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IMPLICIT INTEGRATION ALGORITHM FOR NUMERICAL SOLUTION OF PARTIALLY SATURATED SOILS WITH AN ENHANCED GENERALIZED PLASTICITY CONSTITUTIVE MODEL

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1 INTRODUCTION

The stress update algorithm and the tangent operator for an enhanced generalized plasticity model is investigated within the framework of generalized plasticity. The constitutive model used was developed recently for partially saturated soils[1,2], where on top of the hydrostatic and deviatoric components of the (effective) stress tensor suction has to be considered as an independent variable. The form of the effective stress tensor chosen is thermodynamically consistent. The enhanced generalized constitutive model is derived from a potential energy function, the Helmholtz free energy, and from a dissipation function that has to be positive for every value of their independent variables, the plastic components of the volumetric and shear strain rates, and are defined for the case of associative plasticity and non-associative one[3].

The effective stress σ' and the modified suction (scaled by the porosity) ns are work conjugate with the rate of soil skeleton strain $\dot{\varepsilon}$ and the rate of degree of saturation $-\dot{S}^w$, respectively. The suction plays the role of stress, the degree of water saturation is a strain-like variable. The rate of input work to the soil is equal to the sum of the products of the stresses with their corresponding strain rates.

The Galerkin method is used to obtain a weak form of the governing equations and a "stress-suction coupling matrix" is derived in the discretization in space by the finite element method. The implicit integration algorithm[4] is incorporated in a code for partially saturated soil dynamics. Numerical examples are computed to show the efficiency and validity of the algorithm developed.

2 APPLICATION

An example is the solution of a laboratory experiment performed at IKU Petroleum Research, Trondheim (Papamichos and Schei (1998), Papamichos et al. (1998)) on behalf of AGIP (Italian National Petroleum Company).

The problem deals with a silty consolidated sandstone sample extracted from a gas bearing formation in the Northern Adriatic basin at a depth of 3400 m. Effective porosity, *in situ* water saturation and irreducible saturation of the material were independently obtained. Subsequently the specimen underwent an oedometric test.

The loading process was scheduled as follows: the sample at in situ saturation (0.38-0.45) is firstly stressed with an initial hydrostatic phase presenting σ_r-rate equal to 0.01 MPa/s until σ_r=0.5 MPa. This is followed by a uniaxial phase with σ_z-rate of 0.004 MPa/s until σ_z reaches 35 MPa; the sample is then held at constant stress level and water is injected for 25 hours until full saturation is attained. This procedure is simulated through the specification of change of saturation (suction) from 0.38 to 1.0. During this period of time, volumetric changes of the specimen are recorded, as during the phases of stress changes. Once full saturation is reached, a second uniaxial phase, at constant water content, with stress rate of 0.004 MPa/s till about 110 MPa is performed. The test includes also unloading cycles to determine the elastic behaviour and recoverable deformation. The water injection test (hydric-path) simulates the behaviour of the gas reservoir rock during artificial water injection or during the flooding associated with gas extraction. For this reason the axial stress level at which the sample is injected is representative of the vertical stress in reservoir conditions. In the absence of other information, we assume that gas pressure during the test maintains the same value (reference or zero pressure).

The Figure 1 shows axial stress vs. volumetric strain as results from the experiment. For identification purposes, the experimental curves are slightly changed by eliminating the unloading/reloading cycles. Also the experimental response has been slightly idealised by assuming 30 points, including of the initial known conditions, as shown in Figure 2, together with the model response. The agreement is satisfactory, in particular for the increase of volumetric strain caused by water injection.

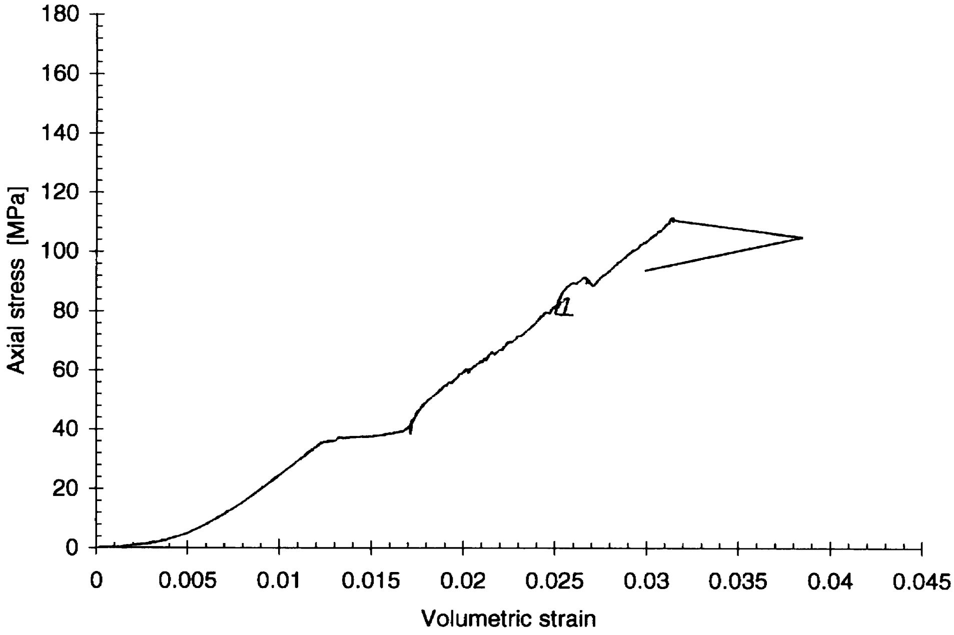

Figure1: Oedometric test with water injection: axial stress vs. volumetric strain. (redrawn from Papamichos and Schei, 1998)

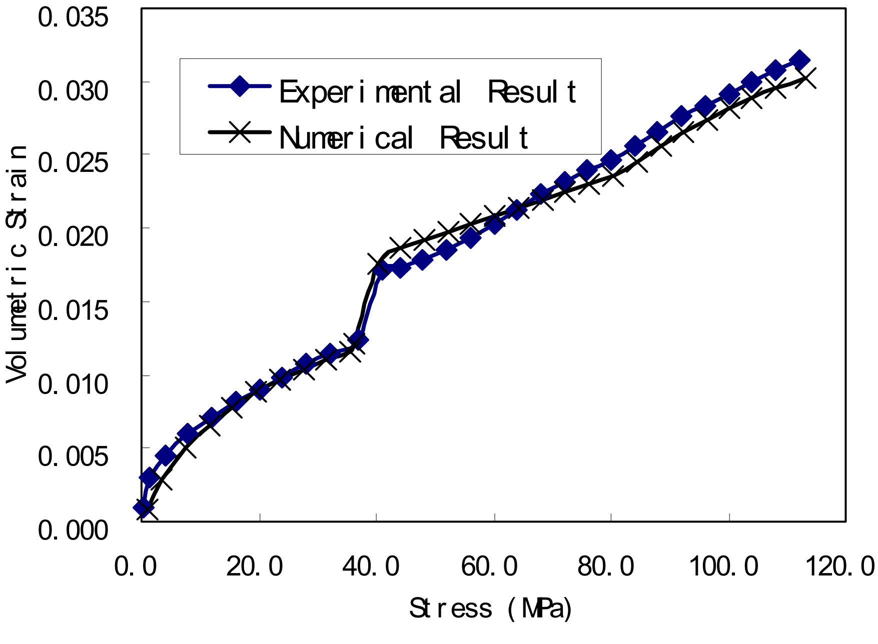

Figure 2: Comparison between numerical results and lab response

3 CONCLUSIONS

This paper shows that the application of an implicit stress update algorithm together with a tangent operator consistent with it can be successfully applied to an enhanced generalized plasticity model which is strain and suction driven for partially saturated soils. The model used is an enhanced Pastor-Zienkiewicz model which takes into account the hydraulic constitutive relationship, hydraulic hysteresis, and a new term of plastic strain to present the mechanical behaviour of partially saturated sands. Numerical examples show also the good convergence behaviour of the algorithm.

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