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EVALUATION OF DESIGN VELOCITY FIELD BY DIRECT DIFFERENTIATION OF SYMBOLICALLY PARAMETERIZED MESH

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Summary. *The paper presents design sensitivity analysis and optimization based on symbolic-numeric approach to evaluation of design velocity field by direct differentiation of symbolically parameterized mesh.*

1 INTRODUCTION

The basic question of the gradient based shape optimization is how to construct the design velocity field. The purpose of design velocity field ($\partial \mathbf{X} / \partial p$) is to characterize the changes of the finite element nodal point coordinates (**X**) with respect to the changes of arbitrary design parameter (p). While the design derivatives of the finite element quantities (residual, tangent matrix, etc..) can be constructed by automatic procedures (see. e.g. Korelc [12]), this is not true for the design velocity field. The main problem is that within the standard approaches to finite element mesh generation, either with the specialized preprocessor or with the CAD tools, there exist no explicit relations between the position of the finite element nodes and the shape design parameter.

2 CONTRUCTION OF DESIGN VELOCITY FIELD

The problem of constructing the design velocity field has attracted a lot of attention and various approaches have been proposed ([1]-[5]). The simplest approach is to evaluate derivatives numerically by the finite difference method. However, the method is prone to large errors for certain type of shape sensitivity problems. Alternatively, the domain of the problem can be divided in smaller parts termed the *design elements* for which analytical design velocity field can be derived and then evaluated at the positions of the finite element nodes. The approach fails when the design parameter relates to some global measure of the structure for which explicit relations to parameters of the *design elements* are hard to derive.

The paper presents a symbolic-numeric approach to evaluation of the design velocity field by computer algebra systems *Mathematica*. Symbolic system can deal with arbitrary formulas. Thus, if we keep the particular shape parameter during the model description and mesh generation in symbolic form, then the nodal coordinates of the mesh will be an explicit function of the parameter involved. The design velocity field is then obtained by the direct differentiation of the symbolically parameterized mesh by a single command (e.g `D[SMSNodes, p]`).

However, for the numerical analysis the computer algebra systems cannot keep up with the run-time efficiency of programming languages such as FORTRAN and C. The key idea of the proposed approach is to use dual symbolic/numeric finite element environment. The first version is written in Mathematica's symbolic language (*MDriver*). Thus, when the design velocity field is derived the symbolic evaluation of FE mesh can be used. The second version is written in C language (*CDriver*) and is connected with *Mathematica* via the *MathLink* protocol so that large-scale problems can be solved at the same time. Both environments operate from *Mathematica* and they also have the same data structures, functions and command language and input data (for the details of the environment see Korelc [8], [9]). The whole procedure is presented in Figure 1 and can be applied on problems with arbitrary complexity.

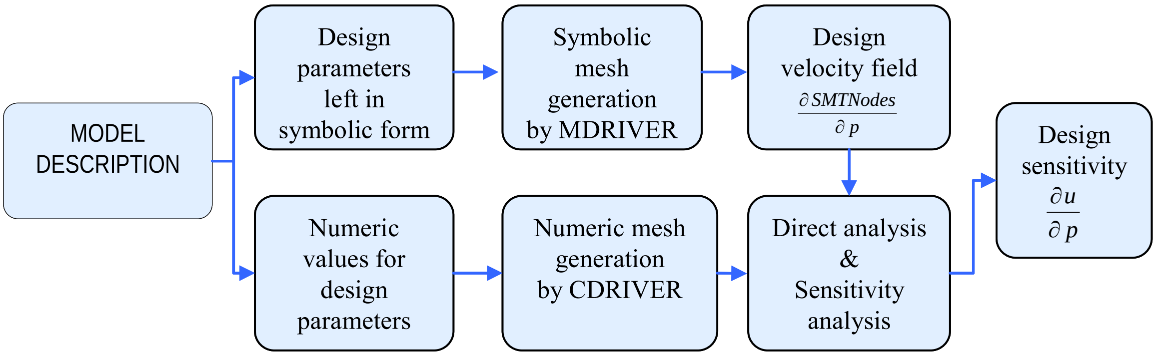


Figure 1: Symbolic-numeric shape sensitivity analysis flow chart

3 SENSITIVITY ANALYSIS OF SINGLE-STOREY STEEL BUILDING

Single-storey steel buildings are used to accommodate many functions such as factories, leisure facilities, and supermarkets. The finite element model of a typical single-storey building is shown in Figure 2.

The model consists of the following parts:

- The main structure consists of four portal frames modeled by the four node shell elements based on finite rotations, 6 parameter shell theory combined with ANS and two enhanced modes for improved performance (Wisniewski, Turska [7])
- The purlins and braced system are modeled by large displacement truss elements.
- Special “loads” elements were generated to apply wind and snow loads.

The exact analytical shape sensitivity pseudo-load vector is derived for all elements by direct differentiation method ([10]) and with the use of symbolic code generation ([9]).

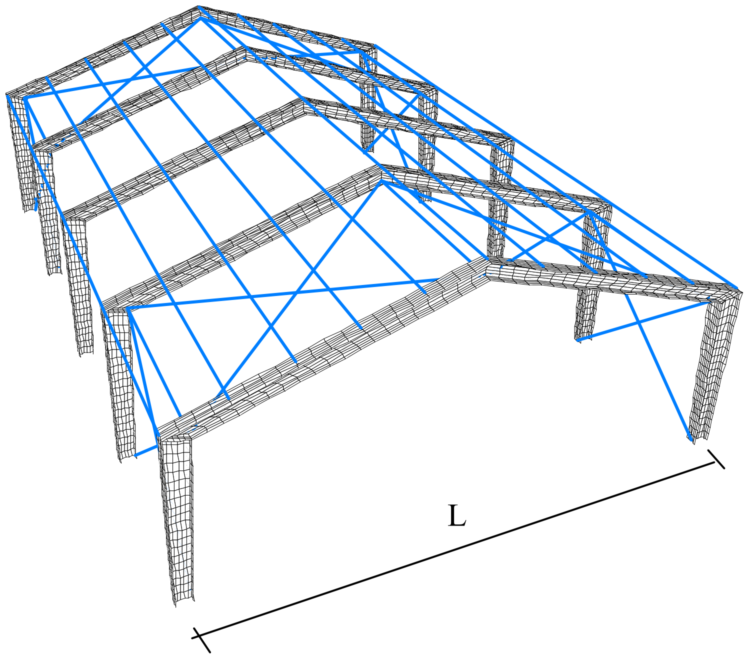
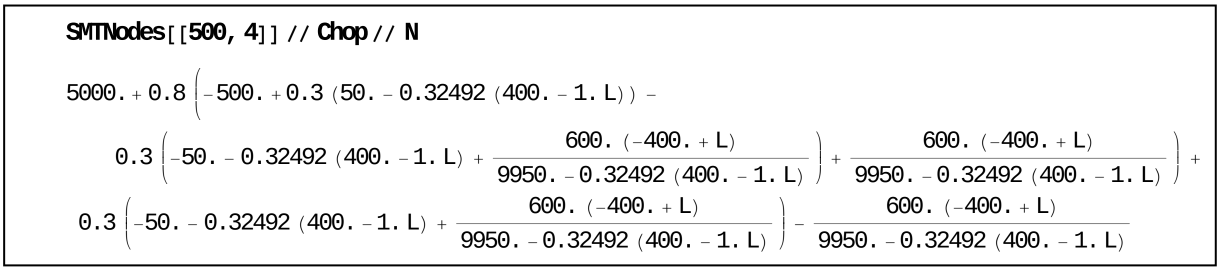


Figure 2: Finite element mesh

In the example the span of the building (L) is used for the design shape parameter. Figure 3 presents the typical symbolic form of the nodal coordinate generated by the *MDriver*. Differentiation of the symbolically parameterized mesh with respect to L results in design velocity field that is then used within standard direct sensitivity anaysis ([10]). The sensitivity of the vertical displacement is presented in Figure 4a. The results of analytical sensitivity analysis are then compared with the results obtained by the finite difference method in Figure 4.b.



```
SMTNodes[[500, 4]] // Chop // N
```

$$5000. + 0.8 \left(-500. + 0.3 \, (50. - 0.32492 \, (400. - 1. \, L)) - 0.3 \left(-50. - 0.32492 \, (400. - 1. \, L) + \frac{600. \, (-400. + L)}{9950. - 0.32492 \, (400. - 1. \, L)} \right) + \frac{600. \, (-400. + L)}{9950. - 0.32492 \, (400. - 1. \, L)} \right) + 0.3 \left(-50. - 0.32492 \, (400. - 1. \, L) + \frac{600. \, (-400. + L)}{9950. - 0.32492 \, (400. - 1. \, L)} \right) - \frac{600. \, (-400. + L)}{9950. - 0.32492 \, (400. - 1. \, L)}$$

Figure 3: Example of a nodal point coordinate in symbolic form

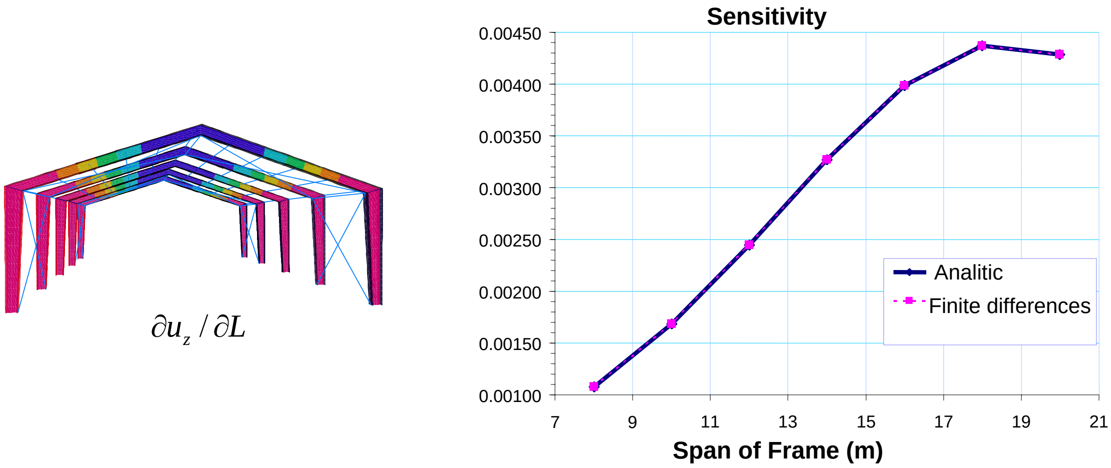


Figure 4: The sensitivity of vertical displacement with respect to the span of the frame (a) and comparison between analytic and FD method

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