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METHOD FOR COMPUTING STEADY STATE RESPONSE OF DYNAMICAL SYSTEMS WITH CLEARANCES

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Summary. *The piecewise full decoupling method is a new numerical procedure for determining the frequency response of vibrating systems with clearances. The method is an extension of classical method of piecing the exact solutions. Numerical examples are given to illustrate the both solution procedures.*

1 PROBLEM FORMULATION

A large class of dynamical systems is subjected to relative motion across the clearance space and impacting between the components. The characteristics of a system with clearances include an abrupt variation of stiffness usually assumed as piecewise linear. Generally, such system is modelled as an $n+1$-degree-of-freedom semi-definite system which consists of $n+1$ mass elements, n linear viscous dampers, $m \le n$ piecewise linear stiffness elements and $n-m$ linear springs. The equation of motion, in nondimensional form, can be written as:

$$\boldsymbol{q}'' + \boldsymbol{Z}\boldsymbol{q}' + \boldsymbol{\Omega}\boldsymbol{h}(\boldsymbol{q}) = \boldsymbol{f}_0 + \boldsymbol{f}_a \cos(\eta\tau) \tag{1}$$

where $\boldsymbol{q}$ is the displacement vector, $\boldsymbol{h}(\boldsymbol{q})$ is the nonlinear displacement vector with piecewise linear and linear terms while $\boldsymbol{Z}$ and $\boldsymbol{\Omega}$ are the damping and stiffness matrices, respectively. Furthermore, $\boldsymbol{f}_0$ and $\boldsymbol{f}_a$ are the amplitude vectors of mean and alternating load and η denotes a nondimensional excitation frequency. Piecewise linear stiffness elements are defined by the piecewise linear displacement function as follows:

$$h(q_i) = \begin{cases} q_i + 1, & q_i < -1 \\ 0, & -1 \le q_i \le 1, \quad i \in \{1,\dots,n\} \\ q_i - 1, & q_i > 1 \end{cases} \tag{2}$$

2 THE METHOD OF PIECING THE EXACT SOLUTION

A motivation for developing the new numerical method for solving piecewise linear equations of motion has been the well-known method of piecing the exact solution which is briefly discussed as follows. Considering a two-degree-of-freedom semi-definite system with clearance, the equation (1) yields:

$$q'' + 2\varsigma q' + h(q) = f_0 + f_a \cos(\eta\tau) \tag{3}$$

The analytical solutions of the above equation, for the domains $q < -1$ and $q > 1$ are:

$$q = \mp 1 + \mathrm{e}^{-\varsigma\tau}\left[A_{1,2}\sin(\sqrt{1-\varsigma^2}\,\tau) + B_{1,2}\cos(\sqrt{1-\varsigma^2}\,\tau)\right] + f_0 + \frac{f_a\cos(\eta\tau+\varphi)}{\sqrt{(1-\eta^2)^2+(2\varsigma\eta)^2}} \tag{4}$$

while the solutions for the domain $-1 \le q \le 1$ takes the form:

$$q = \frac{A_3\,\mathrm{e}^{-2\varsigma\tau}}{2\varsigma} + B_3 + \frac{f_0}{2\varsigma}\tau + \frac{f_a}{\eta\sqrt{\eta^2+4\varsigma^2}}\sin(\eta\tau+\varphi) \tag{5}$$

The coefficients A_l and B_l (l = 1(1)3) depend on initial conditions that they have to be reset at every switchover from one domain to other. The determination of switching points can be done only numerically.

3 THE PIECEWISE FULL DECOUPLING METHOD

The new developed numerical procedure of explicit integration, namely, the piecewise full decoupling method is based on the method of piecing the exact solutions. The method is applicable to multi-degree-of-freedom systems with clearances. The equation (1) can be substituted with a set of linear equations of motion, defined inside each of domain:

$$\boldsymbol{q}'' + \boldsymbol{Z}\boldsymbol{q}' + \boldsymbol{\Pi}_{j\Omega}\boldsymbol{q} = \boldsymbol{p}_{j\Omega} + \boldsymbol{f}_0 + \boldsymbol{f}_a\cos(\eta\tau)\,, \quad j = 1(1)3^m \tag{6}$$

where $\boldsymbol{\Pi}_{j\Omega}$ denotes the local stiffness matrix and $\boldsymbol{p}_{j\Omega}$ is the vector of the breakpoints. The mechanical system starts from an initial position described with one of the local equation of motion. When the system changes a domain, the system is represented with the new local equation of motion.

Local equations of motion are solved by applying the state-space formulation. By employing the state vector $\boldsymbol{y} = \left[\boldsymbol{q}\ \boldsymbol{q}'\right]^{\mathrm{T}}$, the equation (6) can be transformed into the first-order differential equation, with the state matrix $\boldsymbol{A}$ of the form:

$$\boldsymbol{A} = \begin{bmatrix} \boldsymbol{0} & \boldsymbol{I} \\ -\boldsymbol{\Pi}_{j\Omega} & -\boldsymbol{Z} \end{bmatrix} \tag{7}$$

The matrix $\boldsymbol{A}$ is a real nonsymmetric matrix and its eigenvalues can be calculated using one of numerical routines for the nonsymmetric eigenvalue problem. Obtained eigenvalues enable a transformation of the state variable $\boldsymbol{y}$ to the normal coordinate $\boldsymbol{z}$:

$$\boldsymbol{y} = \boldsymbol{V}\,\boldsymbol{z} \tag{8}$$

where $\boldsymbol{V}$ is the matrix of eigenvectors. The coordinate transformation (8) leads to the uncoupled equation of motion:

$$\boldsymbol{z}' = \boldsymbol{\Lambda}\,\boldsymbol{z} + \boldsymbol{g} \tag{9}$$

where $\Lambda = \mathrm{diag}(\lambda_k),\ k = 1(1)2n$ is the matrix of eigenvalues and $\boldsymbol{g}$ is the excitation vector. The uncoupled equation (9) has well-known analytical solutions.

4 NUMERICAL EXAMPLES

Two examples are studied to illustrate the both methods. Using the method of piecing the exact solutions, the frequency response of the system represented by the equation (3) is obtained here and presented in Figure 1. A steady state solution for the single point of excitation frequency is calculated simulating 64 excitation periods for each of two initial conditions; trivial (0,0) and unit (1,1). This procedure is well suited [1] because no another reliable way to distinguish the transient and steady state motion at chaotic responses. Only approximation done in the procedure is the numerical determination of the points of entering in each domain.

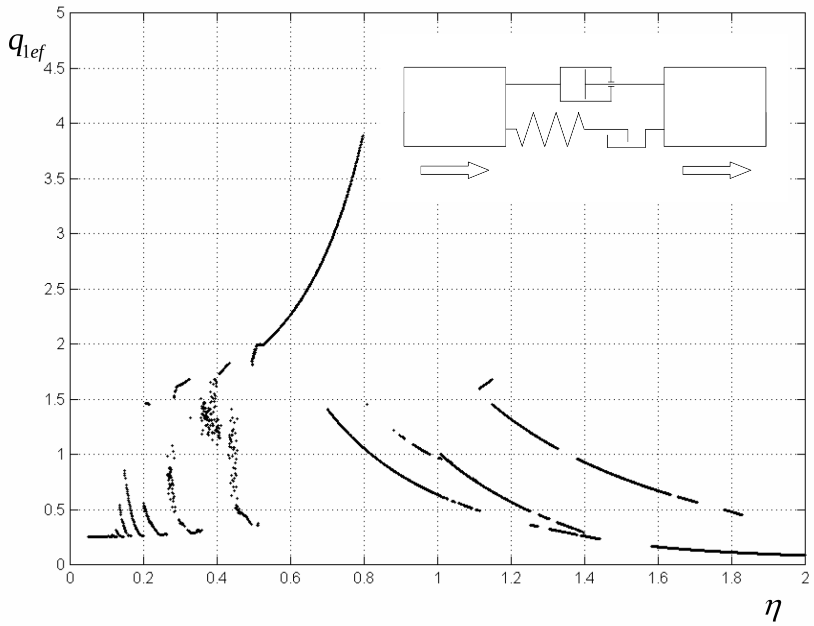

Figure 1: Frequency response of the two-degree-of-freedom semi-definite system with clearance for $f_0 = f_a = 0.25$ and $\varsigma = 0.03$

The piecewise full decoupling method is applied to the three-degree-of-freedom semi-definite system with two clearances. The periodicity of the steady state response is investigated comparing the steady state and effective amplitudes. If the steady state amplitude coincides with the effective amplitude, the response is periodic; otherwise the response is nonperiodic. The frequency response, that is, the steady state and effective amplitudes versus the nondimensional frequency is shown in Figure 2. The results are in agreement with those obtained by the finite element in time method [3].

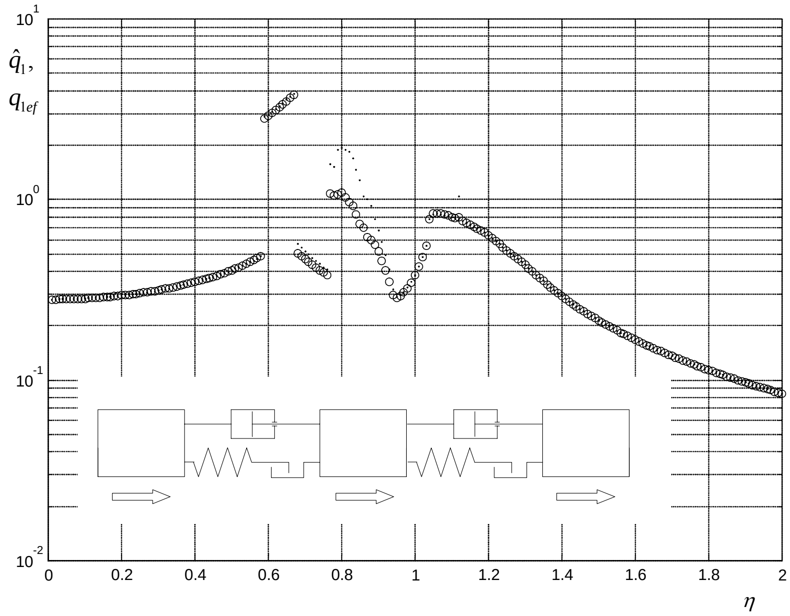

Figure 2: Frequency response of the three-degree-of-freedom semi-definite system with clearances for $\boldsymbol{f}_0 = \begin{bmatrix} 0.5 & 0.25 \end{bmatrix}^T$, $\boldsymbol{f}_a = \begin{bmatrix} 0.25 & 0 \end{bmatrix}^T$, $\varsigma_{11} = \varsigma_{12} = \varsigma_{21} = \varsigma_{22} = 0.05$, $\omega_{12} = \omega_{21} = 0.6$ and $\omega_{22} = 1.1$

(• steady state amplitude $\hat{q}_1$, ∘ effective amplitude q_{1ef})

6 CONCLUSIONS

The piecewise full decoupling method is a robust numerical procedure of explicit integration for predicting the steady state response of piecewise linear dynamical systems under periodic excitations. The method is based on the classical method of piecing the exact solutions. The accuracy of method does not significantly depend on a magnitude of the integration step, since the response inside local domains are obtained by employing analytical solutions. The time step has to be sufficiently small for the reliable numerical determination of the switching points. It is a remarkable advantage with respect to other explicit integration methods such as Runge-Kutta, etc.

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