# NUMERICAL INVESTIGATION ON PLASTIC COLLAPSE OF THIN WALLED BEAMS SUBJECTED TO BIAXIAL BENDING

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**Summary.** The present work reports the main results of a numerical and analytical investigation of the collapse behaviour of thin walled beams subjected to biaxial loads. The objective is to get a fundamental understanding of the phenomenon with particular attention to the test loading conditions and to the identification of the most suitable parameters for the description of such a large rotation collapse.

## **1 INTRODUCTION**

The plastic bending collapse constitutes the more frequent mode of yielding in thin wall tubular members for vehicular and mechanical structures.

The structural members of the vehicle frame designed to dissipate the energy in case of accident are generally subjected to combined loads, in the complex dynamic evolution of the impact, due to the action of external forces or, indirectly, due to the internal loads which take place in the structure after the impact: it is therefore possible the case of a combined loads as a consequence, for example, of a frontal offset impact or of a rollover.

Because of the particularities of the collapse of thin walled structures, which involve wall buckling phenomena and deformation kinematic mechanisms, it is clear that the various types of load interact each other and the collapse cannot be treated by summation of two independent phenomenoa, but if they are contemporaneously present the collapse must be studied as a whole.

## **2** BIAXIAL BENDING

Let's consider a thin walled beam (fig. 1) with transverse rectangular cross section where a and b are the two sides (measured on the middle plane of the walls) and t is the wall thickness; the length of the beam is L. Suppose that the thin walled beam is properly loaded so that to cause the bending about a generical direction not coincident with one of the two principal axes of inertia of the cross section. w (weak) and s (strong) are the two central axes of inertia of the cross section, z the longitudinal axis and x and y is a second reference system, generically rotated af an angle  $\alpha$  respect to the system w-s, which serves to identify the orientation of the beam (see figure 1).

Due to the arbitrary of the load the resistant moment **M**, the neutral axis (N.A.) and consequently the bending plane B.P. orthogonal to the last one are not necessarily parallel to any of the directions defined by the reference systems *w-s* and *x-y*. In order to identify the problem it is necessary to introduce the deviation angle ( $\beta$ ) of the load, that is the angle between the moment vector and the neutral axis N.A., and the angle ( $\gamma$ ) which defines the position of the neutral axis N.A. in the system w-s.



Figure 1 – Rectangular cross section thin walled beam subjected to a biaxial bending solicitation.

#### **3** NUMERICAL ANALYSIS OF BIAXIAL BENDING COLLAPSE

The study of the biaxial bending collapse in the whole field of evolution of the phenomenon (pre-collapse, collapse and post-collapse) may be conveniently performed by means of FE numerical models, paying attention to the construction of models with a sufficient number of elements [9].



Figure 2: Numerical model.

The problem has been studied by performing a serie of analysis with an explicit FE code; three types of beam have been modelled, the dimensions of which are reported in fig. 2 and they have been already adopted in previous experimental works [1,5].

The beams have been modelled with 4 nodes shell elements, the portion next to the plastic hinge is characterized by a more refined mesh, elastic-plastic material characteristics and contact modelling, while the remaining portion is made of purely elastic material without contact algorithms (fig. 2). By rotation of the numerical model about the *z* axis of a known angle  $\alpha$  it is possible to vary the biaxiality of the load. By prescribing for the nodes of the free

edge  $u_x=0$  and  $u_y=f(t)$  a cantilever beam with prescribed bending plane is obtained. Having the displacement  $u_x$  free, a cantilever beam with prescribed direction of the applied moment is obtained. From each simulation the time behaviour of the resultant moments in the constraint section  $M_x$ ,  $M_y$  and  $M_z$ , the hinge rotation angle and the global energies during the phenomenon (particularly the energy dissipated for plastic deformation) have been extracted.

## **3 RESULTS**

In figures 3 there are reported, as example, the results obtained for the numerical model of the square beam  $44.4 \times 44.4 \times 1.6$  for the series of tests with  $\gamma$ =cost.



Figure 3: square section and ≠cost: deformed shapes (a); bending moment vs hinge rotation (b); collapse plane (c); deviation vs hinge rotation (d).

The resultant moment *M* presents the classical behaviour of the uniaxial collapse curves: the maximum value decreases with the value of the angle  $\gamma$  and  $\lambda$  (between 0° and 45°), while during the collapse phase at increasing  $\gamma$ ,  $\lambda$  values correspond higher moments. This means a higher energy absorption during the collapse phase of the biaxial case respect to the uniaxial.

An interesting global evaluation of the phenomenon is possible by considering the *collapse* plane: in figures 3.c there is reported on the planes  $M_w$ - $M_s$  the curves of interaction during the collapse phase, for constant values of the  $\theta$  angle and for  $M=M_{max}$ , relative to performed tests. The bidimensional representation of the collapse locus allows considerations about the

bending moment (resultant moment and components along w and s), but not about the deformations. It is necessary to observe the behaviour of the deviation angle  $\beta$  in function of the hinge rotation angle  $\theta$ (fig. 3.d).

## **5 CONCLUSIONS ON BIAXIAL BENDING COLLAPSE**

From the numerical analysis of the biaxial bending collapse phenomenon some foundamental points come out.

Differently from the uniaxial collapse the resistance bending moment vector and the neutral axis are not parallel each other: the angle ( $\beta$ ) given by the two directions represents the deviation of the biaxial load and generally it is not constant during the deformative process, neither for elementary modes of load. It is present, due to the load deviation, a moment component orthogonal to  $M_{NA}$ ,  $M_{BP}$  that even if it doesn't dissipate energy, produces the rotation of the resultant moment vector with respect to the neutral axis direction.

The collapse mechanism presents two characteristic kinematics: the presence of the first one or of the second one depends on the position of the bending plane; the transition from a mechanism to the other happens approximately around the condition of the passage of the bending plane through the corners of the section.

For the phenomenon characterization 4 variables are necessary, two of load and two of deformation. A possible characterization is that based on the use of the collapse locus. Beside it is necessary to obtain the behaviour of the deviation angle  $\beta$  in function of the angle  $\theta$  since the normality rule can not be applied to the advanced collapse phase.

#### REFERENCES

- [1] Kecman D., "Bending collapse of rectangular steel tubes in relation to the bus rollover problem", *Ph.D. Thesis*, Cranfield Institute of Technology, 1979.
- [2] Wierzbicki T., Recke L., Abramowicz W., Gholami T., Huang J., "Stress profiles in thin-walled prismatic columns subjected to crush loading-II Bending", *Computer & Structures*, 1994, pp. 625-641.
- [3] Todorovska L., Keeman D., "Multi-axial collapse of a rectangular sectioned tube and its modelling in finite element analysis", *Proc. of Int. Conf. on Vehicle Structural Mechanics*, Detroit, USA, (1988).
- [4] Reyes A., Langseth M., Hopperstad O.S., "Crashworthiness of aluminum extrusions subjected to oblique loading: experiments and numerical analyses", *Int. J. of Mech. Sciences* 44, pp. 1965–1984, (2002).
- [5] Brown J. C., G. H. Tidbury, "An investigation of the collapse of thin-walled rectangular beams in biaxial bending", *Int. J. Mech. Sciences*, 1983, pp. 733-746.
- [6] H.S. Kim, T. Wierzbicki, "Numerical and analytical study on deep biaxial bending collapse of thin-walled beams", *International Journal of Mechanical Sciences* 42, pp. 1947-1970,(2000).
- [7] T.H. Kim, S.R. Reid, "Multiaxial softening hinge model for tubular vehicle roll-over protective structures, *International Journal of Mechanical Sciences* 43, pp. 2147–2170, (2001).
- [8] L. Peroni, M. Martella, D. Gastaldin, On the convergence of simulation results in the numerical modelling of the plastic collapse of thin walled structures. COMPLAS 2003.
- [9] L. Peroni, Plastic collapse of thin walled structures: models and experimental investigations, *Ph.D. Thesis*, Politecnico di Torino, Dimec, 2003.