VIII International Conference on Computational Plasticity
COMPLAS VIII
E. Oñate and D.R.J. Owen (Eds)

TWO-SCALE MODELING OF PLASTIC ANISOTROPIES IN METALS

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Key words: Anisotropic Plasticity, Crystallographic Texture, Metal Forming.

1 INTRODUCTION

The distribution and the arrangement of crystal orientations is an important microstructural feature which affects the overall properties of polycrystalline metals. The simplest statistical description of such microstructures is based on the crystallite orientation distribution function (codf). There exist different approaches to the representation of the codf. The classical representation is based on generalized harmonic functions [e.g. 1]. Guidi et al. [3] introduced a tensorial representation of the codf. Although equivalent, the tensorial representation is coordinate-free. Therefore, the tensorial texture coefficients or moment tensors can be used as micro-mechanically defined and experimentally observable internal variables in continuum mechanics.

Compared to higher-order correlation functions, the codf is of primarily importance for homogenization schemes, which aim to predict the mechanical behavior on the macroscopic scale based on the constitutive behavior on the mesoscale and some microstructural information. In this context the elastic-(visco)plastic Taylor model is the prototype model. Taylor type models give a reasonable qualitative approximation of the crystallographic texture evolution in single-phase materials with high stacking-fault energy under proportional loadings. From a numerical point of view the Taylor model has an inherent disadvantage: at the integration points of the finite elements a large amount of internal variables has to be taken into account. If small numbers of crystals are used at the integration points, the mechanical anisotropy is drastically overestimated. This fact considerably limits the number of degrees of freedom that can be handled by standard finite element codes, if Taylor type models are used to simulate metal forming operations. Therefore, there is a need for homogenization strategies which allow to condense the number of degrees of freedom and nevertheless accurately describe the crystallite orientation distribution function.

In the present work an extension of the widely used Mises-Hill anisotropic plasticity model is suggested and discussed. In a first step the Mises-Hill anisotropy tensor - which specifies the quadratic flow potential - is expressed in terms of the the 4th-order moment tensor of the codf. It is well known that specific anisotropies of polycrystalline metals generally cannot be modeled by quadratic flow potentials. Motivated by this fact the concept of anisotropic equivalent stress measures is generalized by incorporating the higher-order moment tensors in a second step. The time-evolution of the moment tensors is modeled based on a rigid-viscoplastic Taylor type model. Other homogenization schemes, for example a rigid-viscoplastic self-consistent one, could also be used for that purpose. In this abstract it is sketched how the Mises-Hill anisotropy tensors can be related to the 4th-order moment tensor of the codf.

2 TENSORIAL FOURIER EXPANSION OF THE CODF

For the subsequent considerations it is assumed that the codf $f(\boldsymbol{Q})$ is square integrable. This property implies the existence of a tensorial Fourier expansion. For aggregates of cubic crystals the Fourier expansion has the following form [2, 3]

$$f(\boldsymbol{Q}) \;=\; 1+\sum_{i=1}^{\infty} f_{\alpha_i}(\boldsymbol{Q}), \tag{1}$$

where $f_{\alpha_i} = \mathbb{V}'_{\langle\alpha_i\rangle} \cdot \mathbb{F}'_{\langle\alpha_i\rangle}(\boldsymbol{Q})$, $\mathbb{F}'_{\langle\alpha_i\rangle}(\boldsymbol{Q}) = \boldsymbol{Q} \star \mathbb{T}'_{\langle\alpha_i\rangle}$ and $\{\alpha_i\} = \{4, 6, 8, 9, 10, 12_1, 12_2, \ldots\}$. The $\mathbb{V}'_{\langle\alpha_i\rangle}$ are called tensorial Fourier coefficients or texture coefficients. The tensors $\mathbb{T}'_{\langle\alpha_i\rangle}$ are called reference tensors, which are normalized without loss of generality $\|\mathbb{T}'_{\langle\alpha_i\rangle}\| = 1$. The $\star$ denotes the rotation of a tensor. The crucial point here is that the quantities $\mathbb{V}'_{\langle\alpha_i\rangle}$ and $\mathbb{T}'_{\langle\alpha_i\rangle}$ are completely symmetric and traceless tensors. E.g., the following relations hold for $\mathbb{V}'_{\langle 4\rangle}$

$$V'_{ijkl} = V'_{jikl} = V'_{klij} = V'_{kjil} = \ldots, \quad V'_{iikl} = 0. \tag{2}$$

Completely symmetric and traceless tensors are called irreducible. An irreducible tensor $\mathbb{V}'_{\langle\alpha_i\rangle}$ has $\dim(\mathbb{V}'_{\langle\alpha_i\rangle}) = 2\alpha_i + 1$ independent components. In the case of a cubic crystal symmetry the independent components of $\mathbb{T}_{\langle 4\rangle}$ are given by

$$\begin{array}{lllllllllllllll} T^4_1 &=& T'_{1111} &=& 2a, & T^4_2 &=& T'_{1112} &=& 0, & T^4_3 &=& T'_{1113} &=& 0, \\ T^4_4 &=& T'_{1122} &=& -a, & T^4_5 &=& T'_{1123} &=& 0, & T^4_6 &=& T'_{1222} &=& 0, \\ T^4_7 &=& T'_{1223} &=& 0, & T^4_8 &=& T'_{2222} &=& 2a, & T^4_9 &=& T'_{2223} &=& 0, \end{array} \tag{3}$$

with $a = 1/\sqrt{30}$. For a set of N discrete crystal orientations and corresponding volume fractions $\{\boldsymbol{Q}_\beta, \nu_\beta\}$ $(\beta = 1, \ldots, N)$ the texture coefficients can be approximated by

$$\mathbb{V}'_{\langle\alpha_i\rangle} = (2\alpha_i + 1)\sum_{\beta=1}^{N} \nu_\beta \boldsymbol{Q}_\beta \star \mathbb{T}'_{\langle\alpha_i\rangle}. \tag{4}$$

3 TEXTURE RELATED FLOW POTENTIALS

The introduction of a quadratic yield function dates back to Mises [5] who introduced a general 4th-order tensor of plastic moduli that establishes a quadratic yield condition in terms of stresses. Viscoplastic large strain material models can be based on a flow potential in terms of an equivalent stress σ_e

$$\Phi = \frac{\sigma_0 \dot{\varepsilon}_0}{m+1} \left(\frac{\sigma_e}{\sigma_0} \right)^{m+1} . \tag{5}$$

If the equivalent stress is identified with $\sigma_e = \sqrt{\frac{3}{2}} \|\boldsymbol{\tau}'\|$ where $\boldsymbol{\tau}'$ is the Kirchhoff stress tensor, the potential is an isotropic function of stress and its gradient is coaxial and proportional to $\boldsymbol{\tau}'$. If the equivalent stress is more generally defined by

$$\sigma_e = \sqrt{\frac{3}{2}} \|\boldsymbol{\tau}'\| \left(1 + \frac{\eta_4}{2} \mathbb{V}'_{\langle 4 \rangle} \cdot (\boldsymbol{N}'_\tau \otimes \boldsymbol{N}'_\tau) + \frac{\eta_6}{3} \mathbb{V}'_{\langle 6 \rangle} \cdot (\boldsymbol{N}'_\tau \otimes \boldsymbol{N}'_\tau \otimes \boldsymbol{N}'_\tau) + \ldots \right) , \tag{6}$$

where $\boldsymbol{N}'_\tau$ denotes the direction of $\boldsymbol{\tau}'$, then an anisotropic material behavior can be modeled. For a nontextured aggregate, i.e. $\mathbb{V}'_{\langle \alpha_i \rangle} = \mathbb{O}$, the classical isotropic v. Mises yield condition is obtained. If the texture is known the $\mathbb{V}'_{\langle \alpha_i \rangle}$ can be computed. The η_i are the phenomenological parameters of the ansatz, which are restricted by the fact that σ_e is a norm of the stress tensor $\boldsymbol{\tau}'$. If only the 4th-order moment tensor is taken into account then one the scalar parameter η_4 has to be identified. This can be done by only one experimental yield stress. For fixed plastic moduli anisotropic quadratic forms are common in plasticity, the novelty here is the link to the codf using moment tensors and the inclusion of higher-order structural or moment tensors.

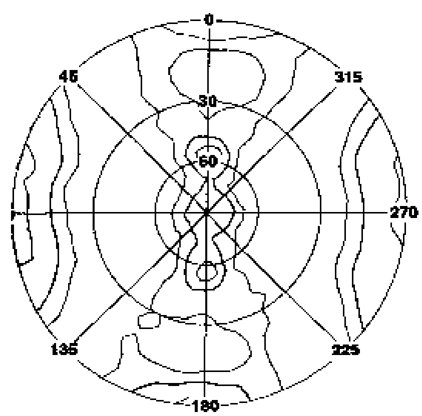

Figure 1: Experimental pole (200) figure Al 2008-T4 automotive sheet sample.

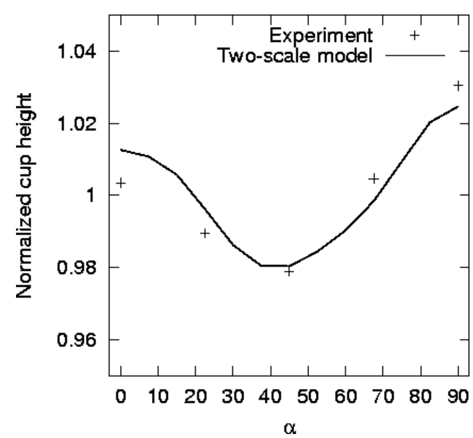

Figure 2: Experimental and simulated earing profile (α: angle from rolling direction).

4 Numerical Example

Lege et al. [4] documented the initial texture and initial yield loci in an Al 2008-T4 automotive sheet sample. The initial texture (Fig. 1 shows the (200) pole figure) can be approximated by 4 texture components and corresponding volume fractions. The Euler angles specifying the texture components and the volume fractions are given in Table 1. In order to ensure an orthorhombic sample symmetry, each texture component is represented by 4 discrete crystal orientations. The texture evolution is taken into account by using a rigid-viscoplastic Taylor model to compute the orientation change of the texture components in each time increment. In the mesoscale model elastic strains can be neglected since only the orientational information is transfered from the grain scale to the macroscale. In this example only the moment tensor $\mathbb{V}'_{\langle 4\rangle}$ is considered. Since $\mathbb{V}'_{\langle 4\rangle}$ is known from the texture, only η_4 has to be determined. This is done by using the measured yield stress in the sheet plane giving $\eta_4 \approx 0.06$. The finite viscoplasticity model has been implemented in ABAQUS using the user routine UMAT. For the finite element simulation of the deep drawing process the elements C3D8H and C3D6H have been used.

i	ν_i	φ_1^i	Φ^i	φ_2^i
1	0.248	1.5532	1.5532	6.2656
2	0.298	0.2564	1.4347	5.7036
3	0.153	0.4664	1.5334	6.0412
4	0.038	1.5549	1.5523	5.9513

Table 1: Parameters specifying the initial texture

5 CONCLUSIONS

It has been discussed that the identification of anisotropy tensors of macroscopic material models with tensorial texture coefficients yields a versatile class of models which is able to describe anisotropy phenomena specific to polycrystalline metals.

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