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A FIRST STRATEGY FOR THE APPLICATION OF THE MODIFIED CONSTITUTIVE ERROR CONCEPT IN NONLINEAR DYNAMICS FOR CORRUPTED MEASUREMENTS.

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Summary. *The objective of this study is to develop an identification strategy of model parameters in the case of test with corrupted measurements. The method was first applied in the case of elasticity where it appears to be very robust [1]. Its extension to the non-linear case is presented here in the case of viscoplasticity. It implies the development of dedicated numerical tools and a first algorithm is proposed. On the first examples, the robustness of the method is confirmed.*

1 INTRODUCTION

This paper is devoted to identification in dynamics in case of very scattered data. It aims at extending a method previously defined in elasticity to the case of non linear constitutive law. Many methods in order to take perturbed measurements into account are proposed in the literature e.g. the Tikhonov regularization methods [2] or the Kalman filter techniques [3]. Nevertheless, the lack of *a priori* knowledge in our context of level of perturbation prevent from using these methods accurately. Therefore, we studied a specific method based on the concept of Modified Error in Constitutive Relation [4]. This concept has been proven to be effective in the case of model updating in vibrations [5] and has been applied in transient dynamics to the identification of the Young's modulus of an elastic bar [1]. It is shown very robust with respect to the corrupted measurements with a perturbation level up to 40%. This method is extended here to the non-linear case.

In the first section, the framework and the proposed strategy are described. The method leads to a minimization under non-linear constraints and needs dedicated solving strategy. Here a method based on the Large Time INcrement (LATIN) method [6] is proposed and applied on a first example.

2 THE PROPOSED IDENTIFICATION STRATEGY

2.1 Description of the inverse problem

The problem we consider here consists in the identification of the parameters k_v and n_v of the evolution law of a viscoplastic material define below, from the measurements on the whole time interval $[0, T]$ at both ends of a 1D bar: measured displacements $\tilde{u}_0(t), \tilde{u}_L(t)$ and measured forces $\tilde{F}_0(t), \tilde{F}_L(t)$. In order to test the robustness of the method, a perturbation in terms of both displacements and forces can be added to the boundary conditions.

2.2 Formulation

The proposed method relies on the guiding principles of the modified error in constitutive relation [4], mainly developed for model updating in vibration [5]. The experimental and theoretical quantities are divided into two groups: the reliable quantities and less reliable quantities. In the proposed method, the verification of the properties which are considered to be reliable is enforced throughout the identification process, whereas the uncertain quantities are taken into account by minimizing a modified constitutive relation error. Considering the case described in 2.1, let us divide the quantities into two groups as shown in Table 1:

Reliable		Less reliable
Equilibrium:	$\rho.\ddot{u} - div\sigma = 0$	Boundary conditions: $\tilde{u}_d$ and $\tilde{f}_d$
State law:	$\sigma = E.(\epsilon - \epsilon_p)$	Evolution law: $\dot{\epsilon}_p = k_v < \lvert\sigma\rvert - R - R_0 >_+^{n_v} .\frac{\sigma}{\lvert\sigma\rvert}$
	$R = h.p$	$-\dot{p} = k_v < \lvert\sigma\rvert - R - R_0 >_+^{n_v} .(-1)$
Initial conditions:	$u(x, 0) = u_0$	
	$\dot{u}(x, 0) = \dot{u}_0$	

Table 1: The reliable and unreliable quantities in the case of 1D viscoplasticity

In the first step of the identification strategy, the experimental data and the model are confronted for fixed parameters k_v, n_v. It defines the basic problem which is the minimization of the sum of a modeling error term and an experimental error term:

Find *the fields* $u, \sigma, R, \epsilon_p, p, u_d, f_d$ *minimizing:*

$$J(u, \sigma, R, \epsilon_p, p, u_d, f_d) = \int_0^T \Big[\int_\Omega d_m(\sigma, R, \dot{\epsilon}_p, \dot{p}).dx + \int_{\partial\Omega_f} d_f(f_d, \tilde{f}_d) + \int_{\partial\Omega_u} d_u(u_d, \tilde{u}_d)\Big] dt$$

under the constraints:

$$u \text{ KA on } u_d, \;\; \sigma \text{ DA on } f_d, \;\; u(0, x) = u_0, \;\; \dot{u}(0, x) = \dot{u}_0, \qquad (1)$$
$$\rho.\ddot{u} - div\sigma = 0, \;\; \sigma = E.(\epsilon - \epsilon_p), \;\; R = h.p$$

The model distance d_m is chosen, among other possibilities, as the quadratic distance between the quantities (ϵ_p, p) which verify the state law and (ϵ_p^e, p^e) which verify the evolution law. As it will be shown, this choice allows to base the solving of the basic problem on the method used in the case of elasticity.

Then, the cost function needed in the identification step of the strategy uses the same functional as in the basic problem, evaluated at the solution fields of the basic problem. Therefore, the identification problem becomes:

$$k_v, n_v = \operatorname{Arg\,min} g(k_v, n_v) \quad \text{with:} \quad g(k_v, n_v) = J(u, \sigma, R, \epsilon_p, p, u_d, f_d, k_v, n_v) \qquad (2)$$

with: $u, \sigma, R, \epsilon_p, p, u_d, f_d$ are the solutions of the basic problem (1).

2.3 Resolution of the basic problem

The basic problem here is a minimization under non-linear constraints in transient dynamic and is therefore global in time. An incremental scheme hence can not be used directly. Since several solving strategies have been studied in the case of linear behavior, one has proposed an iterative scheme that allows to recover the same problem structure. It is based on the LATIN method [6], which is global in time. Its idea is to separate the equations we have to solve into two groups: the local in space variable equation, possibly non-linear, and the linear ones, possibly global in space variable. Then, an iterative procedure is solving at each iteration the first group of equations and the second ones successively. In our case, the two stages of each iteration, where H^+ and H^- are algorithm parameters, are:

The local stage:

$$\begin{bmatrix} \dot{\hat{\epsilon}}_p^e \\ -\dot{\hat{p}}^e \end{bmatrix} = B(\begin{bmatrix} \hat{\sigma} \\ \hat{R} \end{bmatrix}) \quad \text{and} \quad \begin{bmatrix} \dot{\hat{\epsilon}}_p^e - \dot{\epsilon}_{pn}^e \\ -(\dot{\hat{p}}^e - \dot{p}_n^e) \end{bmatrix} = H^+ . \begin{bmatrix} \hat{\sigma} - \sigma_n \\ -(\hat{R} - R_n) \end{bmatrix}$$

The global stage:

Find *the fields* $u, \sigma, R, \epsilon_p, p, \epsilon_p^e, p^e, u_d, f_d$ *minimizing:*

$$J = \int_0^T \left[\frac{1}{2} \int_0^L \left(\epsilon_p - \epsilon_p^e\right)^2 + (p - p^e)^2 + \frac{A}{2} \left| (u_d - \tilde{u}_d)^2 \right|_0^L + \frac{B}{2} \left| (f_d - \tilde{f}_d)^2 \right|_0^L \right] dt$$

under the constraints:

$$u \text{ KA on } u_d, \ \ \sigma \text{ DA on } f_d, \ \ u(0, x) = u_0, \ \ \dot{u}(0, x) = \dot{u}_0,$$
$$\rho.\ddot{u} - div\sigma = 0, \ \ \sigma = E.(\epsilon - \epsilon_p), \ \ R = h.p$$
$$\begin{bmatrix} \dot{\epsilon}_p^e - \dot{\hat{\epsilon}}_p^e \\ -(\dot{p}^e - \dot{\hat{p}}^e) \end{bmatrix} = H^- . \begin{bmatrix} \sigma - \hat{\sigma} \\ -(R - \hat{R}) \end{bmatrix}$$

One can note that the global stage consists in a minimization under constraint, which is not the case when the LATIN method is applied to direct problem. It is solved as in the case of elastic bar [1]. Furthermore, the search direction H^- has to be tangent in order to solve (1).

3 Illustration of the proposed method

The first example deals with the identification of the parameters k_v and n_v in the case of a viscoplastic mass spring. The identification curves are plotted below (Figure 1). One can see that moderate perturbations have little effect on the identification curves, in other words, the proposed method appears to be robust with respect to perturbations on the measurements.

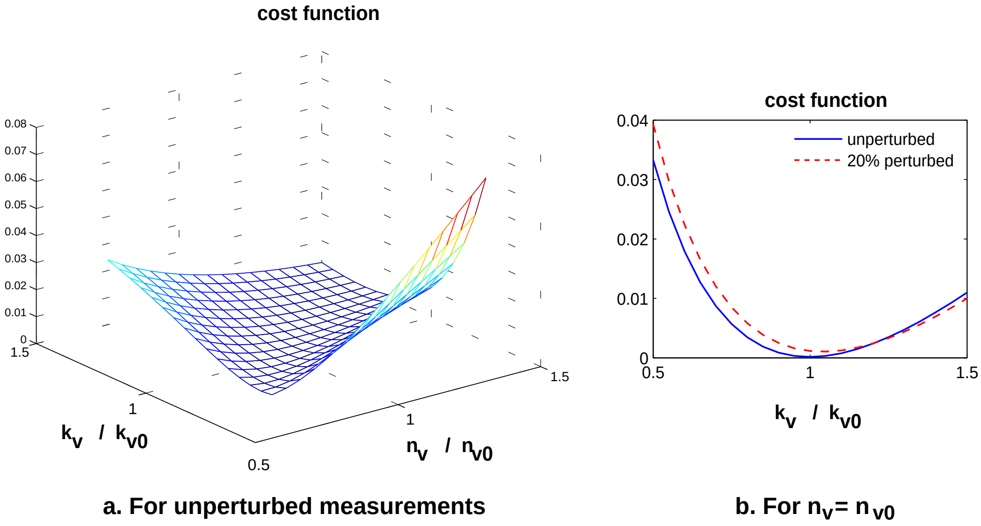

Figure 1: Identification curves at various perturbation levels in the case 0D viscoplasticity

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