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A FIELD DATA TRANSFER PROCEDURE

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Summary. *Analysis of mechanical structures using Finite Element Method in the framework of large elastoplastic strain needs frequently the remeshing of the deformed domain during the computation due to the large geometrical distortion of finite elements or to the adaptation to the physical behavior of the solution. After the remeshing of the computational domain, the different mechanical fields associated with the new mesh of the domain must be interpolated from those associated with the old mesh of the domain which allows us to rerun the computation starting from the new mesh. In this paper, we emphasize the problem of interpolation of the mechanical fields after remeshing and we propose a new field data interpolation method based on the minimisation of the finite element error between the old field and the new one. The method is implemented in two and three dimensions. A numerical example is given to show the efficiency of our approach.*

1 INTRODUCTION

Various kinds of field interpolation techniques were developed and used in these last years. The commun objective of these techniques is to generate a new field data that satisfy mechanical equations verified by the old one. One of these techniques is that used by Ortiz *et al.*[1] in which he formulates three sets of field along the lines of the Hu Washizu variational construct and then introduces some approximations to reduce the weak problem statement to the standard one of weak equilibrium enforcement. We can cite also the super convergent patch recovery technique introduced essentially by Zhu [2] or the method based on the Inverse Isoparametric Mapping technique [3],[4] which provide accurate results but still expensive and computationally time consuming. Constrained and unconstrained optimisation based approaches [5],[6] can be considered here as a new kind of interpolation methods. These ones aims to minimize the gap between the mechanical field associated with the new mesh and that associated with the old one. These methods are of great interest in the analysis of mechanical structures using F.E. Method with remeshing because of their capability to generate a more accurate new field associated with a new mesh and so permitting to rerun the computation with minimum errors.

In this paper, we propose an optimisation based algorithm to minimize the norm of the finite element error between the mechanical field associated with a mesh of computation domain and the searched field associated with a new mesh of the domain. The L^2 and/or H^1 norms are used to quantify this error. The next section describes briefly the idea of our methodology. And we present in the last section a numerical example to show the pertinence of our approach.

2 The main idea of the proposed method

Let U^{Old} be the mechanical field associated with a P^1 mesh τ^{Old} of the computational domain Ω. Consider then a new P^1 mesh τ^{New} of the domain. The problem is to determine the mechanical field U^{New} associated with this new mesh such that the finite element field (τ^{Old},U^{Old}) is similar to the finite element field (τ^{New},U^{New}). The similarity can be quantified by the finite element error between these two fields. The L^2-norm of this error is defined by :

$$\left\|\varepsilon(U^{New})\right\|^2_{L^2_\Omega} = \int_\Omega \left\|U^{New}(X) - U^{Old}(X)\right\|^2 d\Omega \tag{1}$$

The above error can be expressed as :

$$\left\|\varepsilon(U^{New})\right\|^2_{L^2_\Omega} = \sum_{e\in\tau^{New}} \int_e \left\|U^{New}(X) - U^{Old}(X)\right\|^2 de \tag{2}$$

To evaluate the integral over element e, we have to determine the intersection of the two meshes τ^{Old} and τ^{New}. To avoid this expensive calculation, we consider an approximative computation of this integral which consist in partitioning element e on several sub-elements for which the integral is computed using their Gauss points (the function $\left\|U^{New}(X) - U^{Old}(X)\right\|$ is assumed to be constant on these sub-elements). The expression (2) can so be written as :

$$\left\|\varepsilon(U^{New})\right\|^2_{L^2_\Omega} = \sum_{e\in\tau^{New}} \sum_{X_g\in e} \omega_g \left(U^{New}(X_g) - U^{Old}(X_g)\right)^2 \tag{3}$$

where X_g is the Gauss point associated with each sub-element of the partition of e and ω_g is the corresponding weight (i.e. volume of the sub-element).
By minimizing this norm $\left\|\varepsilon(U^{New})\right\|^2_{L^2_\Omega}$, the field U^{New} is uniquely determined and the similarity between the old and new field is guaranteed. In order to obtain the similarity also for the derivatives of the fields, we can consider the H^1-norm of the FE error defined by :

$$\left\|\varepsilon(U^{New})\right\|^2_{H^1_\Omega} = \left\|\varepsilon(U^{New})\right\|^2_{L^2_\Omega} + \sum_{i=1}^{Dim} \left(\int_\Omega \left\|U^{New}_{,x_i}(X) - U^{Old}_{,x_i}(X)\right\|^2 d\Omega \right) \tag{4}$$

where $U^{New}_{,x_i}$ and $U^{Old}_{,x_i}$ are respectively the derivatives of the fields U^{New} and U^{Old} with respect to coordinate x_i, $i = 1, ..,$*Dim* being the dimension of the considered domain.

In the above minimization problems, the field at the boundary Γ of Ω is obtained by considering the error over the domain. To more accurately determining the field at the boundary Γ, we can consider the new L^2 or H^1-norm of the error given below which takes into account separately the field at the boundary Γ :

$$\left\|\varepsilon(U^{New})\right\|^2_{L^2_\Omega,L^2_\Gamma} = \frac{1}{mes(\Omega)}\left\|\varepsilon(U^{New})\right\|^2_{L^2_\Omega} + \frac{1}{mes(\Gamma)}\left\|\varepsilon(U^{New})\right\|^2_{L^2_\Gamma} \tag{5}$$

$$\left\|\varepsilon(U^{New})\right\|^2_{H^1_\Omega,L^2_\Gamma} = \frac{1}{mes(\Omega)}\left\|\varepsilon(U^{New})\right\|^2_{H^1_\Omega} + \frac{1}{mes(\Gamma)}\left\|\varepsilon(U^{New})\right\|^2_{L^2_\Gamma} \tag{6}$$

with : $\left\|\varepsilon(U^{New})\right\|_{L^2_\Gamma}^2 = \int_\Gamma \left\|tr(U^{Old}) - tr(U^{New})\right\|^2 d\Gamma.$
$mes(\Omega)$ and $mes(\Gamma)$ represent here respectively the measure of Ω and Γ and "tr" is the trace operator referring to Γ and which consists on considering the field only at the boundary.

3 Numerical example

we consider a two dimensional domain representing a circle with a radius $r = 3$ and centered at the origin. Two meshes of this domain are generated using The BL2D-V2 mesher [7]. The first mesh (Old) is composed of 146 nodes and 252 triangular elements and the second one (New) is constituted by 2248 nodes and 4343 triangular elements (see fig.1 and 2). The nodal values of the field associated with the old mesh is obtained using an analytical function : $U^{Old}(x, y) = x^2 + y^2$, where x and y are the spacial coordinates of the old-mesh nodes (see fig.3). Figure (4) shows the conventional linear interpolation. Figures (5-8) show the different proposed field interpolation. Notice that the L^2_Ω method present instability on the boundary. This instability does not occurs in the L^2_Ω-L^2_Γ method. The H^1 method, like L^2_Ω method smooth the finite element field and can reduce the field amplitude. As an indication, if we consider the L^2_Ω-L^2_Γ norm, the error norms of fields of figures (4-8) are respectively : 0.037, 0.162, 0.027, 0.141, 0.08.

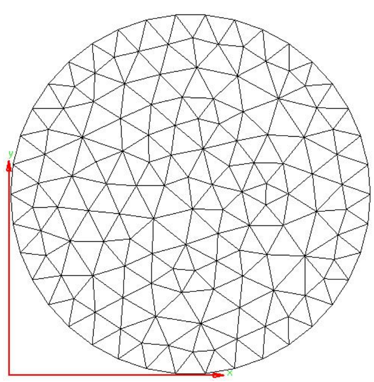

FIG. 1: Old mesh τ^{Old}

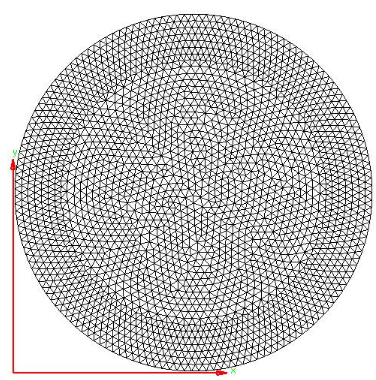

FIG. 2: New mesh τ^{New}

4 Conclusions and future prospects

A new method to interpolate mechanical field is introduced. The method is based on the minimization of finite element error between the initial mechanical field and the interpolated field. Its implementation is remarkably simple. Numerical exemple is given to show its efficiency. The integration of the proposed approach in an adaptive mechanical computation scheme is under development.

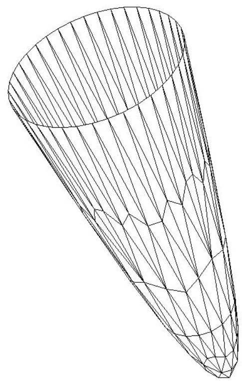

FIG. 3: Field U^{Old}

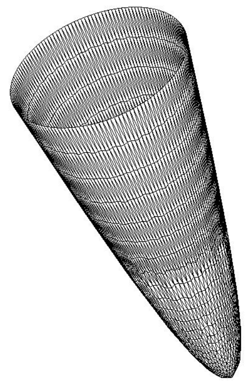

FIG. 4: Field U^{New} using linear interpolation

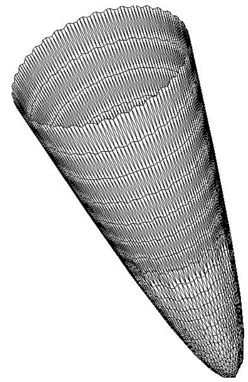

FIG. 5: Field U^{New} using minimisation of L^2_Ω-norm of error

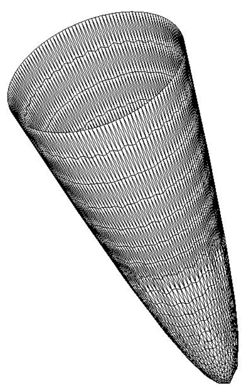

FIG. 6: Field U^{New} using minimisation of (L^2_Ω, L^2_Γ)-norm of error

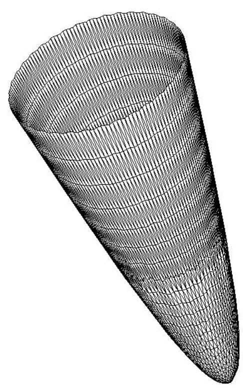

FIG. 7: Field U^{New} using minimisation of H^1_Ω-norm of error

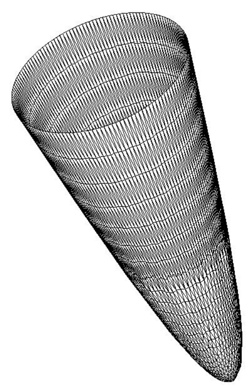

FIG. 8: Field U^{New} using minimisation of (H^1_Ω, L^2_Γ)-norm of error

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