

## **SIMULATION OF RETAINING WALLS BY THE FINITE ELEMENT METHOD AND THE MOHR-COULOMB MODEL**

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**Summary.** *The retaining walls behavior is an interesting problem in the soils mechanics. Many of them are built without a clear knowledge of its mechanic behavior, because the most common theories about retaining walls only are accurate with some types of backfill. This paper presents a simple tool to simulate the retaining walls behavior by the Finite Element Method and numerical methods for non-linear structural problems. The Mohr-Coulomb model is used for soil mechanic behavior, and the interaction backfill soil-wall is simulated by orthotropic materials.*

### **1 INTRODUCTION**

The Finite Element Method (FEM) is one of the most powerful numerical tools that exist nowadays for simulation of mechanical behavior of solids and fluids. The main objective of this paper is to present a simple application of the FEM to the simulation of retaining walls.

It is evident soils do not have a linear-elastic mechanical behavior. There are some models that try to describe the soil's mechanical behavior<sup>1</sup>. The most commonly accepted is the Mohr-Coulomb model, and it is the model used for our approach.

On the other hand, it is well-known that exist a complex interaction between the backfill soil and the backface of the wall<sup>1 2 3</sup>. This behavior can be viewed as a sliding mechanism between the backfill and the backface of the wall (Figure 1). This mechanism is not easy of modeling, and requires re-meshing techniques and sliding surfaces. In this paper, an alternative technique is proposed; which is as simple as effective.

### **2 METHODOLOGY**

For solving the non-linear problem, the Newton and Newton-modified methods are used<sup>4</sup><sup>5</sup>. The problem solution is made by load increments<sup>4 6</sup>, distinguishing between permanent and variable loads. The wall is modeled by a linear-elastic material, only the soil is considered as a non-linear material.

The problem of sliding mechanism between backfill and the wall is solved by using a hypothetical strip of orthotropic elastic-plastic material along the backface of the wall. The

strip is soft in one direction, and stiff in the other. Then, the strip permits large displacements along its weak direction and short displacements in its stiff direction (figure 2). Also, the plastic strip does not resist tension stresses (because the strip is not real, but is a part of the backfill). This gives a good approximation of the real behavior of the interaction soil-wall.

## 2.1 Mohr-Coulomb model description

The well-known Mohr-Coulomb model is a perfect elastic-plastic model, commonly used for geotechnical calculations. The model is represented by the equation

$$t = c + s \tan f \quad (1)$$

The maximum shearing strength is in function of the minimum normal stress, the cohesion (c) and the soil friction angle.

Given a point of the soil mass, whose minimum normal stress is  $\sigma_3$ , the maximum normal strength  $\sigma_1$  is calculated by the following equations <sup>6</sup> (figure 3):

$$\begin{aligned} s_1 &= s_3 + 2R & g &= \tan^{-1}\left(\frac{c}{s_3}\right) \\ L &= \sqrt{c^2 + s_3^2} \frac{\sin(f+g)}{\sin\left(45^\circ - \frac{f}{2}\right)} & R &= L \frac{\sin\left(45^\circ + \frac{f}{2}\right)}{\sin(90^\circ - f)} \end{aligned} \quad (2)$$

If the maximum real normal stress is larger than  $\sigma_1$ , the normal stress is corrected to  $\sigma_1$ . Therefore, the system will not be equilibrated, and it is necessary to make several iterations, until the system reaches the equilibrium <sup>6</sup>.

Also, it is accepted that soils do not resist tension stresses. On the process, any tension stress that is into the soil mass is eliminated.

Note: The normal stresses  $\sigma_1$  and  $\sigma_3$  should be in the space of principal stresses <sup>6</sup>.

## 3 EXAMPLE

The example is a concrete gravity wall, whose backfill material is a sand with  $c=0,5 \text{ ton/m}^2$  y  $\phi=25^\circ$ . The elastic modulus is  $2500 \text{ ton/m}^2$ , and the Poisson's ratio is 0.35. The properties of the plastic strip are  $E_x=20 \text{ ton/m}^2$ ,  $E_y= 3\text{ton/m}^2$ , and all Poisson's ratio are 0.2. Only was considered self weight load, using five load increments. The geometry and materials appears in the figure 4.

The figure 5 shows the principal stresses vectors. Note that there are not tension stresses (blue denotes compression, red denotes tension). Figure 6 shows the displacement vectors. There is a zone like a wedge, where the backfill slid non-vertically.

The figure 7 shows the contour fill of maximum real stress/strength ratio. There is a line where the soil is working to its maximum strength. Finally, the figure 8 shows a deformed configuration of the system, with a scale factor. Also appears the original configuration, to do a comparison.

These results are qualitatively good, because for sands with small cohesion, the backfill soil performed a zone like a wedge, where the active pressure is developed, called Coulomb's failure wedge. This kind of retaining wall is well studied, and there are many results about it <sup>1 2 3</sup>.

#### 4 CONCLUSIONS

- The results for cohesionless soils are good. The method also can be used for cohesive backfills, because the model eliminates tension stresses into the soil mass.
- The model is very simple and inexpensive for calculations, comparing with the quality of results.

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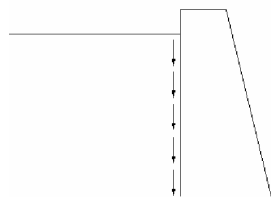


Figure 1

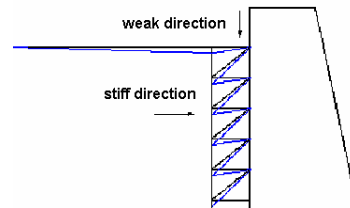


Figure 2

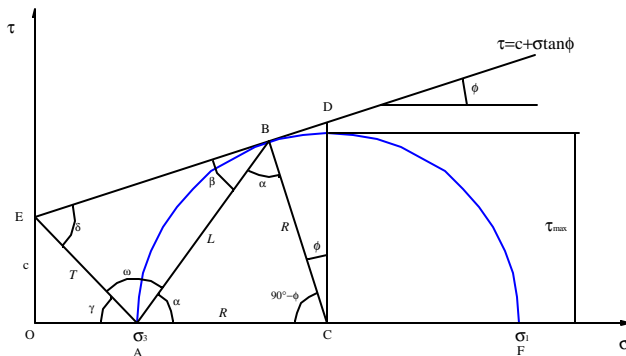


Figure 3

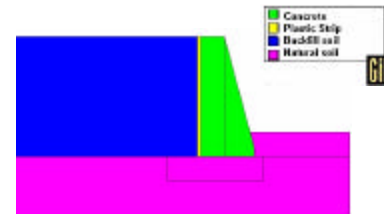


Figure 4

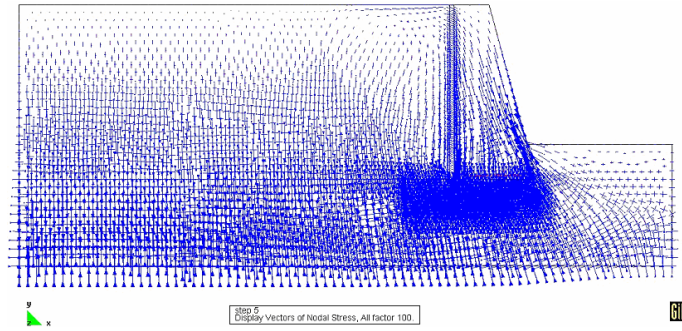


Figure 5

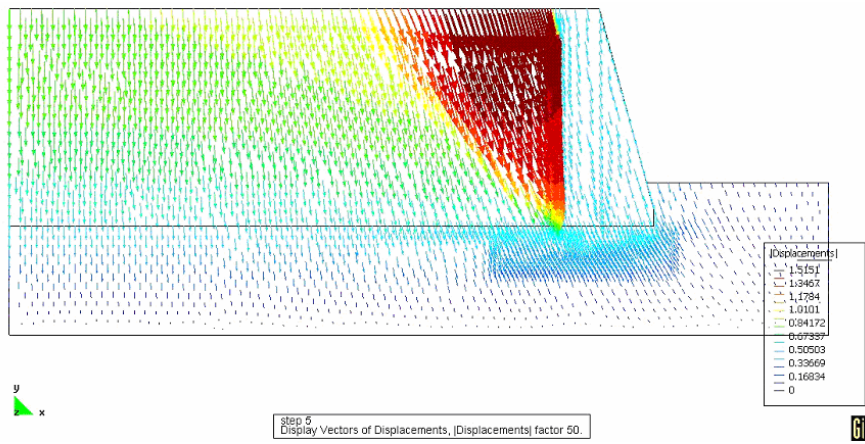


Figure 6

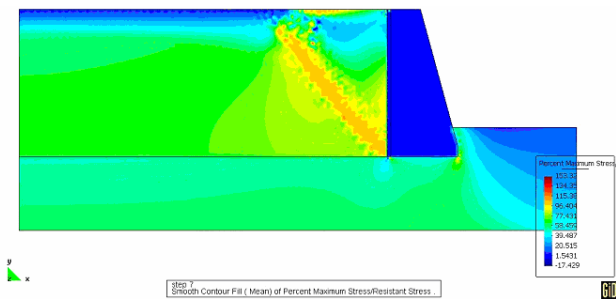


Figure 7

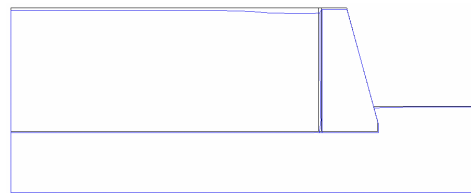


Figure 8