed using Radial Basis Functions (RBF) specifically in the present study inverse multiquadric (for details see [5]).

Using the above interpolation procedure, the stress field generated by the indentation test specified in Section 2 has been computed starting from a new pair of parameters, E=202GPa, σ_Y=405MPa. The same field was computed by the conventional FE analysis through the before mentioned commercial code. The two results exhibit differences of the order of magnitudes of 1/10 percent. The remarkable feature is that the ratio between computing times of the two procedures turned out to be of the order of 10^5 in favour of the POD-RBF.

3 INVERSE ANALYSES BASED ON PROPER ORTHOGONAL DECOMPOSITION

The numerical exercise outlined in Section 2 clearly evidences the significant potential advantages of the use of POD-RBF interpolation for inverse analyses intended to identify material parameters in real life engineering practice on the basis of recurrent non-destructive tests, like instrumented indentations which at present frequently replace the traditional harness tests (see [2,3]).

Figure 2 shows typical experimental data provided by an instrumented indenter and by a laser profilometer, namely force vs. penetration curve (a) and imprint profile (b), respectively.

In the present computational exercise the sought material parameters are Young modulus, yield limit and hardening coefficient. Other meaningful features are as follows: FE model same as in Figure 1; snapshots preliminarily computed by the model concerning the following "nodes" in the parameter space: Young modulus from 130 to 230 GPa, step 20 GPa; yield limit from 150 to 350 MPa step 25 MPa; hardening exponent from 0 to 0.21 with the step 0.01; the chosen RBF are mulriqudric as mentioned earlier. As measurable quantities, 100 penetration depths (corresponding to 100 chosen indentation forces) are derived from the indentation curve, and 82 levels of imprint average profiles for a 82 radial distances from indentation axis.

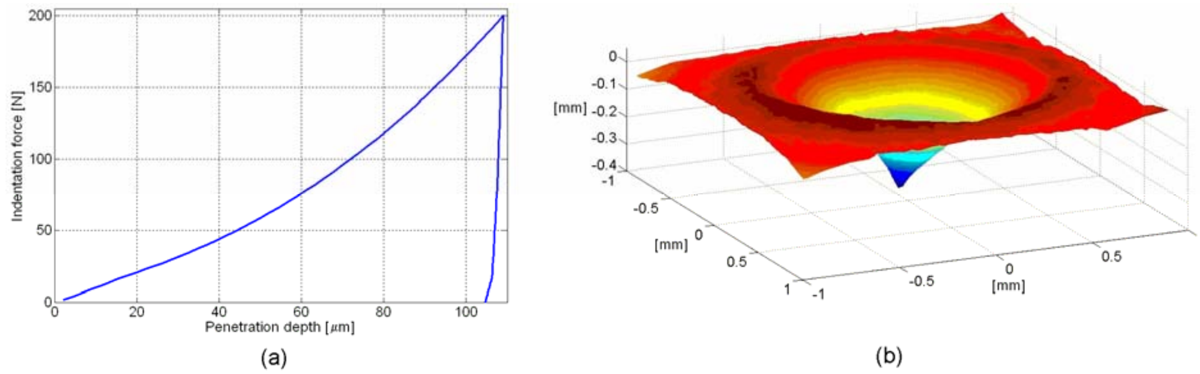

Figure 2: Results from the instrumented indentation test

The identification of three material parameters is here performed by a deterministic approach minimizing the discrepancy function defined as the Euclidean norm of the difference between experimental data and their computed counter-parts depending on the sought parameters as variables. The adopted tool is Trust Region (TR) algorithm, namely a sequence of two dimensional quadratic programming formulated at each step by gradient and Jacobian of the objective function. Convergence in a validation exercise is visualized in Figure 3.

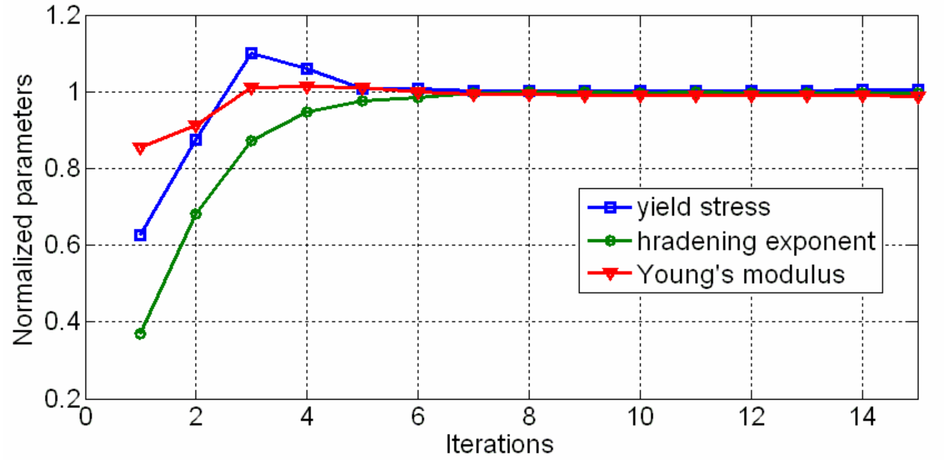

Figure 3: Convergence of the normalized parameters identified by TR algorithm

4 CONCLUSIONS

The significant reduction of computational efforts in inverse analysis when mathematical programming algorithms are combined with POD-RBF has been evidenced in this study with reference to indentation tests on possibly deteriorated metal plant components, with the practical prospect of in-situ repeated fast and inexpensive non-destructive structural diagnoses. Related subjects of research in progress are extensions of the computational technique presented herein to optimizations by genetic algorithms instead of TR, statistical approaches (Monte Carlo method, Kalman filter, etc.) to estimate material parameters together with their estimation uncertainties.

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