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ARBITRARY LAGRANGIAN-EULERIAN METHOD FOR CONSOLIDATION PROBLEMS IN GEOMECHANICS

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1 INTRODUCTION

In geotechnical problems, deformation is usually coupled with flow of pore fluids. A coupled finite element procedure combines the equilibrium equation and the continuity equation through the effective stress principle and the volumetric strain rate[1]. Existing methods for handling large deformations generally lie within the Updated-Lagrangian (UL) framework which may fail to furnish a solution in the case of severe mesh distortion[2-5]. The more advanced Arbitrary Lagrangian-Eulerian (ALE) method, has not yet attracted much attention, mainly due to its complexities. In this paper, the Updated-Lagrangian (UL) method and the Arbitrary Lagrangian-Eulerian (ALE) method are generalised to solve coupled displacement and pore pressure problems. A simple and effective mesh refinement scheme is described for the ALE method. The UL and ALE methods are then used to solve a classical consolidation problem involving large deformations. The results clearly show the advantage and efficiency of the ALE method for the problems studied.

2 UPDATED LAGRANGIAN METHOD

In a coupled displacement and pore water pressure analysis, the governing equations are derived from the principal of virtual displacements and the conservation of mass. The discretised governing equations can be written as

$$\begin{bmatrix} \mathbf{K}^{ep} & \mathbf{L} \\ \mathbf{L}^{T} & \mathbf{0} \end{bmatrix} \begin{Bmatrix} \dot{\mathbf{U}} \\ \dot{\mathbf{P}} \end{Bmatrix} + \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \dot{\mathbf{H}} \end{bmatrix} \begin{Bmatrix} \mathbf{U} \\ \mathbf{P} \end{Bmatrix} = \begin{Bmatrix} \dot{\mathbf{F}}^{ext} \\ \dot{\mathbf{Q}}^{ext} \end{Bmatrix} \tag{1}$$

where $\mathbf{U}$ represents the nodal displacement vector, $\mathbf{P}$ is the nodal pore pressure vector, $\mathbf{K}^{ep}$ is the global elastoplastic stiffness matrix, $\mathbf{L}$ is the coupling matrix, $\mathbf{H}$ is the flow matrix, $\mathbf{F}^{ext}$ is the external force vector, $\mathbf{Q}^{ext}$ is the external flow vector and the superior dot denotes the first order derivative with respect to time. In an Updated-Lagrangian (UL) framework, all the variables and the state variables are known up to time t and the aim is to find the unknowns at time $t+\Delta t$ by solving equation (1). Various time-stepping schemes exist in the literature[6,7] to solve the nonlinear system of equations. In this study, a backward-Euler scheme with

Newton-Raphson iterations is used, see [7] for more details. Moreover, the Jaumann stress rate and an explicit integration scheme[8] are used to find the stress increments for given strain increments. See [8-9] for more details.

3 ARBITRARY LAGRANGIAN-EULERIAN METHOD

The ALE method has been developed based on the idea of separating the material and mesh displacements to eliminate mesh distortion in the UL method. A common form of the ALE method is the operator split technique during which the analysis is performed in two steps: an UL step followed by an Eulerian step. In the UL step, we solve the governing equations to fulfill equilibrium and obtain the material displacements. In the Eulerian step, a new mesh is generated for the deformed domain to obtain the mesh displacements. All kinematic and static variables are then transferred from the distorted mesh to the new mesh. The key issues in the operator-split ALE method thus include the mesh optimisation in the Euler step and the mapping of variables between the two meshes. The latter is performed using a first order expansion of Taylor's series[10], which is also known as the convection equation in the ALE literature. In a coupled displacement-pore water pressure ALE analysis, the state parameters to be transformed at integration points include the effective stresses, hardening parameters, void ratios and permeabilities, while the pore-water pressures are transformed from nodes to nodes. The patch recovery technique[11] is used to recover the nodal values from the values at integration point. The novel mesh refinement was recently developed by the authors [9]. To obtain the mesh displacements, we first re-discretise the deformed boundaries resulting from the UL step. These boundaries include the boundaries of the domain, the material interfaces and the loading boundaries. With known displacements of the nodes on these boundaries, we then perform an elastic analysis using prescribed displacements to obtain the optimal mesh and hence the mesh displacements for all the internal nodes. The method has been implemented for two-dimensional plane strain problems and axi-symmetric problems. However, it can easily be generalised to three-dimensional problems as well. An important advantage of this mesh optimisation method is its independence of element topology and problem dimensions. The method does not require any mesh generation algorithm, does not change the topology of the problem, and hence can be easily implemented in existing finite element codes. For more details see [9].

4 NUMERICAL EXAMPLE

The performance of the UL and the ALE methods are compared via a rigid footing resting the Modified Cam Clay (MCC) soil. The consolidation settlement of the footing is studied, with the finite element mesh shown in Figure 1. The parameters of the MCC soil are

$$\phi = 25^o,\ \lambda = 0.2,\ \kappa = 0.05,\ e_N = 1.8,\ \upsilon = 0.3,\ OCR = 2,\ K_0 = 1.0,\ \gamma = 16kN/m^3,\ k = 10^{-4}\ m/day$$

where λ is the slope of the normal compression line (NCL) in the space of the logarithmic mean stress lnp' versus the void ratio e, κ is the slope of the unloading-reloading line (URL) in the ln$p$ - e space, e_N is the intercept of the NCL on the e axis when ln$p' = 0$, OCR is the over-consolidation ratio of the soil, K_0 is the coefficient of earth pressure at rest, γ is the unit weight of the soil and k represents the permeability of the soil. A thin layer of elastic material is added on top of the MCC soil to prevent a slope instability problem when the settlement of the footing becomes very large. The elastic modulus, unit weight and Poisson's ratio of this layer are assumed to be 10^3 (kPa), 16 (kN/m^3) and 0.3, respectively.

The analysis includes three stages. In the first stage, we use body loading of the self weight of the soil to generate a non-zero initial stress field and a hydrostatic pore pressure profile. Once the initial stresses are established, the initial yield surface locations are determined according to the current stresses and the OCR. In the second stage, a uniform pressure q=100 (kPa) is applied on the footing in 100 days. Finally, the load is kept constant and the soil is allowed to consolidate over time. The settlement of the footing versus time is plotted in Figures 1b. Both the small-deformation and the UL analyses fail to furnish a solution. The small-deformation analysis fails at 35 days, due to the applied load being larger than the small strains collapse load. The UL analysis fails at 75 days, because of negative Jacobian of some elements resulting from mesh distortion. Only the ALE method can finish the analysis and predict the final settlement of the footing. The total settlement of the footing predicted by the ALE method is found to be 1.165 (m) occurring after 3100 days. The deformed meshes at the end of each analysis are shown in Figures 1c, 1d, and 1e for the small strain theory, UL method and ALE method respectively.

5 CONCLUSIONS

Two large deformation methods, the Updated Lagrangian and the Arbitrary-Lagrangian-Eulerian, were generalised for coupled large deformation analysis of geomechanical problems in this paper. The main drawback of the UL method, mesh distortion, can effectively be avoided by the ALE method. The main challenges to the ALE method are the mesh refinement and the remapping of state variables. The mesh refinement scheme adopted in this study works effectively and efficiently for the problem studied. The method is not only independent of problem topology and dimensions, but also requires no mesh generation algorithm.

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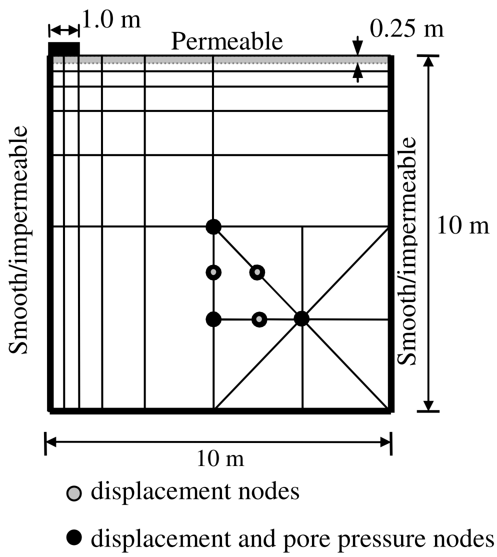
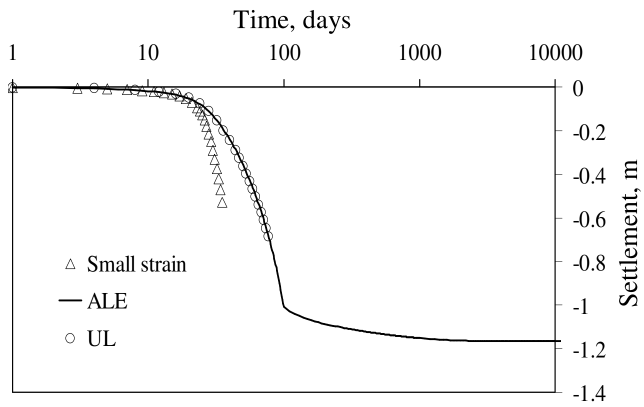
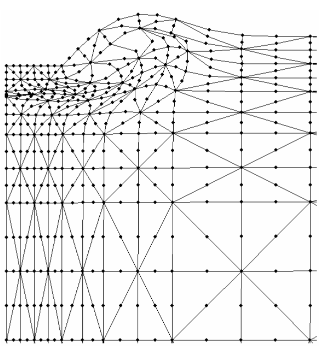


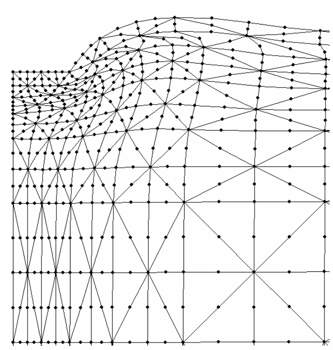
(a) Problem definition and FE mesh



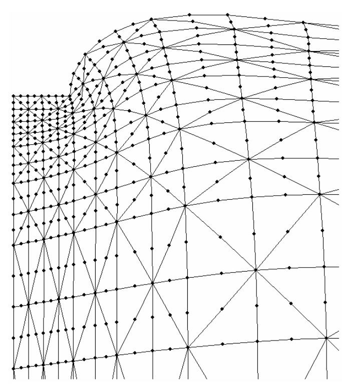
(b) Settlement versus time



(c) Mesh at 35 days, Small deformation



(d) Mesh at 77 days, UL method



(e) Mesh at 3100 days, ALE

Figure 1. Consolidation of a rigid footing on MCC soil.