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A NONLINEAR KINEMATIC HARDENING MODEL FOR THE SIMULATION OF CYCLIC LOADING PATHS IN ANISOTROPIC ALUMINUM ALLOY SHEETS

Rui P.R. Cardoso*, J.W. Yoon†,*, R.A.F. Valente*, J.J. Grácio*, F. Simões* and R.J. Alves de Sousa*

*Center for Mechanical Technology and Automation
University of Aveiro, Campus de Santiago
3810-193, Aveiro, Portugal
e-mail: rcardoso@mec.ua.pt, web page: http://gameua.no.sapo.pt

†Materials Science Division, Alcoa Technical Center
100 Technical Dr., PA 15069-0001, USA

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Summary. *Sheet metal forming processes generally involve complex loadings and material nonlinearities. Combinations of drawing, re-drawing and/or reverse drawing operations commonly induce cyclic loads with different strain paths, leading to Bauschinger effects that can not be predicted by conventional isotropic hardening laws. In order to properly represent such an effect, it is required to accommodate an appropriate kinematic hardening model along with a planar anisotropic yield function. Yld2000 (Barlat et al. [1]), for instance, can accurately capture both yield stress and r-value directionalities. In this work, the Barlat yield function Yld2000 is implemented with a nonlinear kinematic hardening model, based on the definition of two yield surfaces by Dafalias and Popov. The incremental deformation theory is used to properly handle the stress integration for non-quadratic yield functions in elasto-plasticity.*

1 INTRODUCTION

Along last years several efforts were made to model anisotropic aluminum alloys. One of the most appropriate anisotropic yield functions to model aluminum alloys is the function denoted as Yld96 (Barlat *et al.* [2]). The yield function Yld96 uses seven experimental parameters for plane stress conditions. They are computed from σ_0, σ_{45}, σ_{90}, r_0, r_{45} and r_{90} (uniaxial yield stresses and r values measured at 0, 45 and 90^0 from the rolling direction), and σ_b, the balanced biaxial yield stress measured with the bulge test.

Although great accuracy can be achieved with Yld96 yield function, some problems still exist, essentially because there is no proof of convexity and also because the yield function derivatives are difficult to obtain analytically.

Recently, Barlat and co-workers [1] proposed a new yield function for plane stress analysis which overcomes the above mentioned difficulties. The new yield function is called Yld2000 and is obtained from linear transformation of two convex functions, such that convexity can be achieved. The Yld2000 uses eight experimental parameters to account for plastic anisotropy. One more extra parameter (r_b: slope of the biaxial point) in addition to Yld96 is obtained from the disk compression test (Barlat *et al.* [1]).

When sheet parts are removed from tools after forming, material elements experience elastic unloading with springback. During this reverse loading, material elements usually demonstrate the Bauschinger effect, which can be modeled by the translation of the yield stress surface. The isotropic hardening assumption therefore does not properly predict the Bauschinger effect and the springback. Another way to simplify the evolution of the yield stress surface, without changing its shape and size during plastic deformation, is by assuming the initial yield stress surface to translate in the stress field. This corresponds to the kinematic hardening model proposed by Prager [3]. In order to describe the expansion and translation of the yield stress surface during plastic deformation, the combination of the isotropic and kinematic hardening is also commonly used.

Besides the Bauschinger effect, the transient behavior is also observed during reverse loading. In order to account for this, several models were proposed in the works of Dafalias and Popov [4], which are based on two yield surfaces, and also in the work of Chaboche [5] based on one yield surface.

The present work is an extension of previous developments of Yoon et al. [6] and Cardoso [7] for kinematic hardening on Yld96 [2] yield function. The main idea is to combine the two yield surface model of Dafalias and Popov [4] with the non-quadratic anisotropic Yld2000 criterion of Barlat and co-workers [1].

2 YIELD FUNCTION Yld2000

Yld2000 potential is a linear transformation of two convex functions ϕ' and ϕ'' that allows to increase the number of anisotropic coefficients in the formulation in order to better predict plastic anisotropy. The plastic potential is defined as [1]:

$$\phi = \phi' + \phi'' = \underbrace{|X_1' - X_2'|^a}_{\phi'} + \underbrace{|2X_2'' + X_1''|^a + |2X_1'' + X_2''|^a}_{\phi''} = 2\bar{\sigma}^a \tag{1}$$

with X_1', X_2', X_1'' and X_2'' the principal values of the following tensors:

$$\mathbf{X}' = \mathbf{L}' \cdot \boldsymbol{\sigma} = \quad ; \qquad \mathbf{X}'' = \mathbf{L}'' \cdot \boldsymbol{\sigma}. \tag{2}$$

In equation (2), the linear transformation tensors $\mathbf{L}'$ and $\mathbf{L}''$ are:

$$\begin{bmatrix} L'_{11} \\ L'_{12} \\ L'_{21} \\ L'_{22} \\ L'_{66} \end{bmatrix} = \begin{bmatrix} \frac{2}{3} & 0 & 0 \\ -\frac{1}{3} & 0 & 0 \\ 0 & -\frac{1}{3} & 0 \\ 0 & \frac{2}{3} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_7 \end{bmatrix} \qquad \begin{bmatrix} L''_{11} \\ L''_{12} \\ L''_{21} \\ L''_{22} \\ L''_{66} \end{bmatrix} = \frac{1}{9} \begin{bmatrix} -2 & 2 & 8 & -2 & 0 \\ 1 & -4 & -4 & 4 & 0 \\ 4 & -4 & -4 & 1 & 0 \\ -2 & 8 & 2 & -2 & 0 \\ 0 & 0 & 0 & 0 & 9 \end{bmatrix} \begin{bmatrix} \alpha_3 \\ \alpha_4 \\ \alpha_5 \\ \alpha_6 \\ \alpha_8 \end{bmatrix} \tag{3}$$

where all the independent anisotropic experimental coefficients α_k (for k from 1 to 8) reduce to 1 in the isotropic case.

3 KINEMATIC HARDENING MODEL

The kinematic hardening model of this work is based on the original developments of Dafalias and Popov [4] utilizing the two yield surfaces, but now extended to consider anisotropic non-quadratic yield functions, as is the case of Yld2000. The consistency condition requires that the stress field is always on the yield surface. For the two yield model, this applies to the inner (or loading) yield surface and also to the outer (or boundary) yield surface, i.e.:

$$\frac{1}{2}\phi\left(\boldsymbol{\sigma}-\boldsymbol{\alpha}\right)-\bar{\sigma}^{a}=0 \qquad ; \qquad \frac{1}{2}\phi\left(\boldsymbol{\Sigma}-\mathbf{A}\right)-\bar{\Sigma}^{a}=0. \tag{4}$$

In Eq. (4), $\boldsymbol{\alpha}$ is the position vector of the inner yield surface's center (or commonly called back stresses), $\mathbf{A}$ is the position vector of the outer yield surface's center. Also, $\boldsymbol{\sigma}$ is the stress tensor of the inner yield surface and $\boldsymbol{\Sigma}$ is the stress tensor for the outer yield surface. The increment of the back stresses is expressed as,

$$d\boldsymbol{\alpha}=dc\,\mathbf{v}. \tag{5}$$

In Eq. (5), $\mathbf{v}=\boldsymbol{\Sigma}-\boldsymbol{\sigma}$ and defines the direction for the evolution of the back stresses on the stress field. The equivalent uniaxial quantity for the increment of back stresses is calculated by,

$$\phi\left(d\boldsymbol{\alpha}\right)=\phi\left(dc\,\mathbf{v}\right)=dc^{a}\,\phi\left(\mathbf{v}\right)=2d\bar{\alpha}^{a}. \tag{6}$$

Solving Eq. (6) to obtain dc gives,

$$d\boldsymbol{\alpha}=\frac{d\bar{\alpha}}{\bar{\sigma}\left(\mathbf{v}\right)}\,\mathbf{v}=\frac{d\bar{\alpha}}{d\bar{\epsilon}^{p}}\,d\bar{\epsilon}^{p}\frac{\mathbf{v}}{\bar{\sigma}\left(\mathbf{v}\right)}=d\bar{\epsilon}^{p}H^{\alpha}\frac{\mathbf{v}}{\bar{\sigma}\left(\mathbf{v}\right)}. \tag{7}$$

where H^{α} is the plastic modulus and represent the slope on the work-hardening curve for the back stresses. From the uniaxial stress-strain curve, a new variable δ is defined as the distance between the stress on the boundary Σ and the stress on loading σ surfaces, such that $\Sigma=\sigma+\delta$. For multiaxial plasticity, the scalar δ is defined as follows,

$$\delta=\left(\frac{1}{2}\phi\left(\mathbf{v}\right)\right)^{\frac{1}{a}}=\bar{\sigma}\left(\mathbf{v}\right). \tag{8}$$

It is possible to conclude that,

$$\frac{d\Sigma}{d\bar{\epsilon}^{p}}=\frac{d\sigma}{d\bar{\epsilon}^{p}}+\frac{d\delta}{d\bar{\epsilon}^{p}}=\frac{d\sigma}{d\bar{\epsilon}^{p}}-\frac{a}{1+cr^{n}}\left(\frac{\delta}{\delta_{in}-\delta}\right), \tag{9}$$

In (9), the following relationships are satisfied:

- $\frac{d\sigma}{d\bar{\epsilon}^p} = \infty$ for $\delta = \delta_{in}$, which guarantees the continuous hardening slope between the elastic range and the plastic range;
- $\frac{d\sigma}{d\bar{\epsilon}^p} = \frac{d\Sigma}{d\bar{\epsilon}^p}$ for $\delta = 0$, which means that the plastic modulus of the loading surface is the same as the bounding surface. This aspect is in good agreement with experiments.

Here, δ and δ_{in} are the current and initial gap between the outer and inner surfaces. In Eq. (9), a, c and n are material properties, while $r = \frac{\delta_{in}}{\sigma_r}$ with σ_r a reference stress non-dimensionalizing δ_{in}. Equation (9) represents an experimental curve fitting to the uniaxial stress-strain curve.

4 CONCLUSIONS

The proposed two yield surfaces kinematic hardening model is general and appropriate for the modeling of anisotropic non-quadratic yield functions, as is the case of Barlat Yld2000, and suitable for the treatment of the Bauschinger effect in sheet metal forming aluminum alloys. The main advantage of the model is that its formulation is general and easily adapted to any kind of yield surfaces.

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