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MICROSCALE FRACTURING OF CRYSTALLINE ROCK UNDER HOMOGENIZED STRONG DISCONTINUITY AND WATER TRANSPORT

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Summary. *A macroscale fracturing process which results from microscale damaging is discussed on a basis of a homogenization theory under the microscale strong discontinuity kinematics. It is also pointed out that the development of microcracks is a mechano-chemical coupling process of the siloxane materials under water existence, that is, a hydrolysis process of silicate minerals.*

1 INTRODUCTION

Microcrack development and propagation controls the fracturing behavior of crystalline rocks (see Figure 1). Oliver[1,2] showed that the onset and development of displacement discontinuities can be represented by the concept of strong discontinuity. We here extend the strong discontinuity model to a microscale one of the homogenization theory[3] and calculate the macroscale behavior.

Seeking for the reason of microcrack propagation under macroscopically constant stress/strain condition, we next consider a mechano-chemical coupling procedure of siloxane dissolution under water existence[4,5], that is, a hydrolysis of silicate minerals, which is schematically denoted as $\equiv \mathrm{Si}-\mathrm{O}-\mathrm{Si}\equiv +\mathrm{H_2O} \quad \Rightarrow \quad \equiv -\mathrm{Si}-\mathrm{OH}+\mathrm{OH}-\mathrm{Si}\equiv$.

2 HOMOGENIZATION OF MICROCRACKS UNDER STRONG DISCONTINUITY

Let us consider a displacement field $\boldsymbol{u}(\boldsymbol{x},t)$ consisting of a continuous part $\bar{\boldsymbol{u}}(\boldsymbol{x},t)$ and a jump $H_\varphi(\boldsymbol{x})[\![\boldsymbol{u}]\!](\boldsymbol{x},t)$ on a singular surface Σ (Figure 2):

$$\boldsymbol{u}(\boldsymbol{x},t)=\bar{\boldsymbol{u}}(\boldsymbol{x},t)+H_\varphi(\boldsymbol{x})[\![\boldsymbol{u}]\!](\boldsymbol{x},t)$$

where $H_\varphi(\boldsymbol{x})$ is the step function such as $H_\varphi = 0\ \forall\boldsymbol{x} \in \Omega^-,\ H_\varphi = 1\ \forall\boldsymbol{x} \in \Omega^+$. Note that for applying the displacement boundary condition for the continuous part $\hat{\boldsymbol{u}}$ of $\bar{\boldsymbol{u}}$ the displacement is regularized as

$$\boldsymbol{u}(\boldsymbol{x},t) = \hat{\boldsymbol{u}}(\boldsymbol{x},t) + M_\varphi^h(\boldsymbol{x})[\![\boldsymbol{u}]\!](\boldsymbol{x},t)$$
$$\hat{\boldsymbol{u}}(\boldsymbol{x},t) = \bar{\boldsymbol{u}}(\boldsymbol{x},t) + \phi^h(\boldsymbol{x})[\![\boldsymbol{u}]\!](\boldsymbol{x},t), \qquad M_\varphi^h(\boldsymbol{x}) = H_\varphi(\boldsymbol{x}) - \phi^h(\boldsymbol{x}).$$

ϕ^h is a continuous function connecting Ω^- and Ω^+ through a neighborhood Ω_h of Σ.

An isotropic damage model with the damage parameter d is introduced after onset of the displacement discontinuity (Figure 3):

$$\dot{\sigma_{ij}} = C_{ijkl}^D \dot{\varepsilon_{kl}}$$
$$\text{Unloading:} \quad \dot{d} = 0, \quad C_{ijkl}^D = (1-d)C_{ijkl}$$
$$\text{Loading:} \quad \dot{d} \neq 0, \quad C_{ijkl}^D = (1-d)\Big(C_{ijkl} - \frac{1}{1+H}\frac{\tau_0}{(\tau^\sigma)^3}\sigma_{ij}\sigma_{kl}\Big)$$

We next apply a homogenization method[2] for the incremental equilibrium equation under a perturbation for the incremental displacement $\dot{u}_i^\varepsilon$ as

$$\dot{u}_i^\varepsilon(\boldsymbol{x}) = \dot{u}_i^0(\boldsymbol{x}^0, \boldsymbol{x}^1) + \varepsilon \dot{u}_i^1(\boldsymbol{x}^0, \boldsymbol{x}^1) + \varepsilon^2 \dot{u}_i^2(\boldsymbol{x}^0, \boldsymbol{x}^1) + \cdots$$

where $\boldsymbol{x}^0$ and $\boldsymbol{x}^1$ are the global and local coordinate systems, respectively.

Substituting this into the governing equation, we get the following microscale equation

$$\frac{\partial}{\partial x_j^1}\Big[C_{ijrs}^D\Big(\delta_{rk}\delta_{sl} - \frac{\partial \hat{\chi}_k^{rs}}{\partial x_l^1} - [\![\chi_k^{rs}]\!]\frac{\partial M_{\varphi k}}{\partial x_l^1}\Big)\Big] = 0$$

where χ_k^{rs} is the characteristic function, and the macroscale equation is obtained as

$$\frac{\partial}{\partial x_j^0}\Big(C_{ijrs}^H \frac{\partial \dot{u}_r^0}{\partial x_s^0}\Big) + \dot{f}^H = 0,$$
$$C_{ijkl}^H = \frac{1}{|\Omega_1|}\int_{\Omega_1} C_{qnpm}^D(\boldsymbol{x}^1)\Big(\delta_{qi}\delta_{nj} - \frac{\partial \chi_q^{ij}}{\partial x_n^1}\Big)\Big(\delta_{pk}\delta_{ml} - \frac{\partial \chi_p^{kl}}{\partial x_m^1}\Big)d\boldsymbol{x}^1.$$

A finite element scheme is introduced for solving the microscale and macroscale problems. A numerical result is given in Figure 4 for the averaged stress and strain response, and in Figure 5 for microcrack propagation in the local unit cell.

3 WATER TRANSPORT

It is known that microcracks in rock are gradually developed even under loading less than 30% of the uniaxial strength. This may be caused by a mechano-chemical coupling procedure of siloxane dissolution under water existence. Thus, the time-delaying of microscale fracturing is mainly controlled by the water transport and stress intensity at the crack tip.

Water transport is written by the Navier-Stokes equation. The partially-saturated water flow in a single crack is discussed by Bui *et al.*[6] together with the stress intensity factor accounting for the moving crack tip and capillary effects. On the other hand, if a homogenization procedure is applied for a porous media flow problem, the seepage equation is eventually obtained[7]. Thus, for the locally distributed crack problem we will be able to develop a homogenized seepage equation together with the mechano-chemical effect. Note that the mechano-chemical-thermal effects for silicate minerals are discussed by Beeler and Hickman[4] for fracture closure behavior of a single crack developed in a quartz crystal, and by Lehner and Leroy[5] for pressure solution of sandstone grains.

4 CONCLUSIONS

- Microscale crack development of crystalline rock can be simulated by a homogenization theory under the microscale strong discontinuity kinematics concept. A numerical example is shown in Figure 4 and 5.
- Time-dependent microcrack development is controlled by a mechano-chemical coupling procedure of siloxane dissolution under water existence. A coupled mechano-chemical-thermal homogenization analysis is required.

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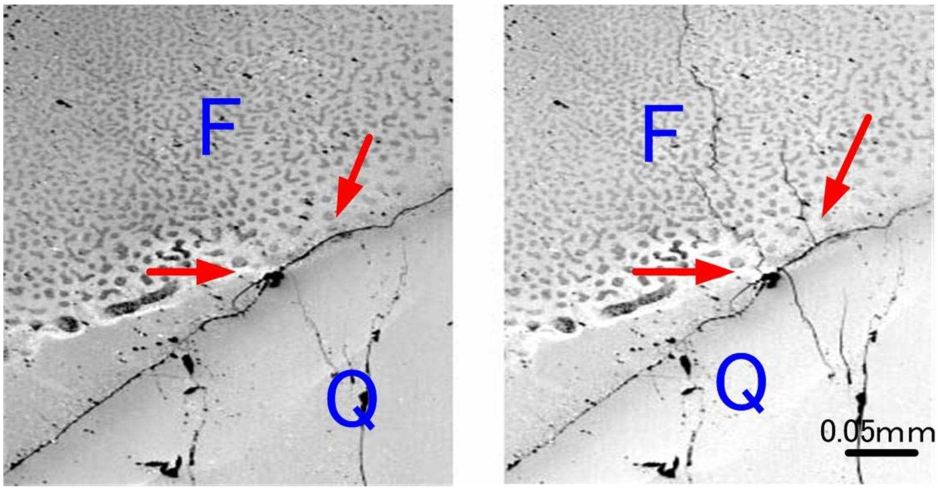

Figure 1: Laser microscope observation of microcrack development in granite before and after loading (Q: quartz, F: feldspar)

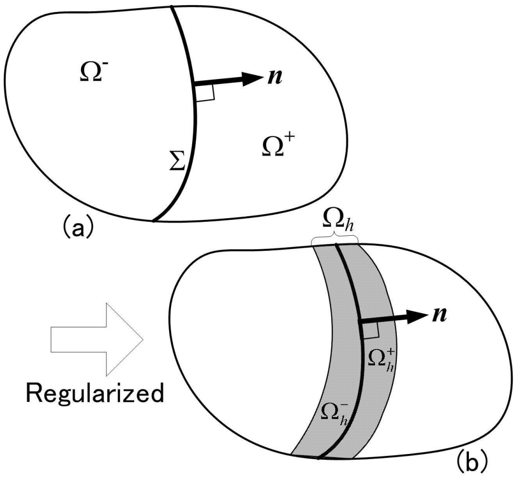

Figure 2: Discontinuity surface Σ.

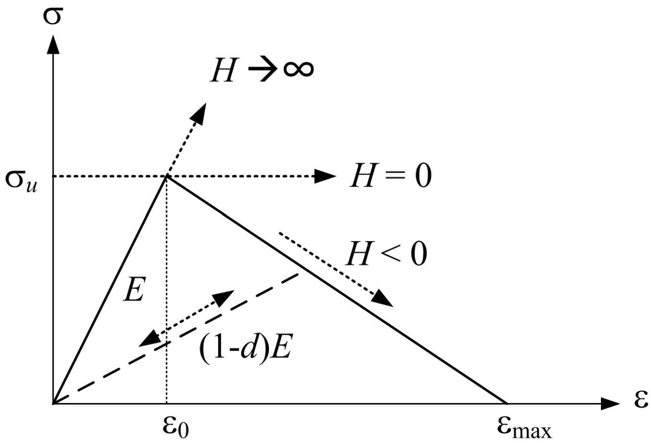

Figure 3: Damage model (Oliver[1])

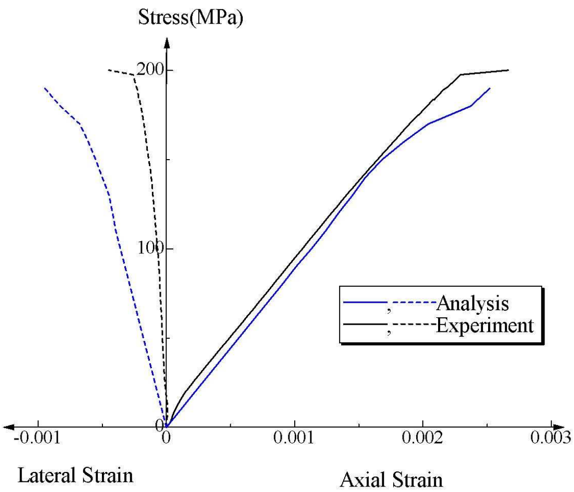

Figure 4: Averaged stress-strain relation.

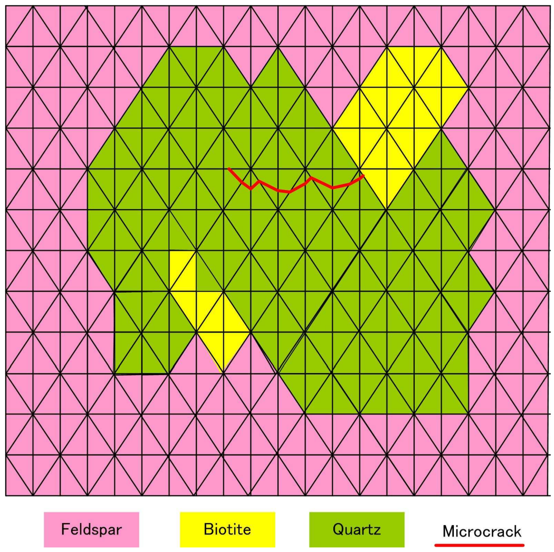

Figure 5: Unit cell and microcrack development.