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MULTISCALE APPROACH FOR LAYERED COMPOSITE SMA PLATES

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Summary. *Aim of the present work is to analyse of smart laminates, obtained as staking sequence of fiber-reinforced composite laminae and composite SMA layers developing a full multiscale approach. In particular, a nonlinear MITC4 laminate finite element, based on the first-order shear deformation theory, is developed. The SMA layer constitutive relationship is determined solving at each nonlinear iteration of each time step for each integration Gauss point a nonlinear homogenization problem.*

1 INTRODUCTION

Layered composite SMA plates are obtained as staking sequence of thin laminae, some of which contain SMA wires. Several are the potential applications of composite SMA laminates, including the control of external shape, stiffness, damage, vibration, buckling and damping properties of the structural elements.

Different studies of the behaviour of structural elements integrated with SMA composite layers have been developed [1]. Micromechanical studies devoted to derive the overall behavior of SMA composites have been developed in Reference [2,3,4].

In the present work, a full multiscale approach is proposed in the framework of the finite element method for the problem of a composite laminate containing layers with SMA wires [4]. A laminate finite element is developed within the Mindlin-Reissner plate theory. The constitutive equations of the laminate are derived at each Gauss point of the mid-plane by integrating in the plate thickness the stress-strain relations of each layer. In particular, taking into account that in a typical point of a layer containing SMA wires the nonlinear mechanical response depends on the specific strain history taken place in that point, the constitutive equations are obtained performing a suitable nonlinear homogenization analysis. To this end, a full 3D SMA model is proposed to reproduce the hysteretic and temperature dependent SMA behavior. A numerical procedures based on the backward Euler time integration algorithm is developed. Numerical examples are performed to assess the proposed model and the developed numerical procedure. Investigations on the influence of the SMA composite layers on the laminate behavior are also reported.

2 MULTISCALE LAMINATE ANALYSIS

A composite laminated plate is defined by a domain V identified by a mid-plane Ω and the thickness h. The laminate is made up of n layers identified by the through-the-thickness coordinates z_k and z_{k+1}, with z_1=-h/2 and z_{n+1}=h/2. Two different types of layers are considered: linear elastic fiber-reinforced laminae and composite shape memory alloy laminae, characterized by a special thermo-mechanical response. In order to describe the structural behaviour of the laminate, the First-order Shear Deformation Theory (FSDT) is considered. It is based on the following assumptions:

- the normal stress in the out-of-plane direction is negligible, i.e. it is assumed σ_z=0;
- the following displacement representation form is considered:

$$
\begin{aligned}
s_1\left(x_1, x_2, z\right) &= u_1\left(x_1, x_2\right) + z\,\varphi_1\left(x_1, x_2\right) \\
s_2\left(x_1, x_2, z\right) &= u_2\left(x_1, x_2\right) + z\,\varphi_2\left(x_1, x_2\right) \\
s_3\left(x_1, x_2, z\right) &= w\left(x_1, x_2\right)
\end{aligned}
\tag{1}
$$

where u_1 and u_2 are the in-plane displacement, w is the transverse deflection and φ_1 and φ_2 are the rotations of the typical fiber orthogonal to the mid-plane.
The in-plane, the curvature and the out-of-plane shear strains in the laminate are computed as:

$$
\mathbf{e} = \begin{Bmatrix} e_1 \\ e_2 \\ e_{12} \end{Bmatrix} = \begin{Bmatrix} u_{1,1} \\ u_{2,2} \\ u_{1,2} + u_{2,1} \end{Bmatrix} \qquad
\boldsymbol{\chi} = \begin{Bmatrix} \chi_1 \\ \chi_2 \\ \chi_{12} \end{Bmatrix} = \begin{Bmatrix} \varphi_{1,1} \\ \varphi_{2,2} \\ \varphi_{1,2} + \varphi_{2,1} \end{Bmatrix} \qquad
\boldsymbol{\gamma} = \begin{Bmatrix} \gamma_{1z} \\ \gamma_{2z} \end{Bmatrix} = \begin{Bmatrix} w_{,1} + \varphi_1 \\ w_{,2} + \varphi_2 \end{Bmatrix}
\tag{2}
$$

while the in-plane and out-of-plane stresses are denoted as $\boldsymbol{\sigma} = \left\{\sigma_1 \quad \sigma_2 \quad \sigma_{12}\right\}^T$ and $\boldsymbol{\tau} = \left\{\tau_{1z} \quad \tau_{2z}\right\}^T$, respectively. The resultants are:

$$
\begin{aligned}
\mathbf{N} &= \int_{-h/2}^{h/2} \boldsymbol{\sigma}\, dz = \mathbf{A}\,\mathbf{e} + \mathbf{B}\,\boldsymbol{\chi} - \sum_{i=1}^{n\_SMA} \int_{z_i}^{z_{i+1}} \overline{\mathbf{E}}^p \overline{\mathbf{q}}\, dz \\
\mathbf{M} &= \int_{-h/2}^{h/2} z\,\boldsymbol{\sigma}\, dz = \mathbf{B}\,\mathbf{e} + \mathbf{D}\,\boldsymbol{\chi} - \sum_{i=1}^{n\_SMA} \int_{z_i}^{z_{i+1}} z\,\overline{\mathbf{E}}^p \overline{\mathbf{q}}\, dz \\
\mathbf{Q} &= \int_{-h/2}^{h/2} \boldsymbol{\tau}\, dz = \mathbf{G}\,\boldsymbol{\gamma} - \sum_{i=1}^{n\_SMA} \int_{z_i}^{z_{i+1}} \overline{\mathbf{E}}^S \overline{\mathbf{g}}\, dz
\end{aligned}
\tag{3}
$$

where:

- n_SMA is the number of composite SMA layers,
- **A**, **B**, **D** and **G** are the membrane, coupling, bending and shear resultant constitutive matrices of the laminate,
- $\overline{\mathbf{E}}^p$ and $\overline{\mathbf{E}}^S$ are the reduced in-plane and out-of-plane parts of the homogenized constitutive matrix $\overline{\mathbf{E}}$ of the composite SMA layer,

- $\overline{\mathbf{q}}$ and $\overline{\mathbf{g}}$ are the in-plane and out-of-plane parts of the inelastic strain $\overline{\mathbf{p}}$ due to the austenite-martensite phase transformations in the composite SMA.

The evaluation of the matrix $\overline{\mathbf{E}}$, and hence of $\overline{\mathbf{E}}^p$ and $\overline{\mathbf{E}}^S$, and of the inelastic strain $\overline{\mathbf{p}}$, and hence of $\overline{\mathbf{q}}$ and $\overline{\mathbf{g}}$, is performed developing a micro-macro approach, within a self consistent procedure.

The development of inelastic strain due to the phase transformation in the SMA fiber is modelled adopting the thermo-mechanically consistent approach proposed by the Souza et al. [5] and modified by Auricchio and Petrini [6] and Marfia [4].

A multiscale nonlinear version of the MITC4 (4-node mixed interpolation of tensorial components) plate element [7,8], for the analysis of composite SMA laminate element, is developed. The MITC element is locking-free, does not contain any spurious zero-energy modes and has a good predictive capability for displacements, bending moments and membrane forces. The main idea in the development of the MITC4 element is the use of a suitable interpolation of the out-of-plane shear strain.

The structural analysis is performed solving a micro-macro problem to evaluate the quantities $\overline{\mathbf{E}}^p$, $\overline{\mathbf{E}}^S$, $\overline{\mathbf{q}}$ and $\overline{\mathbf{g}}$, at each Gauss point of the laminated plate element.

3 NUMERICAL APPLICATIONS

An interesting application of a layered composite SMA plate is presented. In particular, a square cantilever plate, clamped along one edge and free on the other three edges is considered. The plate is characterized by an elastic isotropic layer and two layers made of SMA composite, one on the top and one on the bottom. The SMA composite material properties are reported in [4]. The geometrical properties of the beam are:

$$L = 300\text{mm} \qquad h_{CSMA} = 5\text{mm} \qquad h_{EL} = 10\text{mm} \tag{4}$$

with h_{CSMA} and h_{EL} are the thickness of the CSMA layers and of the elastic layer, respectively, and the material properties of the elastic core are:

$$E_c = 5000MPa \qquad \nu_c = 0.2 \tag{5}$$

The aim is to govern the transversal displacement of the cantilever plate performing temperature cycles in the SMA composite layers. To obtain phase transformation in the SMA wires by temperature changes, the plate is initially subjected to a bending moment. The following loading and temperature history is applied:

t [s]	0	1	2	3
M_1 [Nmm/mm]	0	1333	1333	1333
T [K]	223	223	600	223

The self-consistent homogenization technique is adopted in order to evaluate the overall constitutive law of the composite. The bending moment is applied at constant temperature and it is kept constant during the temperature changes in the SMA composite layers.

In Figure 1 the transversal displacement of the middle point, w_A, and of the corner, w_B, of the free edge is plotted versus time. The maximum transversal displacement is reached at time t=1s, when the maximum value of the bending moment is achieved at temperature T=223K and the austenite-martensite phase transformation has occurred in the SMA fibers. From t = 1 s to t = 2 s the temperature is increased and the martensite-austenite phase transformation occurs in the SMA fibers; as a consequence, the transversal displacement is reduced. From time t = 2 s to t = 3 s the temperature is decreased at the initial value and the plate goes back to the deformed shape reached at t = 1 s.

In Figure 2 the bending moment is plotted versus the transversal displacement of the free edge of the plate. It can be pointed out that the first branch is nonlinear because of the nonlinear behavior of the SMA wires due to the phase transformations. Then, changing the temperature the beam swings between two configuration: the beam is able to recover part of the transversal displacement during heating and it returns to the deformed shape during cooling.

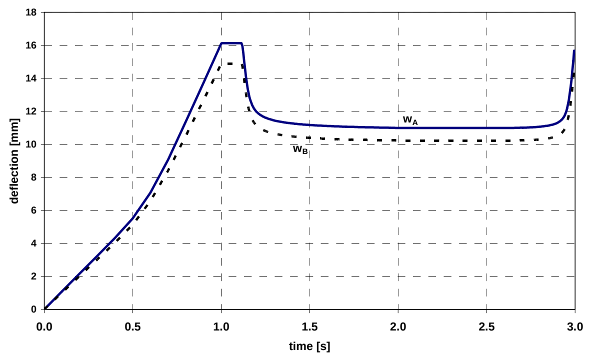

Figure 1: Transversal displacement of the free edge of the cantilever plate versus time.

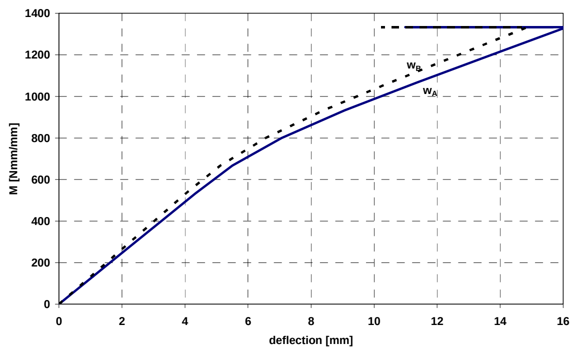

Figure 2: Bending moment versus transversal displacement of the free edge of the cantilever plate

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