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A FORMULATION FOR EMBEDDED DISCONTINUITIES IN 2D PROBLEMS

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Summary. *A formulation for a quadrilateral finite element with an embedded discontinuity, designed for large displacements, is presented. Lagrange multipliers are used to impose the condition that the embedded discontinuity is not activated before crack initiation. Additional degrees of freedom are incorporated in the element, including rotations, which permit the representation of the two states of the crack. A strain enhancement technique is utilised to improve the performance of the element in bending. Some numerical examples are used to illustrate the robustness of the proposed solutions.*

1 INTRODUCTION

The modelling of failure using the Finite Element technology has been an important topic of study in recent years. Failure is often associated with the development of strain localization phenomena which, at the constitutive level is usually associated with strain softening models. The numerical discretization of these models typically results in unstable solutions with numerical pathologies like mesh size and orientation dependency. Non-local models, which include a length scale internal parameter, are among the possible solutions which also include the so-called strong discontinuity models that will be adopted in this work. The use of embedded strong discontinuities, in the displacement field of the finite element solution, allows the treatment of the localization area with a null width, avoiding the need for remeshing as a crack is progressing and making this type of approach very attractive.

In this work a 2D formulation for embedded discontinuities is presented. Departing well established formulations, which were proposed in recent years, a slight different approach is proposed by using Lagrange multipliers to prevent the activation of the embedded discontinuity before crack is opened. Rotations are included, apart from the usual translational degrees of freedom, to represent the crack. A strain enhancement technique is utilised to improve the performance of the element in bending.

2 KINEMATICS OF EMBEDDED DISCONTINUITY

The representation of the embedded discontinuity for a single quadrilateral element[1] is represented in Figure 1.

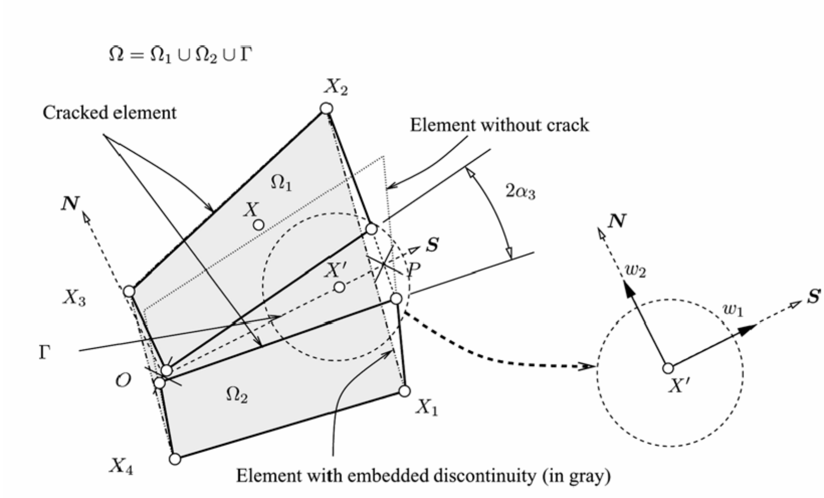

Figure 1: Element with embedded discontinuity.

The displacement vector of a point $X \in \Omega_1 \cup \Omega_2$ due to the crack induced rigid body motion is calculated as:

$$\bar{\boldsymbol{u}}\left(\boldsymbol{X}^*, r\right) = \begin{bmatrix} S_1^* & N_1^* \\ S_2^* & N_2^* \end{bmatrix} \left\{ \left(\underbrace{\begin{bmatrix} \cos\alpha_3 & -r\, sen\alpha_3 \\ r\, sen\alpha_3 & \cos\alpha_3 \end{bmatrix} \begin{bmatrix} O^*X^*.S^* \\ O^*X^*.N^* \end{bmatrix} - \begin{bmatrix} O^*X^*.S^* \\ O^*X^*.N^* \end{bmatrix}}_{rotation} \right) + \underbrace{\begin{bmatrix} r\,\alpha_1 \\ r\,\alpha_2 \end{bmatrix}}_{displacement} \right\} \tag{1}$$

where α_1 and α_2 represent the local displacements of the upper crack face at the point O and α_3 represents the crack face rotation, r is a number belonging to the set $\{-1,1\}$. S and N are, respectively, the tangential and normal vectors at a local frame defined at the crack mid line. The total displacement field, $\boldsymbol{u}$, can then be determined as the sum of the regular displacement and the jump:

$$\boldsymbol{u} = \underbrace{\hat{\boldsymbol{u}}}_{regular} - \underbrace{\bar{\boldsymbol{u}}}_{jump} \tag{2}$$

A constraint must be applied to the α_i variables so that no penetration of the two cracked parts takes place. The conditions to be satisfied are then:

$$l \sin \alpha_3 + \alpha_2 \geq 0 \quad \wedge \ \alpha_2 \geq 0 \tag{3}$$

where l is the length of the crack inside the element. By adopting a “nodal” integration rule along the crack in which the “nodes” are the intersection of the crack with the sides of the element the first condition is reduced to the second one which is therefore included in the discrete crack compliance law in a penalised form.

3 CONSTITUTIVE LAW AT THE CRACK

If the crack is open the following stress vector is defined in the local coordinates of the crack:

$$\boldsymbol{t} = \begin{Bmatrix} t_1 \\ t_2 \end{Bmatrix} = \begin{Bmatrix} 2A_1(k)\,\alpha_1 \\ \left[A_2(k)\,H(\alpha_2) + \rho\,H(-\alpha_2)\right]\alpha_2 \end{Bmatrix}$$

$$A_1(k) = d_{int}\, exp\left[ln\left(\frac{d_1}{d_{int}}\right) k \right];\, A_2(k) = \frac{\tau_{1c}}{k} exp\left(-\frac{\tau_{1c}}{G_f} k \right) \tag{4}$$

where τ_{1c} is the maximum allowable positive principal stress, G_f is the fracture energy, k is the maximum normal displacement, d_{int} is the initial shear stiffness of the crack and ρ a penalty parameter imposing the inequality condition (3).

4 EQUILIBRIUM EQUATIONS

The weak form of equilibrium equations can be expressed as:

$$\int_{Vo} \boldsymbol{\tau}(\boldsymbol{F}) : \nabla \delta \boldsymbol{u}\ dVo + \int_{lo} \delta \overline{\boldsymbol{w}}(\boldsymbol{\alpha}).\boldsymbol{t}\ dlo = \int_{Vo} \boldsymbol{b}.\delta \boldsymbol{u}\ dVo \tag{5}$$

where the first term is associated with the strain energy, the second one with energy at the crack and the last one with the body and external forces[1]. The condition of non existence of the crack is imposed in the weak form by a Lagrange multiplier technique as:

$$\int_{Vo} \boldsymbol{\tau}(\boldsymbol{F}) : \nabla \delta \boldsymbol{u}\ dVo + \int_{lo} \delta \overline{\boldsymbol{w}}.\boldsymbol{t}\ dlo + \underbrace{\delta\left[\boldsymbol{\lambda}\left((1-h)\boldsymbol{\alpha} + h\,\boldsymbol{\lambda}\right)\right]}_{\delta\Psi} = \int_{Vo} \boldsymbol{b}.\delta \boldsymbol{u}\ dVo \tag{6}$$

where $\boldsymbol{\lambda}$ are the Lagrange multipliers, $\boldsymbol{\alpha}$ is the vector of the α_i variables at the crack and h is a crack state parameter which is equal to zero if the crack is closed and equal to one if he crack is opened.

5 NUMERICAL EXAMPLE

The example presented is a double edge-notched tension test with a simultaneous growth of two cracks in mode I. Geometry and boundary conditions are represented on the left side of Figure 2. In the middle of the figure the crack paths are shown for an imposed displacement value of 0.2 mm. The locations of crack initiation are not defined a priori. Two enlarged parts of the cracked zones are presented on the left side of the picture. On the first one the penalty, related to the impenetrability condition in equation (4), is set to zero and it is possible to see that large interpenetration occurs in the right side of the mesh. In the second the penalty value is set to $\rho = \tau_{1c}^{2} \big/ G_f$ preventing the interpenetration of the two sides of the elements to take place.

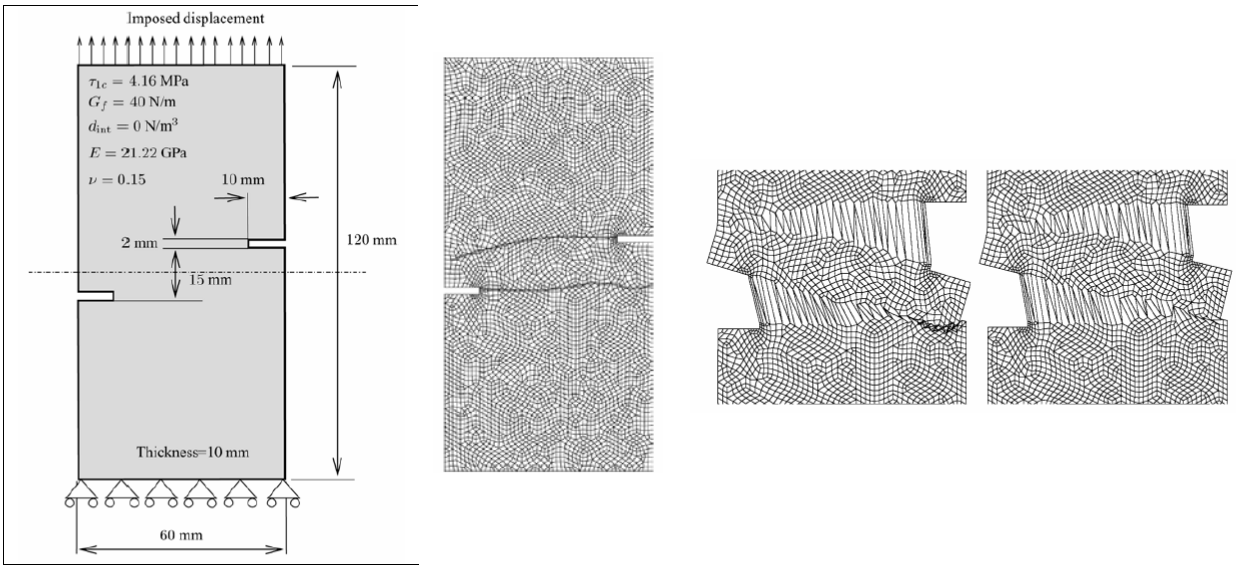

Figure 2: Double edge-notched tension test

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12 CONCLUSIONS

- The incorporation of rotation variables, as well as the symmetric treatment of the rigid body displacement field induced by the crack improved the robustness of the embedded discontinuity formulation.

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