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QUANTIFYING THE LACK OF KNOWLEDGE OF AN INDUSTRIAL MODEL IN STRUCTURAL DYNAMICS

P. Ladevèze[⋆], G. Puel[⋆] and T. Romeuf[†]

[⋆]LMT-Cachan (ENS Cachan/CNRS/Paris 6 University)
61 avenue du Président Wilson, 94235 Cachan Cedex, France
e-mail: {ladeveze,puel}@lmt.ens-cachan.fr, web page: http://www.lmt.ens-cachan.fr

[†]European Aeronautic Defence and Space Company (EADS Space Transportation)
Route de Verneuil BP 96, 78133 Les Mureaux Cedex, France
e-mail: thierry.romeuf@space.eads.net, web page: http://www.eads.net

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1 INTRODUCTION

The quantification of the quality of a structural dynamical model remains a major issue today; with regard to the comparison with an experimental reference, numerous methods have been developed[1, 2], in order to adjust the stiffness and mass properties of dynamic models based on free- or forced-vibration tests. The final result is an updated structural model which, evidently, cannot reproduce the behavior of the family of actual structures under consideration perfectly. In order to deal with these sources of errors, we follow quite a different approach than the classical probabilistic methods[3]. The numerous sources of uncertainties as well as modeling errors are described using the concept of Lack Of Knowledge[4]. The structure being studied is defined as an assembly of substructures E in which the connections can be viewed as special substructures. The LOKs are defined on the substructure level: the basic LOK m_E is a scalar internal variable which quantifies the substructure's LOK state. The LOK theory is herein developed with particular emphasis on two aspects: the meaning of what we call a structural model with LOKs, with its impact on the prediction of the structural response, and the reduction of the basic LOKs using additional information.

2 BASIC LACKS OF KNOWLEDGE

We associate with each substructure E of an actual structure Ω a LOK variable m_E defined over an interval whose bounds $m_\mathrm{E}^+(\theta)$ and $m_\mathrm{E}^-(\theta)$ are formally defined as follows:

$$\left(1 - m_\mathrm{E}^-(\theta)\right) \overline{\mathbf{K}}_\mathrm{E} \leq \mathbf{K}_\mathrm{E} \leq \left(1 + m_\mathrm{E}^+(\theta)\right) \overline{\mathbf{K}}_\mathrm{E} \tag{1}$$

$\overline{\mathbf{K}}_\mathrm{E}$ designates the stiffness matrix of the FE model being used. $\mathbf{K}_\mathrm{E}$ is the stiffness matrix of an actual structure belonging to the family under consideration. In this formal expression, the inequalities can be considered to hold for the eigenvalues associated with these matrices. In practice, these inequalities are expressed using the strain energies.

The quantities $m_E^+(\theta)$ and $m_E^-(\theta)$ are scalar internal variables of Substructure E, which constitute the *basic LOK*. For each substructure, the LOK m_E lies within the interval $[-m_E^-; m_E^+]$, and the bounds of this interval are characterized by probabilistic laws whose nature (Gaussian, uniform, ...) is defined *a priori*.

3 STRUCTURAL MODELING WITH LOKS

Let us recall that we are considering only LOKs of the stiffness type. The basic LOKs are assumed to be known, and so is the FE operator $\overline{\mathbb{A}}$ used to calculate the response $\overline{s}$ of the structure over the time-space domain: $\overline{s} = \overline{\mathbb{A}}\left(\overline{\mathbf{K}}_E; E \in \Omega\right)$; $\{\overline{\mathbf{K}}_E, E \in \Omega\}$ should be viewed as parameters. One assumes that the previous relation holds for any actual structure belonging to the family under consideration if the stiffness matrices are replaced by the actual stiffnesses: $s = \overline{\mathbb{A}}(\mathbf{K}_E; E \in \Omega)$. As the relation between the actual stiffness and the FE stiffness is established through the basic LOKs (1), we are defining an envelope of the actual responses. Let us consider a scalar quantity of interest $\alpha = \widehat{\alpha}(s)$ where $\widehat{\alpha}$ is a given operator. Let us define $\Delta\alpha_{\text{mod}} = \alpha - \overline{\alpha}$ where $\overline{\alpha} = \widehat{\alpha}(\overline{s})$. For the quantity of interest α, one defines the following envelope of the possible actual responses:

$$-[\Delta\alpha_{\text{mod}}^-](\theta) \leq [\Delta\alpha_{\text{mod}}] \leq [\Delta\alpha_{\text{mod}}^+](\theta) \tag{2}$$

For instance, $[\Delta\alpha_{\text{mod}}^+](\theta)$ is determined from the problem:

$$\Delta\alpha_{\text{mod}}^+ = \max_{\substack{-m_E^-\overline{\mathbf{K}}_E \leq \mathbf{K}_E - \overline{\mathbf{K}}_E \leq m_E^+\overline{\mathbf{K}}_E \\ E\in\Omega}} \widehat{\alpha}\left(\overline{\mathbb{A}}(\mathbf{K}_E; E \in \Omega)\right) - \overline{\alpha} \tag{3}$$

Consequently, $\Delta\alpha_{\text{mod}}^+$ can formally be expressed as $\Delta\alpha_{\text{mod}}^+ = \mathbb{Z}^+\left(m_E^+, m_E^-; E \in \Omega\right)$, hence:

$$[\Delta\alpha_{\text{mod}}^+](\theta) = \mathbb{Z}^+\left(m_E^+(\theta), m_E^-(\theta); E \in \Omega\right) \tag{4}$$

The resolution of Problems (3) and (4) is not very difficult, especially if the basic LOKs are small, in which case linearization procedures can be used.

4 IDENTIFICATION AND REDUCTION OF THE BASIC LOKS

Let us consider a structural model with LOKs giving for the quantity of interest α the bounds $[\Delta\alpha_{\text{mod}}^+](\theta), [\Delta\alpha_{\text{mod}}^-](\theta)$. From the distributions of these bounds, we can define the interval $I_{\Delta\alpha_{\text{mod}}}(P)$ as the smallest interval such that $P(\Delta\alpha_{\text{mod}} \in I_{\Delta\alpha_{\text{mod}}}(P))$ is greater than a given percentage P. The bounds of this interval $I_{\Delta\alpha_{\text{mod}}}(P)$, denoted $\Delta\alpha_{\text{mod}}^-(P)$ and $\Delta\alpha_{\text{mod}}^+(P)$, constitute what we call the *effective LOK* on the quantity of interest α. Regarding the family of actual structures, one can determine two values $\Delta\alpha_{\text{exp}}^-(P)$ and $\Delta\alpha_{\text{exp}}^+(P)$ which, for a given probability P, contain $P\%$ of the values of the experimental quantity of interest $\Delta\alpha_{\text{exp}}$. Then one can compare the experimental data with the values given by the LOK model; in order the LOK model to be determined conservatively, we say that the basic LOKs must be such that:

$$\Delta\alpha_{\text{exp}}^-(P) \leq \Delta\alpha_{\text{mod}}^-(P) \text{ and } \Delta\alpha_{\text{exp}}^+(P) \leq \Delta\alpha_{\text{mod}}^+(P) \tag{5}$$

The main idea behind the reduction process of the basic LOKs is that the greater the amount of experimental information available, the more likely one is to reduce the basic LOKs. This principle requires an initial description, which may be coarse but must necessarily be overestimated, of the basic LOKs for each substructure. A set of initial basic LOKs $(m_{\mathrm{E}}^{+0}(\theta), m_{\mathrm{E}}^{-0}(\theta))_{E\in\Omega}$, such that all the constraints (5) are verified for the experimental data available, is obtained. In our present approach, which is conservative, the reduction process consists in using this additional relevant experimental information to reduce the level of LOK one substructure at a time. Let us consider a particular substructure E^*. The problem is to determine a basic LOK $(m_{\mathrm{E}^*}^{-}(\theta), m_{\mathrm{E}^*}^{+}(\theta))$ smaller than the initial LOK $(m_{\mathrm{E}^*}^{-0}(\theta), m_{\mathrm{E}^*}^{+0}(\theta))$, under Constraint (5) associated with the experimental information chosen. Since the reduction is carried out one substructure at a time, the verification of Constraint (5) is not sufficient to guarantee realistic results in all situations. Therefore we build worst-case estimates of $[\Delta\alpha_{\mathrm{mod}}^{+}](\theta)$ and $[\Delta\alpha_{\mathrm{mod}}^{-}](\theta)$, denoted $[\Delta\alpha_{\mathrm{mod}}^{+\,\mathrm{worst}}](\theta)$ and $[\Delta\alpha_{\mathrm{mod}}^{-\,\mathrm{worst}}](\theta)$, such that the associated effective LOKs verify:

$$\Delta\alpha_{\mathrm{exp}}^{-}(P) \leq \Delta\alpha_{\mathrm{mod}}^{-\,\mathrm{worst}}(P) \text{ and } \Delta\alpha_{\mathrm{exp}}^{+}(P) \leq \Delta\alpha_{\mathrm{mod}}^{+\,\mathrm{worst}}(P) \qquad (6)$$

5 STUDY OF AN INDUSTRIAL CASE

Now, let us present the application of our method to an actual industrial structure: the Sylda5 satellite support, developed by the EADS Group, which is capable of carrying two satellites simultaneously (Figure 1). Free-vibration measurements with 260 sensors were carried out by IABG on behalf of DASA/DORNIER under contract with CNES. The model proposed by EADS represents both the support itself and a cylindrical payload which simulates the presence of a satellite resting on the support. As the initial measurements had shown that it was essential to take into account the deformation of the ground under the support, it was modeled very simply using 3 torsional springs, one translational spring and a rigid-body constraint for all interface nodes between the ground and the support. The final model consisted of 27,648 DOFs and 9,728 elements.

Figure 1: Photograph of the Sylda5 support

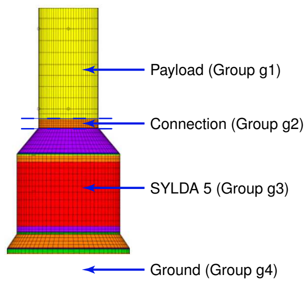

Figure 2: The associated Sylda5 model

First, the model was adjusted using the method described in [2], based on the first 12 modes. The problem was then to determine the remaining LOKs. In order to do that, the structure was divided into 4 main groups of substructures, as described in Figure 2.

The reduction process was initiated by setting *a priori* initial overestimated LOK levels $(\overline{m}_{\mathrm{E}}^{-0}, \overline{m}_{\mathrm{E}}^{+0})$ (and their corresponding laws). The experimental information consisted of the eigenfrequencies of measurements on the actual structure, which are considered as the extreme values that would be obtained if several similar structures had been tested. Table 1 indicates the order in which, and the data with which, the reduction was carried out, just as the results of the process. These results confirm the good quality of the adjusted model of the support ($g3$) and of the model of the connector (both within a few %), whereas the oversimplifications in the model of the ground resulted in a high LOK. In order to evaluate the quality of the results of the reduction process, one can calculate the effective LOK for Mode 1 which was unused and compare it with the corresponding experimental values in Table 2: the results obtained with the other modes are consistent.

Groups	Experimental data	LOKs: laws	LOKs: ranges	LOKs: statistical moments
$g3$	$(\Delta\omega^{2\,+}_{4\,\mathrm{exp}}(0.99), \Delta\omega^{2\,-}_{4\,\mathrm{exp}}(0.99))$	Gaussian	$[-0.016; 0.000]$	$\mu = -0.008 \;/\; \sigma = 0.003$
$g1$	$(\Delta\omega^{2\,+}_{8\,\mathrm{exp}}(0.99), \Delta\omega^{2\,-}_{8\,\mathrm{exp}}(0.99))$	Gaussian	$[0.000; 0.144]$	$\mu = 0.072 \;/\; \sigma = 0.028$
$g4$	$(\Delta\omega^{2\,+}_{6\,\mathrm{exp}}(0.99), \Delta\omega^{2\,-}_{6\,\mathrm{exp}}(0.99))$	uniform	$[0.000; 0.521]$	$\mu = 0.261 \;/\; \sigma = 0.150$
$g2$	$(\Delta\omega^{2\,+}_{3\,\mathrm{exp}}(0.99), \Delta\omega^{2\,-}_{3\,\mathrm{exp}}(0.99))$	uniform	$[-0.060; 0.000]$	$\mu = -0.030 \;/\; \sigma = 0.012$

Table 1: Reduced basic LOKs (μ: mean value / σ: standard deviation)

i	$\overline{\omega}_i^2 + \Delta\omega^{2\,-}_{i\,\mathrm{mod}}$	$\overline{\omega}_i^2 + \Delta\omega^{2\,-}_{i\,\mathrm{exp}}$	$\overline{\omega}_i^2$	$\overline{\omega}_i^2 + \Delta\omega^{2\,+}_{i\,\mathrm{exp}}$	$\overline{\omega}_i^2 + \Delta\omega^{2\,+}_{i\,\mathrm{mod}}$
1	$1.01\,10^3$	$1.02\,10^3$	$1.02\,10^3$	$1.05\,10^3$	$1.06\,10^3$

Table 2: Comparison of 99%-values for Mode 1

6 CONCLUSION

The Lack-Of-Knowledge theory enables one to quantify the uncertainties on the substructure level using quantities of interest defined over the whole structure. The reduction process presented here enables the determination of the basic LOKs for each substructure starting from *a priori* assumptions on their bounds. In order to do that, the experimental data are considered to be information which reduces the LOK on the structure. These investigations constitute a first step toward a general method of reduction of the LOKs.

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