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GEOMETRICAL METHOD FOR LOCALIZATION ANALYSIS IN GRADIENT–DEPENDENT J_2 PLASTICITY

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Summary. *The geometrical method for assessment of discontinuous bifurcation conditions is extended to encompass $J-2$ gradient-dependent plasticity. The gradient-dependent localization condition is cast in form of an elliptical envelope condition in the coordinates of Mohr, see Pijaudier-Cabot and Benallal (1993)[3], Liebe and Willam (2001)[2]. The results of the localization analysis geometrically and numerically demonstrate the capability of the thermodynamically consistent gradient-dependent model formulations to suppress localized failure modes of the related local plasticity formulations*

1 J_2 GRADIENT–DEPENDENT ELASTOPLASTICITY

Under consideration of small strain kinematics, the free energy density of the strain gradient elastoplastic J_2 continuum is expressed in the additive form

$$\rho\Psi(\boldsymbol{\varepsilon}^e, \kappa, \boldsymbol{\nabla}\kappa) = \rho\Psi^e(\boldsymbol{\varepsilon}^e) + \rho\Psi^{p,loc}(\kappa) + \rho\Psi^{p,g}(\boldsymbol{\nabla}\kappa) \tag{1}$$

where ρ is the material density. The elastic free energy density is $\rho\Psi^e(\boldsymbol{\varepsilon}^e) = \frac{1}{2}\boldsymbol{\varepsilon}^e : \boldsymbol{E}^e : \boldsymbol{\varepsilon}^e$, with $\boldsymbol{\varepsilon}^e$ and $\boldsymbol{E}^e$ the elastic strain tensor and the fourth order elastic operator, respectively. The local and gradient free energy density contributions due to inelastic strains $\Psi^{p,loc}$ and $\Psi^{p,g}$ are expressed in terms of the scalar hardening/softening variable κ.

$$\rho\Psi^{p,loc} = \frac{1}{2}H\kappa \quad , \quad \rho\Psi^{p,g} = \frac{1}{2}l^2\boldsymbol{\nabla}\kappa \cdot \boldsymbol{H}^g \cdot \boldsymbol{\nabla}\kappa$$

Two types of state parameters were considered, the *local* hardening/softening modulus H and the second order tensor of *non–local gradient* state parameters $\boldsymbol{H}^g$. The gradient effects are only restricted to hardening/softening behavior via the inclusion of $\boldsymbol{\nabla}\kappa$.

The function corresponding to the $vonMises$ yield criterium is

$$\Phi(\boldsymbol{\sigma}, K) = \sigma_e - \sigma_y - K \quad , \quad \sigma_e = \sqrt{\frac{3}{2}}|\boldsymbol{s}| \tag{2}$$

with σ_y the yield stress, K the dissipative stress and $\boldsymbol{s}$ the deviatoric stress tensor. The explicit expressions of the dissipative stress components K^{loc} and K^g, regarding eq. (2)

result

$$K^{loc} = -\rho \frac{\partial \Psi^{p,loc}}{\partial \kappa} = -H\kappa \quad , \quad K^g = \boldsymbol{\nabla} \cdot \left(\rho \frac{\partial \Psi^{p,g}}{\partial (\boldsymbol{\nabla} \kappa)} \right) = l^2 \boldsymbol{\nabla} \cdot (\boldsymbol{H}^g \cdot \boldsymbol{\nabla} \kappa) \tag{3}$$

within the continuum, while on the boundary $\partial\Omega$ is $K^{(g,b)} = -l^2 \boldsymbol{m} \cdot \boldsymbol{H}^g \cdot \boldsymbol{\nabla}\kappa$ with the (outward) normal $\boldsymbol{m}$ to $\partial\Omega$.

2 CONDITION FOR LOCALIZED FAILURE

In case of localized failure in the form of discontinuous bifurcation we resort to the gradient elastoplastic localization tensor defined as $\boldsymbol{Q}^{epg} = \boldsymbol{Q}^e - \boldsymbol{a}^* \otimes \boldsymbol{a}/(h + h^g)$, with

$$\boldsymbol{Q}^e = \boldsymbol{n}_l \cdot \boldsymbol{E}^e \cdot \boldsymbol{n}_l \quad \text{and} \quad \boldsymbol{a}^* = \frac{\partial \Phi^*}{\partial \boldsymbol{\sigma}} : \boldsymbol{E}^e \cdot \boldsymbol{n}_l, \qquad \boldsymbol{a} = \frac{\partial \Phi}{\partial \boldsymbol{\sigma}} : \boldsymbol{E}^e \cdot \boldsymbol{n}_l \tag{4}$$

The localized failure condition in case of gradient-dependent elastoplasticity $\det(\boldsymbol{Q}^{epg}) = 0$ leads to the analysis of the spectral properties of $\boldsymbol{Q}^{epg}$. Its smallest eigenvalue, with respect to the metric defines by $\boldsymbol{Q}^e$, has the expression

$$\lambda^{(1)} = 1 - \frac{\boldsymbol{a}(\boldsymbol{n}_l) \cdot [\boldsymbol{Q}^e(\boldsymbol{n}_l)]^{-1} \cdot \boldsymbol{a}^*(\boldsymbol{n}_l)}{h + h^g} = 0 \tag{5}$$

In case of gradient isotropy, the explicit form of eq.(5) turns

$$\mathcal{H} + \frac{\partial \Phi}{\partial \boldsymbol{\sigma}} : \boldsymbol{E}^e : \frac{\partial \Phi^*}{\partial \boldsymbol{\sigma}} - \boldsymbol{a} \cdot [\boldsymbol{Q}^e]^{-1} \cdot \boldsymbol{a}^* = 0 \quad \text{with} \quad \mathcal{H} = \bar{H}_c^g (\frac{2\pi l}{\delta})^2 + \bar{H}_c \tag{6}$$

The localization condition in eq.(6) serves as a basis for analytical and numerical evaluations of the localization directions $\boldsymbol{n}_l$ and of the corresponding maximum or critical hardening/softening parameters $\bar{H}_c(\boldsymbol{n}_l) = max[\bar{H}(\boldsymbol{n}_l)]$ in case of local plasticity, and $\bar{H}_c^g(\boldsymbol{n}_l) = max[\bar{H}^g(\boldsymbol{n}_l)]$ in gradient–dependent plasticity.

2.1 GEOMETRICAL LOCALIZATION IN J_2 GRADIENT PLASTICITY

The approach is based on the original proposal by Benallal (1992)[1] , which was further developed by Pijaudier-Cabot and Benallal (1993)[6] and Liebe and Willam (2001)[2] for classical plasticity.

Equation (6) defines an ellipse in the $\sigma - \tau$ coordinates of Mohr

$$\sigma = \boldsymbol{\sigma} \cdot \boldsymbol{n}_l \cdot \boldsymbol{\sigma}, \quad s = \boldsymbol{n}_l \cdot \boldsymbol{s} \cdot \boldsymbol{n}_l, \quad \tau = (\boldsymbol{n}_l \cdot \boldsymbol{s}) \cdot (\boldsymbol{n}_l \cdot \boldsymbol{s}) - (\boldsymbol{n}_l \cdot \boldsymbol{s} \cdot \boldsymbol{n}_l)^2 \tag{7}$$

The critical direction $\boldsymbol{n}_l$, normal to the plane where the Mohr components are evaluated, and the critical hardening/softening parameters $\bar{H}_c$ and $\bar{H}_c^g$ for localization are obtained when the Mohr circle of stresses contacts the elliptical localization envelope $\frac{(\sigma-\sigma_0)^2}{A^2} - \frac{\tau^2}{B^2} = 1$. Considering the expression $\boldsymbol{E}^e = 2G\boldsymbol{I}_4 + \Lambda \boldsymbol{I} \otimes \boldsymbol{I}$ for the elastic tensor, with the shear module G and the Lamé's constant Λ, the traction vectors can then be rewritten as $\boldsymbol{a}^* = \boldsymbol{a} = GJ_2^{-\frac{1}{2}}\boldsymbol{n}_l \cdot \boldsymbol{s}$. Thus, from eq.(4), follows

$$[\boldsymbol{Q}^e]^{-1} = \frac{1}{G}[\boldsymbol{I} - \frac{1}{2(1-\nu)}\boldsymbol{n}_l \otimes \boldsymbol{n}_l] \tag{8}$$

Replacing eq. (8) in eq.(6), and combining with eq.(7), the center σ_0 and the half axes A and B of the localization ellipse result

$$\sigma_0 = \frac{1}{3}I_1 \quad B^2 = J_2(\tfrac{\mathcal{H}}{G} + 1) \quad A^2 = 2\frac{1-\nu}{1-2\nu}B^2 \tag{9}$$

So, the thermodynamically consistent gradient–dependent plasticity formulation allows a simple extension of the geometrical localization method for local plasticity as demonstrated in this section. Thereby, the non-local effects in terms of the characteristic length and of the gradient hardening/softening modulus only affect the expression of the localization ellipse half axes A and B.

3 GOMETRICAL LOCALIZATION ASSESSMENTS

The localization properties of the thermodynamically consistent gradient–dependent J_2 elastoplastic model are analyzed for the plane strain condition when $\sigma_z = \nu(\sigma_x + \sigma_y)$ and $\bar{H} = \bar{H}_c$, being $\bar{H}$ the particular hardening/softening modulus of the gradient-dependent model and $\bar{H}_c$ the critical one for localization of the local elastoplastic model.

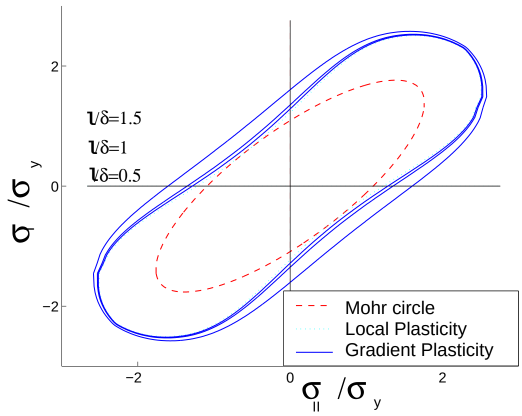

Figure 1: Localization in local and gradient *von Mises* yield criterion in the principal stress space.

Figure 1 illustrate the results of numerical localization analysis performed for the stress states belonging to the *von Mises* ellipse of limit stress states in the plane strain regime. The distance between the different curves and the $J-2$ limit stress ellipse in its normal direction, is proportional to the normalized localization indicator. Thereby, outward normal to the ellipse corresponds to positive values of this failure indicator, indicating that diffuse mode of failure takes place for the considered limit stress state. In this analysis both local and non-local gradient J_2 material models were considered. In case of the non-local model, the adopted internal material length equals the width of the localization zone, i.e. $l = \delta$. The results in Figure 1 indicate that the gradient-regularized plasticity suppresses the localized failure modes of the local model along the entire set of limit stress states of the $J - 2$ material, in the plane strain regime.

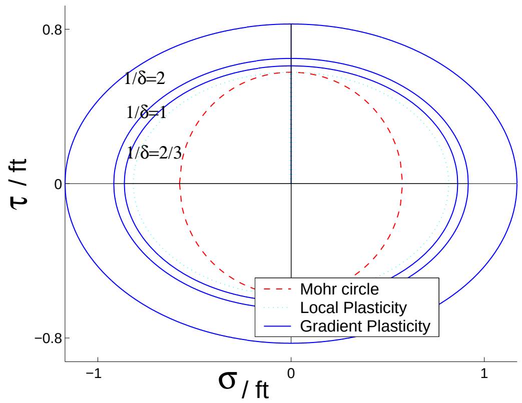

Figure 2: Geometrical (a) and numerical (b) localization analysis at peak of the simple shear test. Local and gradient-dependent J_2 plasticity.

The geometrical localization analysis of the gradient-based J_2 model is performed at peak stress of the simple shear and the results are shown in Figure 2(a). These results illustrate the influence of the characteristic length l in the mode of failure. When $l > 0$, no contact is observed between the localization ellipses of the gradient–dependent plasticity model and the Mohr circle corresponding to the considered limit stress state. Thus, diffuse failure mode takes place for all three limit stress states. However, as $l/\delta \to 0$ discontinuous bifurcation takes place.

4 CONCLUSIONS

The geometrical localization analysis in this work demonstrates that the non-local effects of the gradient plasticity formulation only affects the half axes of the localization ellipse. The results show that the J_2 gradient-based elastoplastic model has the capacity to suppress the discontinuous bifurcations of the related classical elastoplastic model, when the selected hardening/softening modulus $\bar{H}$ equals the critical one for localization of the local material formulation $\bar{H}_c$. Its regularization capability reduces as $l/\delta \to 0$. When l approaches zero, a continuous transition from diffuse to localized failure modes is obtained.

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