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STATIC VS. KINETIC LIMIT LOADS OF SHELLS OF REVOLUTION

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Summary. *The paper deals with the stability of perturbation sensitive shells. The perturbation energy concept developed by Dinkler/Kröplin [2] is applied to estimate the static and kinetic limit loads of different perturbation sensitive shells by a single energy value, the perturbation energy. The material behaviour is described by a rate-independent model for elasto-plasticity.*

1 INTRODUCTION

Generally, shell buckling behaviour is highly sensitive with respect to perturbations and imperfections, respectively. A common approach for the evaluation of limit loads of shells is to consider different kinds of perturbations and to assess their influence on the load-carrying capacity. This approach suffers from finding the worst perturbation. For time–dependent perturbations even several time step analyses are required. A more sophisticated method is the perturbation energy concept which describes the origin of buckling in very fundamental way.

2 PERTURBATION ENERGY CONCEPT

It is well known from experiments that the postbuckling region of perturbation sensitive shells is characterised by a variety of adjacent load-deformation paths.

The basic idea of the perturbation energy concept is to assess the influence of perturbations on the stability of a fundamental state of equilibium in the prebuckling range by the perturbation energy. The calculation of the critical perturbation energy which causes buckling requires the identification of the critical state at the same load level as the fundamental state, if perturbations are defined to be normal to the fundamental load. Considering a fundamental state F at a certain load level P_0, a sufficiently large perturbation p_p may cause a snap-through of the system, see figure 1 left. For time-invariant perturbations, the critical state is the statically indifferent state of equilibrium M with respect to the perturbations. In case of short-time perturbations the critical state is described by the energy level of the statically unstable state of equilibrium N.

The energy distance between the fundamental state and a critical state is defined as

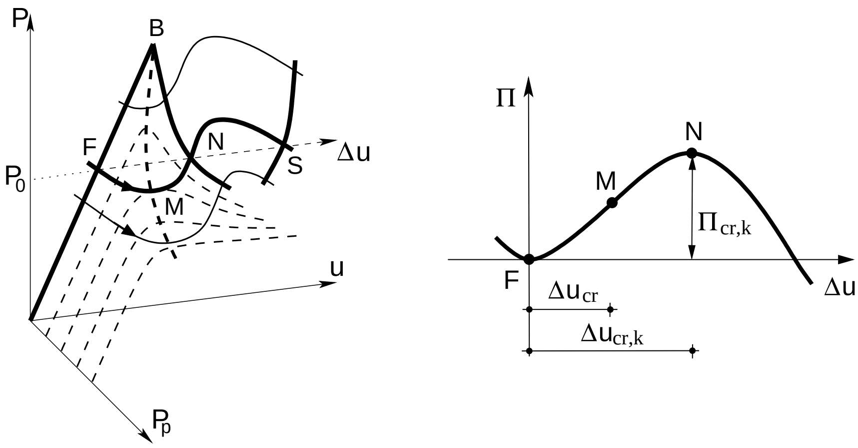

Figure 1: Load-deformation behaviour of a perturbation sensitive shell and energy distribution

the critical perturbation energy. Since the distance between F and M or N varies with respect to the load level, the size and shape of the critical perturbation varies with the load level, too. Formulating the incremental potential at a certain fundamental state F, its first and second variation

$$\delta\Delta\Pi(\Delta u_{cr,k}) = 0 \quad \text{and} \quad \delta^2\Delta\Pi(\Delta u_{cr}) = 0 \tag{1}$$

are the fundamental equations to compute the critical states with respect to kinetic and static loads. Its solutions $\Delta u_{cr,k}$ and Δu_{cr} represent the distances between the fundamental state and the critical states. The corresponding strain energy and the dissipation done along the path between the fundamental state F and the critical states are referred to as the critical perturbation energies $\Pi_{cr,k}$ and Π_{cr}.

For comparison of the perturbation sensitivity of different buckling cases, the perturbation energy is normalised by the bending stiffness B of the shell continuum, resulting in $\pi_{cr} = \Pi_{cr}/B$. The influence of the plastification of the shell continuum on the perturbation sensitivity of shells is described by the elasto-plastic model developed by Eggers/Kröplin [3].

To evaluate the impact of a short–time perturbation, the total energy V induced into the system is to compare with the critical perturbation energy $\Pi_{cr,k}$. Therefore, an integration in time domain is only necessary while the perturbation load is acting. Compared to a standard kinetic stability analysis, the proposed procedure reduces the numerical effort. This advantage becomes obvious, if the stability of a system is to be analysed at different load levels and against different short-time perturbations.

The classification of the perturbation energy concept into the family of stability criteria is as follows. The critical perturbation energy is a criterion for the stability of systems under static and short-time perturbations. Furthermore, the perturbation energy concept provides for conservative systems the worst perturbation which the kinetic stability criterion according to Ziegler is based on.

3 LIMIT LOADS OF SHELLS

The perturbation sensitivity of cylindrical shells under time-invariant axial pressure depends on several parameters, e.g. the load level, the slenderness r/t, the ratio l/r, and the material behaviour. For several buckling cases, the critical state is characterised by a locally limited buckle, as depicted in figure 2a. The consideration of elasto-plastic material behaviour for compact shells reduces the load-carrying capacity and influences the perturbation sensitivity as well.

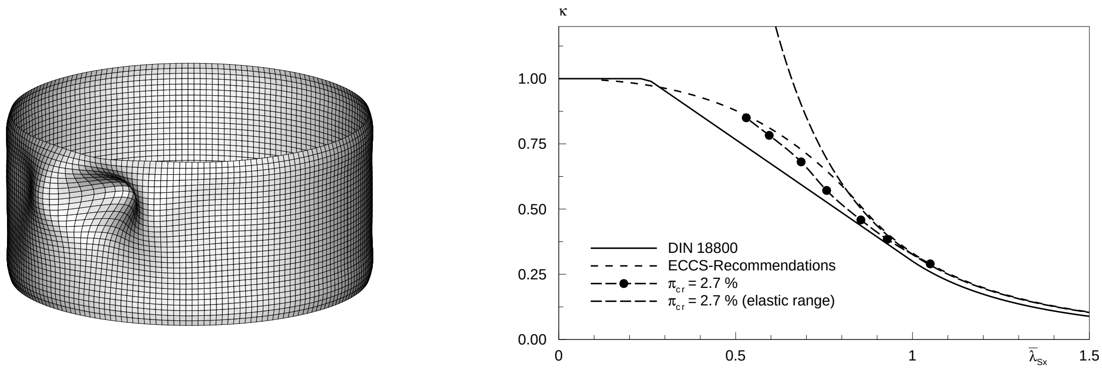

Figure 2: (a) Critical state and (b) comparison with design rules [1]

A value of the critical perturbation energy $\pi_{cr} = 2.7\,\%$ defining static limit loads may be derived by comparing the distribution of the critical perturbation energy with experimental data and design rules, see figure 2b.

In the following, the static load-deformation behaviour of an ideal spherical shell subjected to a time-invariant constantly distributed pressure field is computed. The system data of the analysed spherical shell are the same as in Dinkler/Pontow [4].

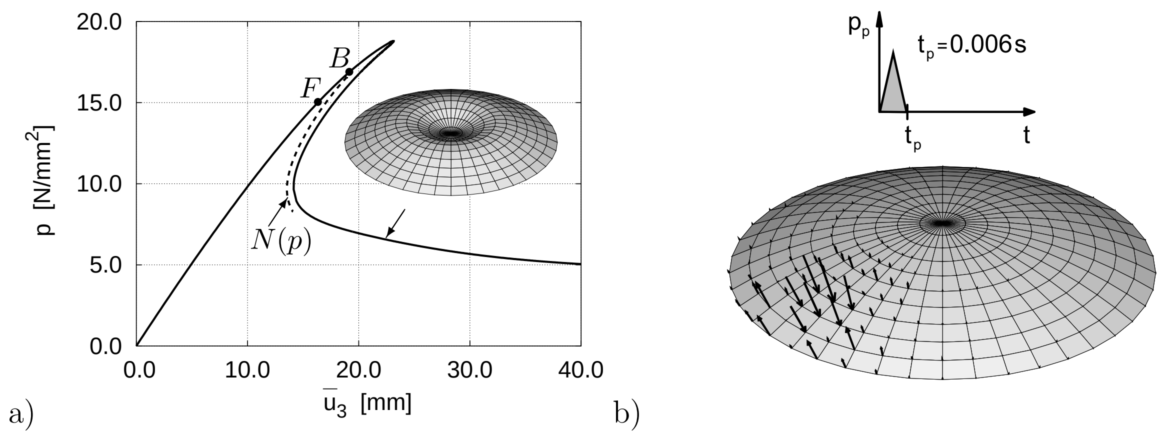

Figure 3: (a) load-deformation diagram and (b) distribution in space and time of perturbation load

Hereafter, the limit states M and N are determined with respect to a fundamental state F at load level $p_0 = 15\,\mathrm{N/mm^2}$, see figure 3a. At this load level, the static limit load is described by a perturbation energy $\Pi_{cr} = 1.26 \cdot 10^7\,\mathrm{Nmm}$. The corresponding perturbation load may be computed by equations (1) and is locally concentrated as pictured in figure 3b. For a short-time perturbation, the critical perturbation energy with respect to the fundamental state F is $\Pi_{cr,k} = 2.53 \cdot 10^7\,\mathrm{Nmm}$.

For a kinetic perturbation distributed in space and time according to figure 3b, a perturbation energy $V_1 = 2.88 \cdot 10^7\,\mathrm{Nmm}$ is induced into the system causing shell buckling asymmetrically. The reason for the slight difference between $\Pi_{cr,k}$ and the required total energy V_1 is that the perturbation energy is distributed to the whole variety of all possible modes. Therefore, the perturbation energy concept describes the stability of shells against short-time perturbations as a lower bound.
In contrary, a perturbation load consisting of a spatially constantly distributed pressure field as the fundamental load and distributed in time as pictured in figure 3b may initiate symmetric buckling by a perturbation energy of at least $V_2 = 8.0 \cdot 10^8\,\mathrm{Nmm}$ which is significantly greater than $\Pi_{cr,k}$. For smaller values of V_2 but still greater than $\Pi_{cr,k}$, the shell does not buckle. Therefore, the resistence of shells against perturbations depends on the type of the perturbations and the proposed perturbation energy concept is a lower bound concept for perturbation sensitive buckling cases.

4 CONCLUSIONS

- The perturbation energy concept allows to estimate static and kinetic limit loads of perturbation sensitive shells by a single control paramter, the perturbation energy.
- For short-time perturbations, the limit loads are described as a lower bound.
- Considering the associated perturbation energies, static limit loads are more conservative than kinetic limit loads.

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