VIII International Conference on Computational Plasticity
COMPLAS VIII
E. Oñate and D.R.J. Owen (Eds)


# MODELLING OF COHESIVE AND NON-COHESIVE CRACKS VIA X-FEM BASED ON GLOBAL ENERGY CRITERIA

**P. Dumstorff and G. Meschke***

Institute for Structural Mechanics
Ruhr-Universität Bochum
Universitätsstr. 150 D-44780 Bochum, Germany
e-mails : peter.dumstorff@rub.de, guenther.meschke@rub.de, Web page: http://www.sd.rub.de

**Key words:** X-FEM, discrete crack model, cohesive crack, crack growth

**Summary.** *In this paper, the Extended Finite Element Method [1] is used for modelling cohesive and non-cohesive cracks in brittle materials. For the determination of the direction of crack propagation as well as of the length of new crack segments a global energy based criterion is used. The numerical assessment of the model includes comparisons of crack paths obtained from non-cohesive analyses and from analyses using stress intensity factors. Furthermore, the influence of the interface law is investigated.*

## 1 INTRODUCTION

Numerical analysis of cracks in quasi-brittle materials require models which adequately represent the discontinuous character of the problem. In this paper, the Extended Finite Element Method [1] is used for modelling cohesive as well as non-cohesive cracks in brittle materials. Numerous investigations by different authors have shown that the Extended Finite Element Method is capable of simulating cracking and crack propagation independent of the discretization of the finite element mesh [1, 2, 3]. The X-FEM was first introduced in the context of linear elastic fracture mechanics [1] and has been extended for the modelling of cohesive cracks [3, 2].

In the proposed model, cohesive cracks are considered by introducing modified enrichment functions to enhance the resolution of the displacement field in the vicinity of the crack tip and by the Sign function if elements are fully penetrated by cracks. In contrast to the enrichment functions used in Linear Elastic Fracture Mechanics [1], the proposed functions do not exhibit stress singularities but yield bounded values of stresses at the crack tip.

The analysis of crack propagation using discrete crack models crucially depends on the crack growth criterion. Incorrect predictions of the crack propagation direction may lead to locking and, consequently, to unreasonable results. In this paper, a global energy based criterion, proposed in [4], is extended to determine simultaneously the direction of crack propagation and the length of new crack segments. Starting from a variational formulation in terms of the displacements, the direction and the length of new crack segments,

a coupled format of the discretized tangential algebraic equation system analogous to multifield problems is solved simultaneously by means of the NEWTON iteration scheme.

## 2 EXTENDED FINITE ELEMENT MODEL

The displacement field $\boldsymbol{u}$ of a cracked body $\mathcal{B}$ can be decomposed into a continuous part $\bar{\boldsymbol{u}}$ and a discontinuous part $\check{\boldsymbol{u}}$

$$\boldsymbol{u}(\boldsymbol{x}) = \bar{\boldsymbol{u}}(\boldsymbol{x}) + \check{\boldsymbol{u}}(\boldsymbol{x}), \quad \forall\, \boldsymbol{x} \in \Omega, \quad \text{with} \quad \check{\boldsymbol{u}}(\boldsymbol{x}) = S_s(\boldsymbol{x})\, \hat{\boldsymbol{u}}(\boldsymbol{x}), \tag{1}$$

where $\bar{\boldsymbol{u}}$ and $\hat{\boldsymbol{u}}$ are continuous functions in the domain $\Omega$ and $S_s$ is the Sign function defined on the crack surface $\partial_s\Omega$ [1].

In the X-FEM the Partition of Unity Method is used to locally enhance the displacement approximation where a crack has opened. In the presented formulation not only the Sign function but also the crack tip functions $\{F_i\}$, as proposed by [1], are used to enhance the approximation of the displacement field in the cracked solids:

$$\{F_i(\theta, r)\} = \left\{ f(r)\cos\frac{\theta}{2}\,\sin\theta,\; f(r)\sin\frac{\theta}{2}\,\sin\theta,\; f(r)\cos\frac{\theta}{2},\; f(r)\sin\frac{\theta}{2} \right\}. \tag{2}$$

$\theta$ and $r$ are the local polar coordinates at the crack tip. Using crack tip enhancement functions, crack tips do not have to be located on element boundaries, but can be located arbitrarily in the finite element mesh. For the simulation of non-cohesive crack growth the function $f(r)$ is defined as $f(r) = \sqrt{r}$ [1]. In contrast, for the simulation of cohesive crack growth, characterized by bounded stress states at the crack tip, a linear function $f(r) = r$ is used [4, 5]. An interface law as suggested by [3] is employed for the cohesive crack model.

Using standard finite element shape functions as a partition of unity, a finite element approximation which includes three parts - the standard finite element approximation, the Sign function $S_s$ and the crack tip enhancement functions $\{F_i\}$ - can be written as

$$\boldsymbol{u} = \sum_{i=1}^{n_r} N_i\, \boldsymbol{u}_i^{er} + \sum_{i=1}^{n_c} N_i S_s\, \boldsymbol{u}_i^{ec} + \sum_{i=1}^{n_t}\sum_{j=1}^{4} N_i F_j\, \boldsymbol{u}_{ij}^{et}, \tag{3}$$

where $\boldsymbol{u}_i^{er}$ are the regular degrees of freedom, $\boldsymbol{u}_i^{ec}$ are the enhanced degrees of freedom associated with the Sign function $S_s$ and $\boldsymbol{u}_i^{et}$ are the enhanced degrees of freedom associated with the crack tip enhancement functions $F_j$.

## 3 ENERGY BASED MODELLING OF CRACK PROPAGATION

The basic assumption of the proposed energy based crack model is that the fracture process is governed by a minimization of the total potential energy $\Pi$ of the structure. The necessary condition for a minimum of the total potential energy is that the first variation vanishes:

$$\delta\Pi(\boldsymbol{u}, r_c, \theta_c) = \frac{\partial\Pi(\boldsymbol{u}, r_c, \theta_c)}{\partial \boldsymbol{u}}\,\delta\boldsymbol{u} + \frac{\partial\Pi(\boldsymbol{u}, r_c, \theta_c)}{\partial r_c}\,\delta r_c + \frac{\partial\Pi(\boldsymbol{u}, r_c, \theta_c)}{\partial \theta_c}\,\delta\theta_c = 0\,. \tag{4}$$

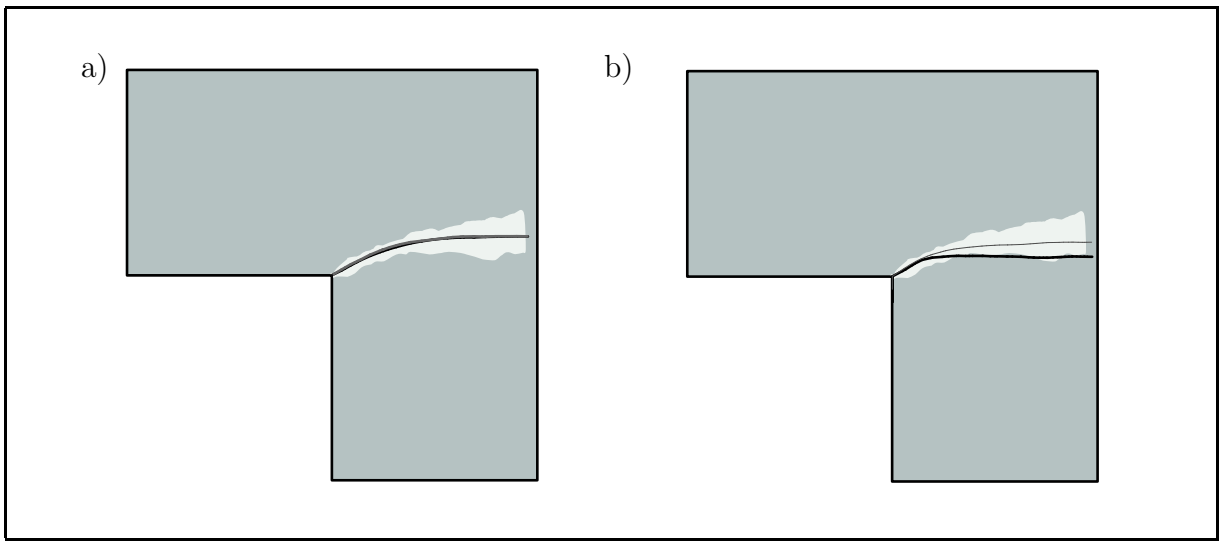


Figure 1: Numerical analysis of an L-shaped panel: a) crack paths from LEFM-based analyses using $K$-factors (thick line) and the proposed energy-based criterion (thin line); b) crack paths from the cohesive crack model using a mode I (thick line) and a mixed mode interface law (thin line)

This leads to a global system of equations which is enhanced by two additional global degrees of freedom associated with the crack growth direction $\theta_c$ and the length of the new crack segment $r_c$. The derivatives of the total potential with respect to the additional degrees of freedom $\theta_c$ and $r_c$ are calculated numerically. In the end the global energy based criterion leads to a coupled stiffness matrix $\boldsymbol{K}$ and the internal load vector $\boldsymbol{f}_{int}$ which have the general format:

$$\boldsymbol{K} = \begin{bmatrix} \boldsymbol{K}^{u\,u} & \boldsymbol{K}^{u\,\theta_c} & \boldsymbol{K}^{u\,r_c} \\ \boldsymbol{K}^{\theta_c\,u} & K^{\theta_c\,\theta_c} & K^{\theta_c\,r_c} \\ \boldsymbol{K}^{r_c\,u} & K^{r_c\,\theta_c} & K^{r_c\,r_c} \end{bmatrix}, \qquad \boldsymbol{f}_{int} = \begin{bmatrix} \boldsymbol{f}^{u} \\ f^{\theta_c} \\ f^{r_c} \end{bmatrix}. \tag{5}$$

## 4 NUMERICAL ASSESSMENT

For the numerical assessment of the proposed method, a L-shaped panel [6] is used as a benchmark example. Two analyses were based upon linear elastic fracture mechanics (LEFM). In one analysis the crack direction was calculated using the stress intensity factors according to LEFM while in the second calculation the proposed global energy criterion has been applied. The calculated crack paths shown in Figure 1a) are almost identical.

Two additional analyses were based upon the cohesive crack model (Figure 1b): In one analysis, no transfer of shear stresses along the crack faces is taken into account. In another analysis, a mixed mode interface law [3] is used. Figure 1b) demonstrates, that the chosen interface law has a considerable effect on the calculated crack path. In contrast

to the mode-I analysis (thick line), the crack path calculated with consideration of the mixed mode-model (thin line) lies perfectly within the range of the experiments.

## 5 CONCLUDING REMARKS

A global energy criterion originally proposed for the determination of the crack propagation direction in the context of the Extended Finite Element Method [4] has been extended to the simultaneous determination of the direction as well as of the length of propagating crack segments. Minimization of the total energy of the structure leads to a coupled problem solved for the displacements, the crack direction and the segment length. It was shown, that for LEFM, the computed crack path is identical with the classical maximum circumferential stress criterion using $K_I$ and $K_{II}$. It also has been shown, that consideration of mixed mode in the interface law is crucial to obtain realistic paths of propagating cracks.

## REFERENCES

[1] N. Moës, J. Dolbow, T. Belytschko. A finite element method for crack growth without remeshing. *Int. Journal for Numerical Methods in Engineering*, **46**, 131–150, 1999.

[2] N. Moës, T. Belytschko. Extended finite element method for cohesive crack growth. *Engineering Fracture Mechanics*, **69**, 813–833, 2002.

[3] G.N. Wells, L.J. Sluys. A new method for modelling cohesive cracks using finite elements. *Int. Journal for Numerical Methods in Engineering*, **50**, 2667–2682, 2001.

[4] G. Meschke and P. Dumstorff. Discontinuous numerical representation of brittle failure. In P.A. Vemeer and W. Ehlers and H.J. Herrmann and E. Ramm, editors, *Continuous and Discontinuous Modelling of Cohesive-Frictional Materials*, 339–352, 2004.

[5] P. Dumstorff and G. Meschke. A global energy criterion for crack propagation using the Extended Finite Element Method. *Proceedings of the International Conference on Fracture (ICF 11)*, 2005.

[6] B.J. Winkler. Traglastuntersuchungen von unbewehrten und bewehrten Betonstrukturen. *Insitut für Baustatik, Festigkeitlehre und Tragwerkslehre, Universität Insbruck*, 2001.