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ELASTIC/PLASTIC SHAKEDOWN BOUNDARY DETERMINATION ACCOUNTING FOR LIMITED DUCTILITY STRUCTURES

Francesco Giambanco, Luigi Palizzolo and Marianna Zito

Dipartimento di Ingegneria Strutturale e Geotecnica, DISeG
Università degli Studi di Palermo
Viale delle Scienze, 90128 – Palermo - Italy
e-mail: fgiamban@diseg.unipa.it, palizzol@diseg.unipa.it, zito@diseg.unipa.it

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Summary. *The problem related to the determination of the borderline between elastic/plastic shakedown and incremental/instantaneous collapse domains in the Bree diagram for elastic perfectly plastic structures subjected to a combination of fixed and perfect cyclic loads is studied. The relevant boundary is determined taking into account suitable limits related to the structure ductility features. In particular, basing on a special bounding theorem, it is possible to introduce appropriate constraints on chosen measure of the plastic strains occurring at the end of the transient phase of the structural response.*

1 INTRODUCTION

In many cases of practical interest, elastic plastic structures are subjected to the action of loads which can be described as a combination of fixed and cyclic loads arbitrarily varying within given limits. Under such conditions, and for load intensities not exceeding suitable limits, the elastic shakedown theory provides useful tools in studying the behaviour of the relevant structure, as well as the so-called bounding techniques provide limits on suitably chosen measures of the plastic deformations related to the transient phase of the elastic shakedown response of the structure. Furthermore, if the load multipliers exceed the elastic shakedown limit, then the structure is addressed towards a collapse condition, either due to a plastic shakedown behaviour or to a ratchetting behaviour. Finally, for increasing values of the load multipliers the structure is eventually addressed towards an instantaneous collapse.

Above the elastic shakedown limit (and below the instantaneous collapse) it is preferable that the structure behaves in condition of plastic shakedown, rather than in condition of ratchetting; as a consequence, the knowledge of the borderline between elastic/plastic shakedown and incremental/instantaneous collapse domains on the Bree diagram is of crucial importance to establish if the assigned structure/load system safely operates with potentially different load conditions, as well as the plastic strain response might be known in order to check the respect of some ductility and/or functionality limits for the structure.

As known, the plastic shakedown steady-state structural response possesses the same periodicity features as the loads and it can be determined by solving a sequence of linear complementarity problems related to the given load condition. The kinematical part of such a response provides the steady-state plastic strain history during the cycle and, as a consequence, it is possible to compute any chosen measure of such deformations, but, unfortunately, being unforeseeable the real loading path, the same response does not provide any information about the plastic deformations occurring during the initial transient phase. Therefore, in order to obtain even rough information about the plastic deformations occurring at the end of the transient phase, it is necessary to make reference to suitable bounding techniques. These techniques allows us to evaluate bounds on some prefixed measures of the plastic deformations, whatever the real loading history is during the transient phase.

The present paper, therefore, is devoted to the formulation of a sequence of maximum fixed load multiplier problems for the determination of the above described shakedown boundary, appropriately constrained in order to take into account suitable bounds on chosen measures of the plastic deformations characterizing the transient phase structural response.

2 ELASTIC/PLASTIC SHAKEDOWN BOUNDARY DETERMINATION

Let us consider a finite element elastic perfectly plastic structure subjected to quasi-statical loads defined as the combination of a reference mechanical fixed load $\boldsymbol{F}_0$ and a reference mechanical perfect cyclic load identifying with a convex polygonal shaped path with vertices corresponding to an even number b of mutually independent load vectors, $\boldsymbol{F}_{ci}$, $\forall i \in I(b)$. Furthermore, let us introduce the scalars $\xi_0 \geq 0$ and $\xi_c \geq 0$, representing the fixed and the cyclic load multipliers, respectively.

As it is well known [1,2], in this load condition the structural response is characterized at first by a transient response and, eventually, by a steady-state response exhibiting the same periodicity features as the cyclic loads and independent of the initial conditions and of the chosen load path. Actually, for each cycle, the steady-state response just depends on the sequence of the b amplified basic load conditions $\boldsymbol{F}_i = \xi_0 \boldsymbol{F}_0 + \xi_c \boldsymbol{F}_{ci}$, $\forall i \in I(b)$.

As a consequence, the elastic plastic steady-state response of the structure in the cycle can be obtained by an analysis effected just for the b basic conditions, i.e.:

$$\boldsymbol{Z}_i = \boldsymbol{S}\boldsymbol{Y}_i + \boldsymbol{b}_i \qquad \forall i \in I(b) \tag{1a}$$

$$\boldsymbol{Z}_i \geq \boldsymbol{0}, \quad \boldsymbol{Y}_i \geq \boldsymbol{0}, \quad \widetilde{\boldsymbol{Y}}_i \boldsymbol{Z}_i = 0 \qquad \forall i \in I(b) \tag{1b}$$

where $\boldsymbol{Z}_i$ is the opposite of the plastic potential vector, $\boldsymbol{S} = -\widetilde{\boldsymbol{N}}\left(\widetilde{\boldsymbol{B}}\boldsymbol{K}^{-1}\boldsymbol{B} - \boldsymbol{D}\right)\boldsymbol{N}$, with $\boldsymbol{N}$ block diagonal matrix of unit external normals to the yield surface, $\boldsymbol{B}$ pseudo-force matrix, $\boldsymbol{K}$ external stiffness matrix and $\boldsymbol{D}$ block diagonal stiffness matrix related to the strain points, $\boldsymbol{Y}_i$ is the vector of plastic activation intensities, $\boldsymbol{b}_i = \boldsymbol{R} - \widetilde{\boldsymbol{N}}\,\widetilde{\boldsymbol{B}}\,\boldsymbol{K}^{-1}\boldsymbol{F}_i - \widetilde{\boldsymbol{N}}\,\boldsymbol{P}_i^*$ is a known term vector, with $\boldsymbol{R}$ plastic resistance vector and $\boldsymbol{P}_i^*$ generalized stresses, evaluated at the strain points, due to the loads acting on the elements.

In order to determine, on the Bree diagram (Fig. 1), the borderline between the shakedown domains and the incremental/instantaneous collapse regions it is useful to consider the steady-state elastic plastic response just to the amplified perfect cyclic loads, and separately the elastic response to the amplified fixed loads, solving the following problem [3]:

$$\boldsymbol{K}\boldsymbol{u}_{ci} - \boldsymbol{F}_{ci} = \boldsymbol{0} \;, \quad \boldsymbol{P}_{ci} = \widetilde{\boldsymbol{B}}\boldsymbol{u}_{ci} + \boldsymbol{P}_{ci}^* \qquad \forall i \in I(b) \tag{2a}$$

$$\boldsymbol{Z}_{ci} \equiv -\boldsymbol{\varphi}_{ci} = \boldsymbol{R} - \xi_c^a \widetilde{\boldsymbol{N}}\boldsymbol{P}_{ci} + \boldsymbol{S}\boldsymbol{Y}_{ci} \qquad \forall i \in I(b) \tag{2b}$$

$$\boldsymbol{Z}_{ci} \geq \boldsymbol{0}, \quad \boldsymbol{Y}_{ci} \geq \boldsymbol{0}, \quad \widetilde{\boldsymbol{Y}}_{ci}\boldsymbol{Z}_{ci} = 0 \qquad \forall i \in I(b) \tag{2c}$$

$$\boldsymbol{K}\boldsymbol{u}_0 - \boldsymbol{F}_0 = \boldsymbol{0}, \quad \boldsymbol{P}_0 = \widetilde{\boldsymbol{B}}\boldsymbol{u}_0 + \boldsymbol{P}_0^* \tag{3a}$$

$$\xi_0^S\left(\xi_c^a\right) = \max_{(\xi_0, \boldsymbol{Y}_0)} \xi_0 \qquad \text{subject to} \tag{3b}$$

$$-\boldsymbol{\varphi}_i^S = \boldsymbol{Z}_{ci} - \xi_0 \widetilde{\boldsymbol{N}}\boldsymbol{P}_0 + \boldsymbol{S}\boldsymbol{Y}_0 \geq \boldsymbol{0}, \quad \boldsymbol{Y}_0 \geq \boldsymbol{0} \qquad \forall i \in I(b) \tag{3c}$$

for a suitably chosen set of assigned cyclic load multiplier values ξ_c^a, lower than the instantaneous collapse cyclic limit load multiplier ξ_c^I, being $\boldsymbol{Y}_0$ some plastic activation intensities related to appropriate fields of selfstresses. If $0 \leq \xi_c^a \leq \xi_c^S$ is assumed, being ξ_c^S the elastic shakedown cyclic limit load multiplier, then Eqs. (2) admit the vanishing solution $\boldsymbol{Y}_{ci} = \boldsymbol{0}$, $\forall i \in I(b)$, and in the steady-state phase the whole structural behaviour is eventually elastic. In this case the couple of values $\left[\xi_0^S\left(\xi_c^a\right), \xi_c^a\right]$, deduced solving problem (3), represents a point of the boundary of the elastic shakedown domain. Otherwise, if $\xi_c^S < \xi_c^a < \xi_c^I$ is assumed, then Eqs. (2) admit a non-vanishing solution, $\boldsymbol{Y}_{ci}$, at least for some $i \in I(b)$, and the structure eventually exhibits a steady-state elastic plastic behaviour, so that the couple of values $\left[\xi_0^S\left(\xi_c^a\right), \xi_c^a\right]$ represents a point of the boundary of the plastic shakedown domain.

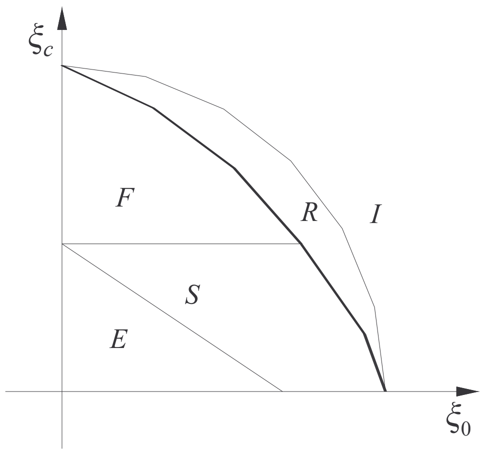

Figure 1: Typical borderline (thick line) between the elastic/plastic shakedown domains (zones *S*+*F*) and the incremental/instantaneous collapse regions (zones *R*+*I*).

In order to take into account appropriate limits related with the ductility features of the relevant structure, the above formulated problem (2-3) must be suitably specialized.

With this aim, basing, on the perturbation method of the bounding theory, let us introduce the linear perturbation mode vector $\hat{\boldsymbol{R}}$ and the related perturbation multiplier $\omega > 0$. It is worth noticing that suitably choosing the perturbation vector $\hat{\boldsymbol{R}}$ it is possible to impose bounds on different quantities related to the actual process, while the value of ω influence the stringentness of the relevant bounds.

According with an appropriate bounding theorem [4], again for a suitably chosen set of assigned values ξ_c^a, the following problem must be solved:

$$\boldsymbol{K}\boldsymbol{u}_{ci} - \boldsymbol{F}_{ci} = \boldsymbol{0} \quad , \quad \boldsymbol{P}_{ci} = \widetilde{\boldsymbol{B}}\boldsymbol{u}_{ci} + \boldsymbol{P}_{ci}^{*} \qquad \forall i \in I(b) \tag{4a}$$

$$\boldsymbol{Z}_{ci} \equiv -\boldsymbol{\varphi}_{ci} = \boldsymbol{R} - \xi_c^a \widetilde{\boldsymbol{N}}\boldsymbol{P}_{ci} + \boldsymbol{S}\boldsymbol{Y}_{ci} \qquad \forall i \in I(b) \tag{4b}$$

$$\boldsymbol{Z}_{ci} \geq \boldsymbol{0}, \ \boldsymbol{Y}_{ci} \geq \boldsymbol{0}, \ \widetilde{\boldsymbol{Y}}_{ci}\boldsymbol{Z}_{ci} = 0 \qquad \forall i \in I(b) \tag{4c}$$

$$\boldsymbol{K}\boldsymbol{u}_0 - \boldsymbol{F}_0 = \boldsymbol{0}, \quad \boldsymbol{P}_0 = \widetilde{\boldsymbol{B}}\boldsymbol{u}_0 + \boldsymbol{P}_0^{*} \tag{5a}$$

$$\hat{\xi}_0^S\left(\xi_c^a\right) = \max_{\left(\xi_0, \hat{\boldsymbol{Y}}_0\right)} \xi_0 \qquad \text{subject to} \tag{5b}$$

$$-\hat{\boldsymbol{\varphi}}_i^S = \boldsymbol{R} - \omega\hat{\boldsymbol{R}} - \widetilde{\boldsymbol{N}}\left(\xi_c^a \boldsymbol{P}_{ci} + \xi_0 \boldsymbol{P}_0\right) + \boldsymbol{S}\boldsymbol{Y}_{ci} + \boldsymbol{S}\hat{\boldsymbol{Y}}_0 \geq \boldsymbol{0}, \ \hat{\boldsymbol{Y}}_0 \geq \boldsymbol{0} \quad \forall i \in I(b) \tag{5c}$$

where $\hat{\boldsymbol{\varphi}}_i^S$ is the perturbed yield function and $\hat{\boldsymbol{Y}}_0$ the related plastic activation intensities.

It is worth noticing that Eqs. (4) represent the *actual* elastic plastic response of the structure to the cyclic loads, while Eqs. (5c) represent the *fictitious* elastic/plastic shakedown conditions, depending on the chosen value for ξ_c^a, related to the perturbed plastic potential.

3 CONCLUSIONS

- A special formulation of maximum problems devoted to the determination of the borderline between elastic/plastic shakedown and incremental/instantaneous collapse domains in the Bree diagram for elastic perfectly plastic structures subjected to a combination of fixed and perfect cyclic loads has been proposed.
- All the numerical application effected, related to plane steel frames and here not reported for the sake of brevity, confirmed the theoretical expectations.

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