VIII International Conference on Computational Plasticity
COMPLAS VIII
E. Oñate and D. R. J. Owen (Eds)

ABSORBING BOUNDARY LAYERS FOR ELASTIC WAVE PROPAGATION

Håkan Lane*, Per Kettil*, Mikael Enelund*, Torbjörn Ekevid† and Nils-Erik Wiberg*

* Department of Applied Mechanics
Chalmers University of Technology
S-412 96 Gothenburg, Sweden
e-mail: Hakan.Lane@sm.chalmers.se, web page: http://www.sm.chalmers.se

† School of Technology and Design
Växjö University
S-351 95 Växjö, Sweden
Email: Torbjorn.Ekevid@vxu.se Web page: http://www.vxu.se/td/

Key words: Solid Wave Propagation, Viscoelastic materials, Absorbing Boundaries

Summary. *For finite element analysis of wave propagation in an elastic medium surrounded by an adjacent "infinite" region, the introduction of external boundaries introduces severe reflection problems, which may ultimately distort the solution. Analysis based solely on finite elements is not sufficient, as the boundary will add unwanted reflections to the grid. This paper explores the approach of introducing a virtual element layer to absorb the outgoing waves. The results indicate that reflections could be reduced to a large extent.*

1 INTRODUCTION

Computational modeling of the rail vehicle – track structure – subgrade involves the merging of several subsystems. One of the most pronounced challenges inherent arises from the vast ("infinite") regions of predominantly clay (in Sweden) making up the surrounding medium. The shortcoming of the finite element method for unbounded domains introduces a need for special methods. Waves need to leave the mesh without reflections at the external boundaries.

There are several examples of applications where a viscoelastic material has been used as an important component of a device mounted to shield the environment from the vibrations generated by the opration of a structure or vice versa. The floating slab systems used in some rail track systems[1] is an important example. Such an approach has been used on the Barbican line on the London underground[2]. These *passive energy dissipation* systems commonly rely on a layer of a viscoelastic material converting elastic energy to heat.

A computationally efficient local way to fulfill the radiation condition is to introduce an artificial layer of elements outside the grid with a viscoelastic material. If the characteristics can be adjusted so as to constitute critical damping, the waves coming in from the elastic material will be attenuated inside the fictitious region. Hence, the energy of the outgoing waves will be dissipated inside the absorbing boundary layer.

2 ABSORBING BOUNDARY LAYER

In this paper, a layer consisting of a viscoelastic material will be placed surrounding the otherwise elastic solid FEM mesh. The target of this application is to adjust the parameters of the layer such that the equivalent damping constant corresponds to a state of critical damping. If this can be achieved, the wave energy would be dissipated inside the artificial domain without returning to the interface to the elastic region. In this manner, the external boundary of the finite element domain representing the system under analysis will be non-reflecting. Waves will be damped out as they reach the end of the viscoelastic domain. The concept is illustrated in Figure 1.

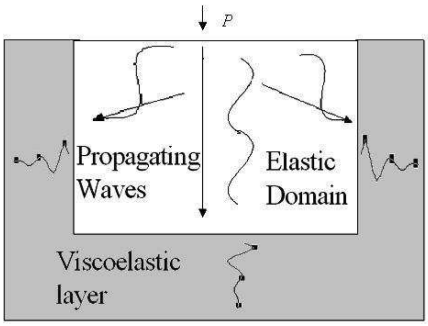

Figure 1. Absorbing boundary layer.

3 SIMULATIONS

3.1 Test Case 1: 1D wave propagation

In the first test, a wave was sent through a bar by a rectangular pulse. The geometry of the bar under consideration is defined in Figure 2 and the vertical load time history in Figure 3. The aim was to find optimal values of E_l, η_l and h_l.

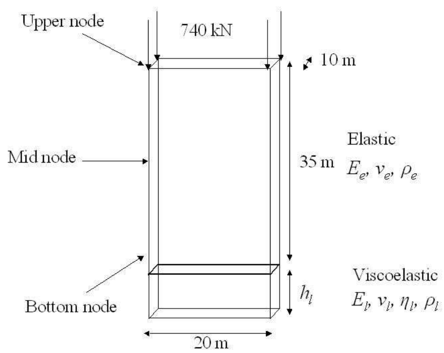

Figure 2. Test configuration 1.

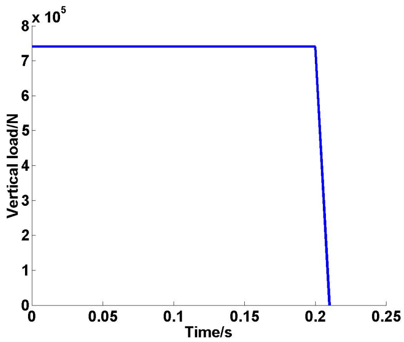

Figure 3. Load history.

The values of the elastic constants in the “real” region are based on typical clay conditions in Sweden and defined in Table 1.

Entity	Value
E_e	$1\cdot10^7$ Pa
ν_e	0
ρ_e	2000 kg / m^3

Table 1. Properties of the elastic region..

Displacements are monitored at (see Figure 2) the surface, 15 and 30 m down in the bar. In order to find the best configuration of the layer, a surface displacement time history analogous to a state of critical damping[3]. Tests on the bar in Figure 2 with the set of elastic parameters in Table 1 yielded an optimal viscosity η_l of 2.7 MNs/m, a ratio of $\kappa = \frac{E_l}{E_e} = \frac{1}{9}$ and a height h_l = 9 m. Further tests with altering elastic parameters revealed the dependence

$$\frac{\eta_l}{h_l} \propto \sqrt{E_e \frac{(1+\nu)(1-2\nu)}{(1-\nu)} \cdot \rho_e} \tag{1}$$

Figure 4 presents the displacement time histories at the different locations.

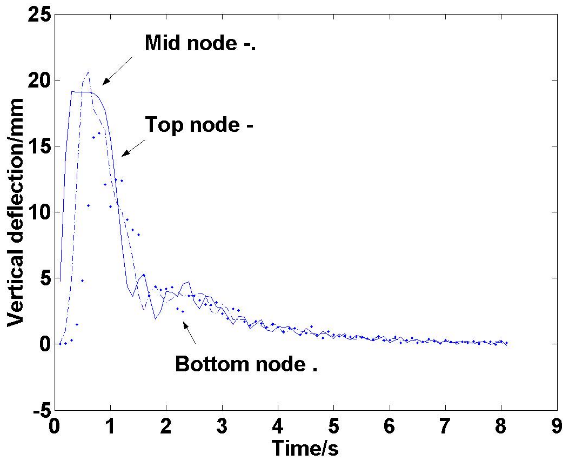

Figure 4. Displacements monitored at different locations.

Finally, it was investigated whether the number of elements in the layer had an impact. A series of simulations with different numbers of elements was performed. Although the difference is minor, one element with a length of 9 m seems to be optimal.

Tests in 2D also indicate a strong reduction of reflections from external boundaries.

4 CONCLUSIONS

A Kelvin material model has been implemented and integrated in the multi-purpose finite element software FEM90[4]. An absorbing viscoelastic boundary layer has been added surrounding a mesh in test geometries for elastic wave propagation. The influence of the respective layers were evaluated by comparing displacements to the same configuration with only elastic material.

The conclusions are:

- The relationship between the optimal viscosity and height of the boundary layer and Young modulus, height and density parameters of the elastic region was found.
- For the best set of parameters, reflections were reduced to a rather small fraction of the results with fixed boundaries in the 1D and 2D test cases.
- The numerical dependence and wave behaviour indicate the observance of critical damping in the viscoelastic subdomain.

5 ACKNOWLEDGMENTS

The generous financial and administrative support afforded the supporting organisations Swedish Railroad Administration (Banverket), FORMAS (Swedish Research Council for Environment, Agricultural Sciences and Spatial Planning) and CHARMEC (Chalmers Railway Mechanics Competence Centre) is gratefully recognised. The libraries BLAS[5], HSL (Rutherford Appleton Laboratory[6]), LAPACK[7], SuperLU[8] and METIS[9] are included in the software.

6 REFERENCES

1 *Vibration Problems in Structures. Practical guidelines*, Comite Euro-International du Beton, Lausanne, Switzerland, (1977).
2 P. Grootenhuis, "Floating Track Slab Isolation for Railways," *Journal of Sound and Vibration*, **51**, 443-448, (1977).
3 http://www.efunda.com/formulae/vibrations/sdof_free_damped.cfm?search_string=Damping - SDOF systems: Free Vibrations with Viscous Damping. Latest access 9/5 – 2005.
4 P. Kettil, *Computational engineering. Development and implementation of computational methods for structural analysis and design*. Ph. D. thesis, Department of Structural Mechanics, Chalmers University of Technology, Gothenburg, Sweden, (2001).
5 http://www.netlib.org/blas/ - BLAS, Latest access 9/5-2005.
6 http://www.cse.clrc.ac.uk/nag/hsl/ - Welcome to CLRC's Computational Science and Engineering Department, Latest access 9/5-2005.
7 http://www.netlib.org/lapack/ - LAPACK – Linear Algebra PACKage, Latest access 9/5-2005.
8 http://crd.lbl.gov/~xiaoye/SuperLU/index.html - SuperLU, Latest access 9/5-2005.
9 http://www-users.cs.umn.edu/~karypis/metis/ - METIS: Family of Multilevel Partitioning Algorithms, Latest access 9/5-2005.