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Energy absorption in plastic expansion of circular metal tubes

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Summary. This paper investigates a new method of dissipating energy in which a rigid tube is driven into the deformable tube under axial compression. Since there is a specific clearance between the rigid and deformable tubes, the deformable tube is expanded during axial compression, and the impact energy is absorbed by plastic expansion of deformable tube and frictional energy between deformable and rigid tubes. Quasi-static axial compression tests with different values of friction coefficient between rigid and deformable tubes are performed. Theoretical formulation with consideration of strain hardening effect is presented to predict the mean crushing load. Theoretical and experimental studies show that this method of dissipating energy has high crush force efficiency and favorable crashworthiness characteristics.

1 INTRODUCTION

The protection of structures under impact loading often necessitates the need for energy absorber which dissipates impact energy in a variety of ways (e.g. by friction, fracture, plastic bending, crushing, cyclic plastic deformation, metal cutting, etc) [1]. Circular metal tubes are used extensively as energy absorbing elements, their ready availability in a wide range of dimensions and materials as well as the wide range of deformation modes which can be generated are the main reasons of their extensive applications for energy dissipating purposes [2].

Axial crushing of tubes between two flat plates, inversion of tubes against shaped die and axial splitting and curling of cylindrical tubes against canonical dies are the most common energy dissipating methods, which have been realized and studied by several researchers so far [3-5]. However, the efficiency of these realized energy absorbing methods are strongly sensitive to external parameters like loading direction and conditions, which causes the limitation of their applications in protection of some structures where collision may cause serious damages.

In this paper, expansion of deformable tubes by a rigid tube under axial compression is introduced as a very efficient process for absorbing impact energy, which has not been taken into attention so far.

2 THEORITICAL FORMULATION

According to the deformation mechanism of deformable tubes, a simple model with the following assumptions is used for the analysis:

1. The elastic strain is neglected.
2. Coulomb friction model is considered.
3. The material flow curve obeys equation $\overline{\sigma} = k\overline{\varepsilon}^{n}$
4. There is no variation in tube thickness during expansion process.
5. Horizontal and vertical directions are considered as the principal axes of strain and stress.

The equilibrium equation in the horizontal direction is given by:

$$\sigma_r = P\left(1-\mu \tan\alpha\right) \qquad (1)$$

Where p is the axial load, and μ is friction (Fig.1).

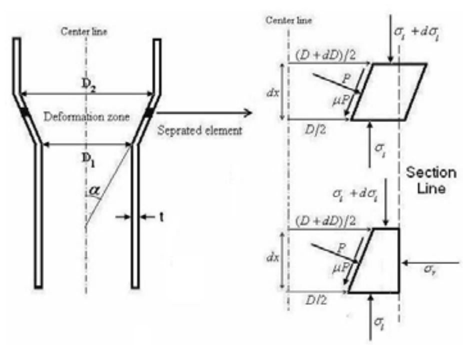

Figure 1: analytic model of expansion of deformable tube

Using Tresca yield criterion and assumption number 3, we will have:

$$\sigma_l - \sigma_r = k\overline{\varepsilon}^{n} \qquad (2)$$

The radial and hoop strain is $\varepsilon_r = \varepsilon_\theta = Ln(D/D_1)$, and with incompressibility $\varepsilon_l = -2Ln(D/D_1)$.

By inserting effective plastic strain ($\overline{\varepsilon} = 2Ln(D/D_1)$) one can have $\sigma_l - \sigma_r = k\left(2Ln(D/D_1)\right)^{n}$.

The equilibrium equation in the vertical direction can be written as below:

$$\frac{d\sigma_l}{dD} + \frac{\sigma_l}{D} - \frac{1}{t}\left(\frac{\mu + \tan\alpha}{2\tan\alpha\left(1-\mu\tan\alpha\right)}\right)\sigma_l = -\frac{k}{t}\left(\frac{\mu + \tan\alpha}{2\tan\alpha\left(1-\mu\tan\alpha\right)}\right)\left(2Ln(\frac{D}{D_1})\right)^{n} \qquad (3)$$

The value of longitudinal stress in expanded diameter of deformable tube ($D = D_2$) is zero. Using this boundary condition, and substituting $D = D_1$ in to the solution expression of differential equation (3), the value of longitudinal stress in the unexpanded part of the deformable tube during expansion process is derived from the following expression:

$$\sigma_d = \frac{kAe^{\frac{AD_1}{t}}}{tD_1}\int_{D_1}^{D_2} De^{-\frac{A}{t}D}[2Ln(\frac{D}{D_1})]^n dD \tag{4}$$

where $A = \left([\mu + \tan\alpha]/[2\tan\alpha(1-\mu\tan\alpha)]\right)$

Using three-point Gaussian quadrature method to solve the integral in Eq.4 numerically, this equation is estimated as follows:

$$\sigma_d = \frac{kmAe^{-\frac{mA}{t}}}{9tD_1}\left\{\left[5e^{-0.7m}(d-0.7m)\left(2Ln(\frac{d-0.7m}{D_1})\right)^n\right] + \left[5e^{0.7m}(d+0.7m)\left(2Ln(\frac{d+0.7m}{D_1})\right)^n\right] + 8d\left(2Ln(\frac{d}{D_1})\right)^n\right\} \tag{5}$$

where, $m = (D_2 - D_1)/2 \qquad d = (D_2 + D_1)/2$

Finally, the mean crush force of the shock absorber can be easily obtained from multiplying the value of longitude stress in unexpanded part of the deformable tube by its area.

Eq.5 indicates that the mean crush load is a function of unknown parameter of expansion angle of deformation α. According to the minimum energy approach, which suggests that the stable expansion process propagates when the value of expansion angle of deformation is such that the total force is rendered to a minimum, this unknown parameter is derived from the following expression $dP_m/d\alpha = 0$.

3 RESULTS AND DISCUSSION

Experimental results show that expansion of deformable tube with rigid one is an efficient energy dissipating method which results in a high crush force efficiency and completely constant crush force. Mean crush force in this method of dissipating energy can be affected by the value of friction coefficient, where increasing of the friction coefficient increases the value of mean crush load considerably. Hence, friction coefficient is one of the important parameters which can be used by designers to obtain their required values of mean crush force for protection of the crashworthy structure. Other parameters like the thickness of deformable tube and the clearance between rigid and deformable tubes, which their effects on energy absorption behaviors of the shock absorber are not studied in the present paper, can be used to control the values of crush force. Experimental results have also shown that the shock absorber has very low sensitivity to external parameters.

The experimental and analytical values of mean crush force against friction coefficient are plotted in Fig. 2. It is worth mentioning that if the strain hardening effect is employed in the mathematical model with k = 485 Mpa and n = 0.23 (the values corresponding to the material of deformable tube) the theoretical predictions give better correlation with experimental data, compared to the case that effect of the strain hardening is neglected ($\overline{\sigma} = \sigma_y$ or k = 340 Mpa and n = 0). Also as it is shown in this figure, strain hardening has an effect in higher values of mean crush load. These observations indicate the effects of strain hardening on deformation behavior of deformable tube.

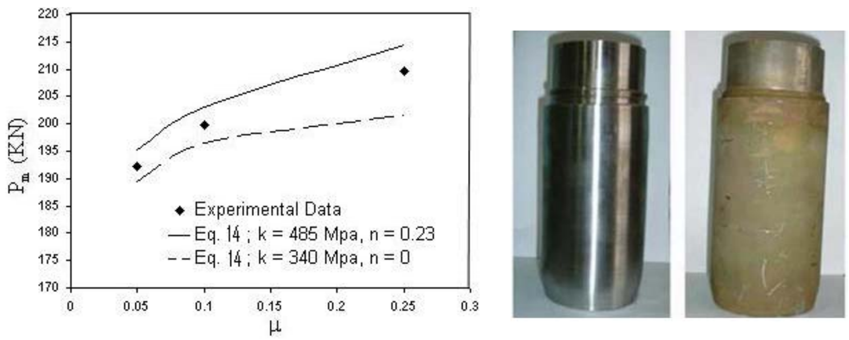

Figure 2: experimental and analytical mean crushing load of all specimens

4 CONCLUSIONS

Experimental results show that expansion of circular metal tubes by rigid one is an efficient energy dissipating method, which has the following advantages from the view point of energy absorption:

- It results in a high crush force efficiency and constant crush load.
- The value of mean crush load can be easily controlled by designers.
- It is less sensitive to external parameters like loading conditions and directions in comparison to other methods of dissipating energy by metal tubes.

The theoretical formulations performed in this study is in a good agreement with experimental findings. These theoretical studies indicate that consideration of strain hardening effect leads to more accurate predictions.

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