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SIMULATION OF COMPLEX CRASHWORTHINESS PROBLEMS BY USING A COMBINE IMPLICIT/EXPLICIT TIME INTEGRATION

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Summary. *In this work we will combine the α-generalized explicit time integration scheme with the Energy-Dissipating Momentum-Conserving (EDMC) implicit time-integration scheme.*

1 INTRODUCTION

The optimal solution to simulate crashworthiness problems is to be able to combine an implicit and an explicit time integration. Automatic criteria are used to switch automatically from a method to another one[1]. A problem is to developed a method to shift, in a stable way, from an explicit method to an implicit one. We will proceed as we have proposed[2] by balanced the last explicit steps but in this work we will combine the α-generalized explicit scheme[3] with the Energy-Dissipating Momentum-Conserving (EDMC) implicit scheme[4].

2 TIME INTEGRATION

Chung and Hulbert[3] have proposed an explicit scheme exhibiting numerical dissipation. It yields:

$$\left[\ddot{x}^{n+1}\right]^{\xi} = \frac{1}{1-\alpha_M}\left[M^{-1}\right]^{\xi\mu}\left[\vec{F}^{n}_{ext}-\vec{F}^{n}_{int}\right]^{\mu} - \frac{\alpha_M}{1-\alpha_M}\left[\ddot{x}^{n}\right]^{\xi} \tag{1}$$

$$\left[\dot{\vec{x}}^{n+1}\right]^{\xi} = \left[\dot{\vec{x}}^{n}\right]^{\xi} + \Delta t\left[1-\gamma\right]\left[\ddot{\vec{x}}^{n}\right]^{\xi} + \Delta t\gamma\left[\ddot{\vec{x}}^{n+1}\right]^{\xi} \tag{2}$$

$$\left[\vec{x}^{n+1}\right]^{\xi} = \left[\vec{x}^{n}\right]^{\xi} + \Delta t\left[\dot{\vec{x}}^{n}\right]^{\xi} + \Delta t^2\left[\frac{1}{2}-\beta\right]\left[\ddot{\vec{x}}^{n}\right]^{\xi} + \Delta t^2\beta\left[\ddot{\vec{x}}^{n+1}\right]^{\xi} \tag{3}$$

with M the mass matrix, $\vec{F}_{ext}$ the external forces and $\vec{F}_{int}$ the external forces. This scheme is characterized by a spectral radius at bifurcation pulsation $\rho_b < 1$.

Armero and Romero[4] have introduced velocities dissipation $\vec{G}_{diss}$ and forces dissipation $\vec{F}_{diss}$ in Simo and Tarnow EMCA implicit scheme. Equations at node ξ becomes:

$$\left[\vec{x}^{n+1}\right]^{\xi} = \left[\vec{x}^{n}\right]^{\xi} + \frac{\Delta t}{2}\left[\dot{\vec{x}}^{n+1}\right]^{\xi} + \frac{\Delta t}{2}\left[\dot{\vec{x}}^{n}\right]^{\xi} + \Delta t\left[\vec{G}^{n+\frac{1}{2}}_{diss}\right]^{\xi} \tag{4}$$

$$\left[\dot{\vec{x}}^{n+1}\right]^{\xi} = \left[\dot{\vec{x}}^{n}\right]^{\xi} + \frac{\Delta t}{2}\left[\ddot{\vec{x}}^{n+1}\right]^{\xi} + \frac{\Delta t}{2}\left[\ddot{\vec{x}}^{n}\right]^{\xi} \tag{5}$$

$$\frac{1}{2}M^{\xi\mu}\left[\ddot{\vec{x}}^{n+1}+\ddot{\vec{x}}^{n}\right]^{\mu} = \left[\vec{F}^{n+\frac{1}{2}}_{ext}-\vec{F}^{n+\frac{1}{2}}_{int}-\vec{F}^{n+\frac{1}{2}}_{diss}\right]^{\xi} \tag{6}$$

Expression of the internal forces and of the dissipation terms for hyper-elastic models can be found in[4] and for elasto-plastic hypo-elastic model in[5]. This scheme is characterized by a spectral radius at infinite pulsation $\rho_\infty < 1$.

3 COMBINED IMPLICIT/EXPLICIT METHOD

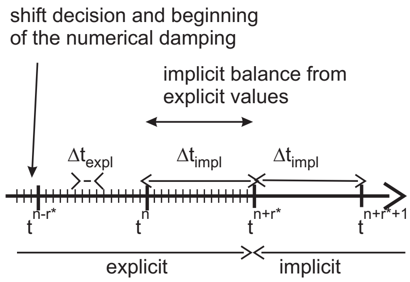

Figure 1: Transition scheme from an explicit scheme to an implicit one.

Details about the automatic criteria to be able to shift from a method to another one can be found in[1]. Let us define r^* the ratio between the implicit time step and the explicit one. Figure 1 illustrates the transition between the explicit algorithm and the implicit one. Let us assume that at time t^{n-r^*} we will shift from an explicit algorithm to an implicit one. Using numerical dissipation property of the generalized-α explicit scheme, numerical oscillations resulting from the explicit scheme are first annihilated between time t^{n-r^*} and time t^n with r^* explicit steps occur with a spectral radius ρ_b set equal to zero. The second step in the algorithm is to determine a balanced configuration at time t^{n+r^*}. Therefore, we act in two stages. First an explicit solution using r^* explicit steps are computed. This solution results in the displacements $\vec{x}_{expl}^{n+r^*}$. Then proceeding as we have proposed[2] we will use a predictor corrector algorithms, to reach a balance implicit step between time t^n and time t^{n+r^*}. Initial predictions are:

$$\begin{aligned}
\left[\vec{x}_{impl}^{n+r^*,0}\right]^\xi &= \left[\vec{x}^n + \Delta t_{impl}\dot{\vec{x}}^n + \frac{\Delta t_{impl}^2}{4}\ddot{\vec{x}}^n + \Delta t_{impl}\vec{G}_{diss}\left(\dot{\vec{x}}_{impl}^{n+r^*,0}\right)\right]^\xi \\
\left[\dot{\vec{x}}_{impl}^{n+r^*,0}\right]^\xi &= \left[\dot{\vec{x}}^n + \frac{\Delta t_{impl}}{2}\ddot{\vec{x}}^n\right]^\xi \\
\left[\ddot{\vec{x}}_{impl}^{n+r^*,0}\right]^\xi &= 0
\end{aligned} \tag{7}$$

with $\vec{G}_{diss}\left(\dot{\vec{x}}_{impl}^{n+r^*,i}\right)$ the dissipation velocities. Since relations (4) and (5) have to be always satisfied. Therefore, one has:

$$\begin{aligned}
\left[\vec{x}_{impl}^{n+r^*}\right]^\xi &= \left[\vec{x}^n\right]^\xi + \frac{\Delta t_{impl}}{2}\left[\dot{\vec{x}}_{impl}^{n+r^*} + \dot{\vec{x}}^n\right]^\xi + \Delta t_{impl}\left[\vec{G}_{diss}\left(\dot{\vec{x}}_{impl}^{n+r^*}\right)\right]^\xi \\
\left[\dot{\vec{x}}_{impl}^{n+r^*}\right]^\xi &= \left[\dot{\vec{x}}^n\right]^\xi + \frac{\Delta t_{impl}}{2}\left[\ddot{\vec{x}}_{impl}^{n+r^*}\right]^\xi + \frac{\Delta t_{impl}}{2}\left[\ddot{\vec{x}}^n\right]^\xi
\end{aligned} \tag{8}$$

At this point we will use the explicit displacements $\vec{x}_{expl}^{n+r^*}$ to be closer from the balance solution. First iteration of the correction algorithm is then obtained by assuming:

$$\begin{aligned}
\left[\Delta\vec{x}^{1}\right]^{\xi} &= \left[\vec{x}_{expl}^{n+r^*} + \Delta t_{impl}\vec{G}_{diss}\left(\dot{\vec{x}}_{impl}^{n+r^*,1}\right) - \vec{x}_{impl}^{n+r^*,0}\right]^{\xi} \\
\left[\vec{x}_{impl}^{n+r^*,1}\right]^{\xi} &= \left[\vec{x}_{expl}^{n+r^*} + \Delta t_{impl}\vec{G}_{diss}\left(\dot{\vec{x}}_{impl}^{n+r^*,1}\right)\right]^{\xi} \\
\left[\dot{\vec{x}}_{impl}^{n+r^*,1}\right]^{\xi} &= \left[\dot{\vec{x}}_{impl}^{n+r^*,0}\right]^{\xi} + \frac{2}{\Delta t_{impl}}\left[\vec{x}_{expl}^{n+r^*} - \vec{x}_{impl}^{n+r^*,0} + \Delta t_{impl}\vec{G}_{diss}\left(\dot{\vec{x}}_{impl}^{n+r^*,0}\right)\right]^{\xi} \\
\left[\ddot{\vec{x}}_{impl}^{n+r^*,1}\right]^{\xi} &= \left[\ddot{\vec{x}}_{impl}^{n+r^*,0}\right]^{\xi} + \frac{4}{\Delta t_{impl}^{2}}\left[\vec{x}_{expl}^{n+r^*} - \vec{x}_{impl}^{n+r^*,0} + \Delta t_{impl}\vec{G}_{diss}\left(\dot{\vec{x}}_{impl}^{n+r^*,0}\right)\right]^{\xi}
\end{aligned} \tag{9}$$

that satisfy (8). The next iterations are solved using a classical Newton-Raphson scheme with the balance equation at node ξ between time t^n and t^{n+r^*}.

4 BLADE OFF SIMULATION

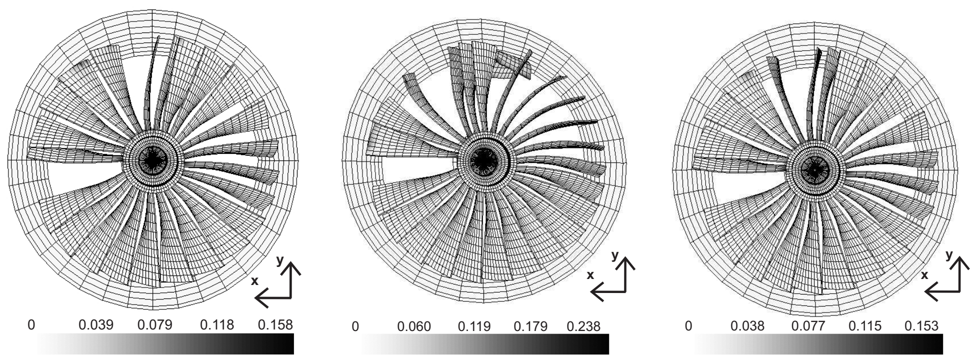

Figure 2: Configuration and equivalent plastic strain after one revoltion (a) Implicit method (b) Explicit method (c) Combined method.

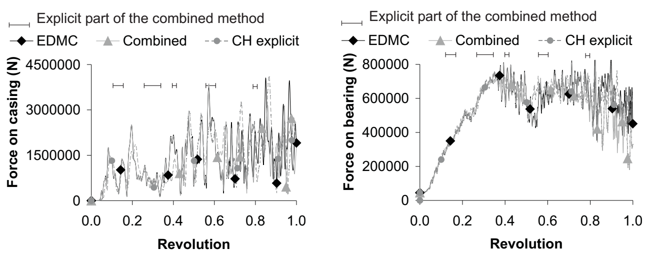

Figure 3: Evolution of the clamped forces (a) on the casing, (b) on the bearing.

Let us study a blade off simulation. Full description of this model can be found in[5]. At time $t = 0s$, the initial configuration of the rotor is equilibrated for a rotation velocity of $4775rpm$ and a blade is released from the disk. Let us compare the solution obtained by (1) the EDMC implicit scheme; (2) The α-generalized explicit algorithm; (3) The combined method proposed. The implicit scheme uses $\rho_\infty = 0.8$ and the α-generalized explicit scheme uses $\rho_b = 0.4$. Figures 2 illustrate the configuration obtained after one revolution of simulation. It appears that the threes methods give a similar configuration, but the explicit method overestimates the plastic deformations. Figure 3a and b respectively illustrates the evolution of clamped forces on the casing and on the bearing for the three computations. The solutions obtained by the three methods are identical. On theses figures we have reported the explicit interval of the combined method. Finally if we study the CPU cost needed for each simulation it appears that the combined method (10.9 days) is 50% less expensive that the explicit one (22.9 days) and 30% less expensive that the implicit one (15.6 days).

5 CONCLUSIONS

When shifting from an explicit method to an implicit one, we have proposed a predictor-corrector algorithm that gives stable initial condition for the implicit simulation. These developments leads to an accurate scheme in the non-linear range able to reduce the CPU cost.

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