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PARAMETER IDENTIFICATION OF NONLINEAR CONSTITUTIVE LAWS BY AN UNSCENTED KALMAN FILTER

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Summary. *In this paper we deal with the calibration of nonlinear constitutive laws via the recently proposed unscented Kalman filter (UKF). This nonlinear filter does not require the computation of the gradient of the equations governing the dynamics of the structural system; furthermore, the statistics of the system state variables -model parameters and nodal displacements- are accurately propagated in time up to the third order.*
In case of the numerical simulation of spalling in ceramics, we compare the performances of the UKF and of the usually employed extended Kalman filter (EKF).

1 INTRODUCTION

To simulate highly localized phenomena in continua, like e.g. crack growth in quasi-brittle materials and delamination in composites, the concept of interface modeling can be resorted to [1, 2]. Interfaces describe what actually happens in very narrow regions, as compared to the characteristic size of the continuum. The relevant traction vs displacement discontinuity relationship can be based on micromechanical considerations or it can be established a priori and then calibrated on the basis of experimental data [3, 4, 5, 6]. In this work we follow the latter approach and we propose a methodology based on the UKF [7] to identify the interface constitutive parameters.

The UKF has been recently proposed to deal with highly nonlinear system dynamics. Within a stochastic framework, instead of linearizing the evolution equations around the most recent expected state, as done with the EKF, a population of so-called sigma points is generated around the current expected state allowing for the information gathered in the state covariance matrix. This procedure has two main advantages: it does not require the computation of the gradient of the evolution equations; in a Taylor series expansion of the statistics of the state variables, a second/third order accuracy (depending on the initial statistical distribution) is achieved, whereas only a first order accuracy is guaranteed by the EKF.

At time t_i, for $i = 1, ..., N$

- Predictor phase:

$$\hat{\boldsymbol{\mathcal{X}}}_i = \left[\hat{\boldsymbol{x}}_i \quad \hat{\boldsymbol{x}}_i + \mu\sqrt{\boldsymbol{P}_i} \quad \hat{\boldsymbol{x}}_i - \mu\sqrt{\boldsymbol{P}_i}\right] \qquad \hat{\boldsymbol{\mathcal{X}}}^-_{i+1} = \boldsymbol{f}_i(\hat{\boldsymbol{\mathcal{X}}}_i)$$

$$\begin{aligned} \hat{\boldsymbol{x}}^-_{i+1} =& \sum_{\ell=0}^{2L} w^{(m)}_\ell \, \hat{\mathcal{X}}^-_{i+1,\ell} \\ \boldsymbol{P}^-_{i+1} =& \boldsymbol{R}^-_{i+1} + \boldsymbol{V} \end{aligned}$$

where $\boldsymbol{R}^-_{i+1} = \sum_{\ell=0}^{2L} w^{(c)}_\ell \left[\hat{\mathcal{X}}^-_{i+1,\ell} - \hat{\boldsymbol{x}}^-_{i+1}\right] \left[\hat{\mathcal{X}}^-_{i+1,\ell} - \hat{\boldsymbol{x}}^-_{i+1}\right]^{\mathrm{T}}$

- Corrector phase:

$$\begin{aligned} \hat{\boldsymbol{x}}_i =& \hat{\boldsymbol{x}}^-_i + \boldsymbol{R}^-_i \boldsymbol{H}^{\mathrm{T}} (\boldsymbol{H} \boldsymbol{R}^-_i \boldsymbol{H}^{\mathrm{T}} + \boldsymbol{W})^{-1} \left(\boldsymbol{y}_i - \boldsymbol{H} \hat{\boldsymbol{x}}^-_i\right) \\ \boldsymbol{P}_i =& \boldsymbol{P}^-_i - \boldsymbol{R}^-_i \boldsymbol{H}^{\mathrm{T}} (\boldsymbol{H} \boldsymbol{R}^-_i \boldsymbol{H}^{\mathrm{T}} + \boldsymbol{W})^{-1} \boldsymbol{H} \boldsymbol{R}^-_i \end{aligned}$$

Table 1: unscented Kalman filter.

To compare the performances of the UKF and of the EKF, we show some results concerning the calibration of SiC toughness under impact loading.

2 UNSCENTED KALMAN FILTER

The stochastic dynamics of a structural system can be formulated, after space and time discretization, according to:

$$\begin{aligned} \boldsymbol{z}_{i+1} =& \boldsymbol{f}^z_i \left(\boldsymbol{z}_i; \boldsymbol{\vartheta}_i\right) + \boldsymbol{v}^z_i, & (1) \\ \boldsymbol{\vartheta}_{i+1} =& \boldsymbol{\vartheta}_i + \boldsymbol{v}^\vartheta_i, & (2) \\ \boldsymbol{y}_i =& \boldsymbol{H}^z \boldsymbol{z}_i + \boldsymbol{w}_i, & (3) \end{aligned}$$

where: $\boldsymbol{z}$ is the state vector, which gathers the nodal displacements; $\boldsymbol{\vartheta}$ is the vector of model parameters; $\boldsymbol{y}$ is the observation vector; $\boldsymbol{v}^z$ and $\boldsymbol{v}^\vartheta$ are the process noises; $\boldsymbol{w}$ is the measurement noise. The state equation (1) describes the evolution in time of $\boldsymbol{z}$, as caused by the nonlinear mapping $\boldsymbol{f}^z$; the time invariance of model parameters is given by (2); the observation equation (3) defines a linear relationship between the observation vector $\boldsymbol{y}$ and the state vector $\boldsymbol{z}$.

By introducing a joint state vector $\boldsymbol{x}_i$

$$\boldsymbol{x}_i \equiv \begin{Bmatrix} \boldsymbol{z}_i \\ \boldsymbol{\vartheta}_i \end{Bmatrix}, \tag{4}$$

Eqs. (1)-(3) transform into:

$$\begin{aligned} \boldsymbol{x}_{i+1} =& \boldsymbol{f}_i(\boldsymbol{x}_i) + \boldsymbol{v}_i, & (5) \\ \boldsymbol{y}_i =& \boldsymbol{H} \boldsymbol{x}_i + \boldsymbol{w}_i. & (6) \end{aligned}$$

In Table 1 we summarize the improvement at instant t_i of the estimates of the joint state vector $\boldsymbol{x}$ and of the relevant covariance matrix $\boldsymbol{P}$, as furnished by the UKF. Here: $\boldsymbol{V}$ and $\boldsymbol{W}$ are the covariance matrices of the process and measurement noises, respectively; L is the length of vector $\boldsymbol{x}$; $\hat{\boldsymbol{\mathcal{X}}}_i$ are the sigma points; μ is a scaling factor, which affects the distribution of the sigma points around the current mean $\hat{\boldsymbol{x}}_i$; $w_\ell^{(m)}$ and $w_\ell^{(c)}$ are appropriately defined weights [7].

3 IDENTIFICATION OF SiC TOUGHNESS THROUGH IMPACT TESTS

SiC is a high strength, low density material; under shock loading it fails due to tensile stresses exceeding its strength. In [8] several SiC specimens were tested in order to produce plane shocks in the through-the-thickness direction and to evaluate the spall strength. Fig. 1 displays a typical experimental evolution of the free surface velocity (experiment 519 in [8]) recorded at the rear surface of the specimen via a VISAR.

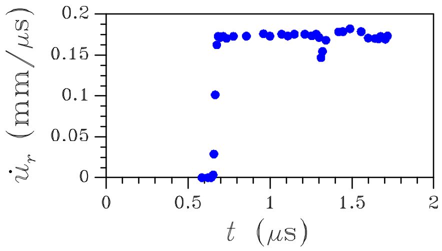
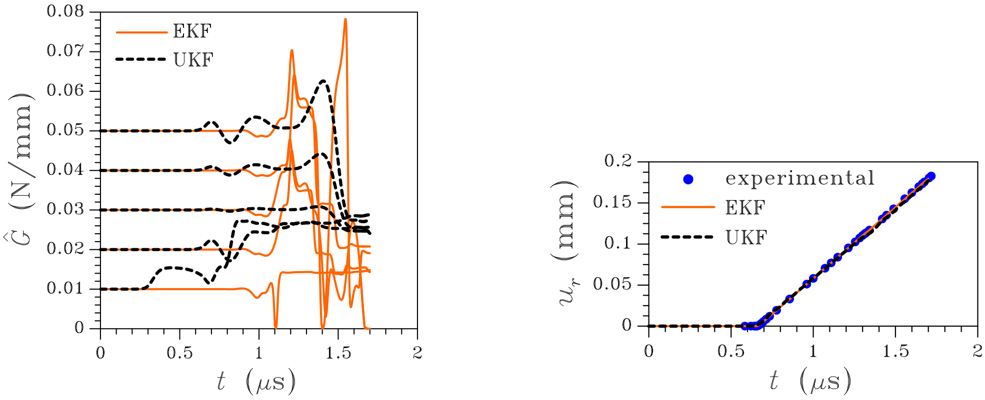


Figure 1: impact tests on SiC. Experimental free surface velocity record.



(a) (b)

Figure 2: impact tests on SiC. (a): current estimated value of fracture energy G_c (at fixed tensile strength $\sigma^t = 540$ MPa); (b): current tracked free surface displacement u_r.

Due to the brittle behavior of SiC, damage development can be disregarded and the location of the spall plane can be determined with a simple elastic analysis. Disregarding also possible rate-dependency effects and assuming a tensile strength $\sigma^t = 540$ MPa, according to [8], we aim at identifying the fracture toughness G_c. In Fig. 2 we compare the outcomes obtained with the UKF and the EKF in terms of currently estimated G_c (Fig. 2a) and tracked free surface displacement u_r (Fig. 2b). It can be seen that only the UKF furnishes stable (i.e. no oscillatory) estimations and provides final solutions almost independent of the initialization values.

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