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A MIXED TEMPERATURE HEAT/FLUX FORMULATION FOR SOLVING UNSTEADY THERMAL PROBLEMS. APPLICATION TO HOT FORMING PROCESS SIMULATION

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Summary. *The thermal analysis using standard linear tetrahedral finite elements may be affected by spurious local extrema in the regions affected by thermal shocks, in such a severe way to discourage the use of these elements. The present work proposes a mixed continuous temperature/heat flux formulation to solve the unsteady thermal problem. This new numerical model should allow to improve the thermomechanical coupling effects during the simulation of 3D forming processes (for example during hot or cold forging processes and during heat treatment. The spatial model is based on the Galerkin approach with the linear tetrahedral P1/P1 mixed finite elements. Time integration is based on an implicit scheme. The performance of this method is evaluated by means of test case with analytical solution, as well as an industrial application, for which a well-behaved numerical solution is available, and by comparisons with two discontinuous Galerkin : the explicit Taylor Discontinuous Galerkin scheme, DTG (P0/P0 interpolation and third degree Taylor time integration) and the implicit Discontinuous Galerkin model, IMPGD (P0/P0+ interpolation with implicit Euler scheme). For this study, the 3D finite element FORGE3® software is required.*

1 INTRODUCTION

During industrial forming processes, important thermal phenomena occur. So, the main purpose of this paper is to present an improved method for solving the thermal problem efficiently. We should deliver a good compromise between results accuracy and the corresponding calculation time, taking into account the strong mechanical couplings deriving from the mechanical problem. There are several resolution techniques of the thermal problem in the literature. The Standard Galerkin approach (SG) applied to diffusion problems (temperature being the only unknown)[1,2], is certainly the best-known method, but it nevertheless generates difficulties in treating thermal shocks with the presence of oscillations in the FEM solution inside the regions affected by thermal shocks[3]. If an asynchronous time step is associated to the Galerkin version[3] , the thermal shocks will be smoothed. This strategy is now used in the thermal solver FORGE3® and gives satisfactory results for linear or slightly non-linear problems, as long as there aren't too severe thermal shocks. In order to avoid these limitations, a mixed temperature/heat flux formulation of the thermal problem is then introduced which enables to capture high temperature gradients without any polluting oscillations of the solution. After studying the constant P0 finite element and establishing several comparisons between two discontinuous methods (DTG and IMPDG schemes), we naturally became interested in the P1 continuous interpolation: our new model will be called the Mixed continuous P1/P1 formulation[4]. In this domain, few investigations have been conducted. We can quote those of Zienkiewicz and al.[1] and those of Manzari and al.[5] that consider hyperbolic heat conduction equations with the non-Fourier hypothesis but this relevant scheme is only validated with 1D and 2D test cases. In this paper, after recalling the governing equations of the unsteady thermal problem, we will first describe our numerical model based on the Mixed continuous temperature/heat flux formulation, before delivering some numerical results. The three methods (DTG, IMPDG and Mixed) on which our investigation relies are implemented in the Forge3® software (able to simulate strongly coupled thermomechanical problems and steel quenching), validated and compared in purely thermal test cases, for which the analytical solution is known, and in one case of hot forging process simulation.

2 GOVERNING EQUATIONS OF THERMAL MODELING

The thermal equilibrium, the initial conditions and the boundary conditions are written in the following classical equations:

$$\left\{\begin{array}{ll} \text{Heat equation} & \rho c \dfrac{dT}{dt} + div(\vec{q}) = \dot{W} \\ \text{Fourier law} & \vec{q} = -k \; gradT \\ \text{Initial conditions} & \mathrm{T} = \mathrm{T}_0 \text{ and } \vec{\mathrm{q}} = \vec{0} \text{ at t} = 0 \\ \text{Dirichlet boundary conditions} & T = T_{imp} \;\; on \;\; \Gamma_1 \\ \text{Neuman boundary conditions} & \vec{q}.\vec{n} = \Phi_{imp} on \;\; \Gamma_2 \\ \text{Conduction boundary condition} & \vec{q}.\vec{n} = h_{cd}(T - T_{tool}) \\ \text{Convection/radiation bc} & \left\{\begin{array}{l} \vec{q}.\vec{n} = h(T - T_{ext}) \\ h = h_{cv} + h_r \\ h_r = \varepsilon_r \sigma_r (T + T_{ext})(T^2 + T_{ext}^2) \end{array}\right. \end{array}\right. \tag{1}$$

where T is the temperature, $\vec{q}$ the heat flux, t the time variable, $\dot{W}$ the power dissipated by the plastic deformation, k the thermal conductivity, ρ the density and c the specific heat. Then the unsteady thermal problem (1) have two fields of unknowns: the temperature T and the heat flux $\vec{q}$.

3 A MIXED CONTINUOUS FORMULATION

Inspired by the Manzari's work[5], our numerical model uses mixed continuous P1/P1 finite element combined with an implicit scheme for time integration[4]. In this work, we must be able to choose compatible mixed finite element on temperature and heat flux. So linear element (P1) are used with the same interpolation functions employed for the temperature and each heat flux component. In each element Ω_e, we have, by denoting $\{U_j\} = {}^t\{q_{xj}, q_{yj}, q_{zj}, T_j\}$ the unknown vector at the j node of the mesh, the local spatial discretization of the thermal problem can be written by :

$$[M^e]\frac{d\{U^e\}}{dt} + [K^e]\{U^e\} = \{F^e\} \tag{2}$$

Finally, a system of (4*node number) equations at (4*node number) unknowns $\{U_j\}$ is obtained for the global spatial discretization.

For the time discretization, the Dupont implicit scheme[2] is required. It's a second order scheme with 3 time steps: the system (2) is discretized at the time $\hat{t} = \alpha_1 t_{n-1} + \alpha_2 t_n + \alpha_3 t_{n+1}$ with $\alpha_1 + \alpha_2 + \alpha_3 = 1$. So the following linear symmetric system is obtained :

$$\left(M\frac{\gamma_2}{\alpha_3 dt} + K\right)\hat{U} = F + M\left[\left(\frac{\alpha_1\gamma_2}{\alpha_3 dt} - \frac{\beta_1}{dt}\right)U_{n-1} + \left(\frac{\alpha_2\gamma_2}{\alpha_3 dt} - \frac{\beta_2}{dt} - \frac{\gamma_1}{dt}\right)U_n\right] \tag{3}$$

with $\alpha_1 = \alpha_2 = 0$, $\alpha_3 = 1$, $\gamma_2 = \frac{3}{2}$ and $\beta_1 = \frac{1}{2}$. After solving this linear system (3), the unknowns U_{n+1} at time $n+1$ are estimated by :

$$U_{n+1} = \frac{\hat{U} - (\alpha_1 U_{n-1} + \alpha_2 U_n)}{\alpha_3} \tag{4}$$

4 APPLICATION TO A COMPRESSION OF ONE SIXTH OF CYLINDER

For the hot forging simulations, a Norton-off viscoplastic behavior power law is used. We are interested in the mechanical work coupled with the various thermal exchanges. The cylinder was initially at the temperature T0=980°C and its material data are $\rho = 7870$ kg/m^3, c = 681J/(kg°C), k = 27.5W/(m/C). It's placed between two tools of T_{tool}=200°C : the upper tool moves while the lower tool is fixed. Without all heat transfers (conduction and convection) and without frictions between the part and the tooling (Tresca friction law is required), only the self-heating phenomenon occurs during all the compression phase. In this anisothermal case, an analytical solution may be established and evaluated by our continuous model. Fig.1a allows to conclude that the internal heat source $\dot{W}$ is correctly estimated with our method.

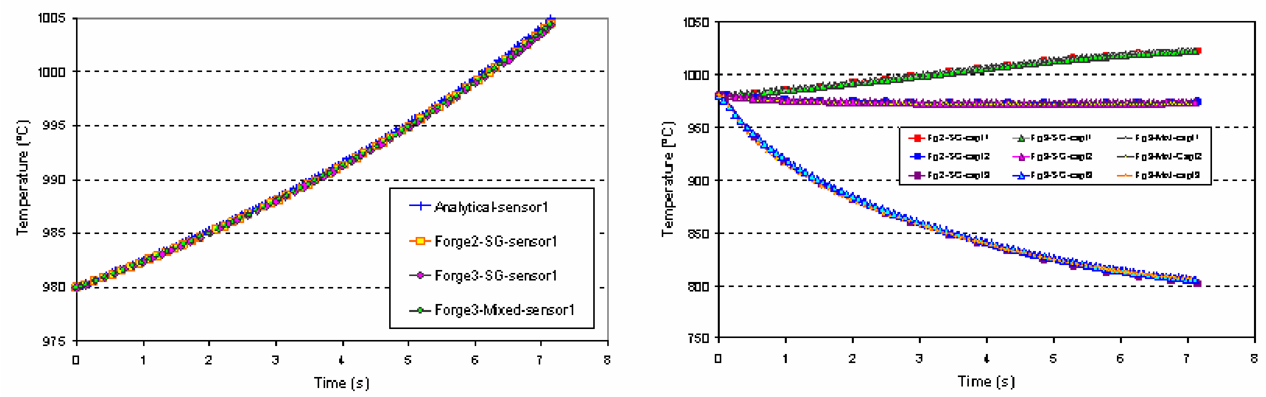

Figure1. Comparison between the SG Forge2®, the SG and the Mixed Forge3® *: a)Anisothermal case with analytical solution and sensor1, b) Compression with heat transfer and 3 sensors.*

In a second test, friction and thermal exchanges are taken into account: exchanges occur as well between the part and the tools (h_{cd}=2000W/(m°C)) as between the part and the air (Text=50°C, hext=10W/(m°C)). Three virtual sensors are placed in order to describe correctly these phenomena. The sensor 1 is on centre of the piece for the self-heating, sensor 2 on the surface of the piece for the convection and/or radiation and the sensor 3 on the part having contact with the upper tool (conduction). Presented in Fig.1b, the results are very relevant with the Mixed curves matching the SG Forge2® and Forge3® solutions which serve as the reference values in absence of analytical solutions or experimental results.

5 CONCLUSION

The comparison of the various methods shows that this Mixed method is stable, robust and rather well adapted to the thermomechanical coupling. Our formulation offers an excellent compromise between the precision of the estimations of the temperature field and the calculation time. Therefore, our Mixed method should be preferred to solve as well the thermal treatment problems and the thermomechanical problems.

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