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COMPUTATIONAL STUDY OF DYNAMIC FAILURE OF CONCRETE INCLUDING VISCOELASTICITY, VISCOPLASTICITY AND DAMAGE

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Summary. *Concrete in dynamics shows viscoelastic material behaviour because of a frequency dependent stiffness and attenuation effects. We study two viscoelastic constitutive models applied to the description of concrete under impact loading. The first model is based on a chemo-plastic formulation, while the second model is a general viscoelastic model using fractional time derivatives. Both models are used to reproduce results from Split Hopkinson Bar tests.*

1 INTRODUCTION

The behaviour of concrete under impact loading is object of extensive studies. Concrete is a highly rate-dependent material at loading rates exceeding 15 GPa/s. This means that the apparent macroscopic mechanical properties of concrete depend on the applied loading rate. This has been determined, experimentally, for the material strength and, to a smaller extent, for the stiffness and the fracture energy. To reproduce the response of concrete structures exposed to extreme dynamic loading, reliable material data and numerical material models, with rate effect properly taken into account, are crucial. Rate-dependency in high-rate dynamics is mainly caused by inertia effects. Moisture in nano- and micro pores contributes to an increase of the material parameters for moderate loading rates.

In this contribution, we present two viscoelastic constitutive models to account for the strengthening effect associated with viscous phenomenon due to moisture. A viscoelastic material model for the bulk material is necessary in order to take into account the dispersion and attenuation effect and a frequency dependent stiffness. We elaborate the viscoelastic plastic model described in [1–3], in which retardation of micro-crack growth is taken into account. A general viscoelastic model based on fractional time derivatives [4] is also presented. Both models are coupled to a damage model. Results from simulations of a Split Hopkinson Bar test are compared.

2 CHEMO-PLASTIC MODEL

Realistic modelling of concrete is difficult due to its porous micro-structure. Different mechanisms take place in micro- and nano-pores, and the mutual dependency of these mechanisms complicates the material modelling. To improve a constitutive model, chemical kinetics at the macro level can be considered by using the theory of reactive porous media [5]. The hydration process and alkali silicate reactions in early-age concrete or viscous phenomena due to moisture in the nano-pores in high-rate dynamics are two examples of chemical reactions. Both reactions influence the mechanical material properties of concrete.

We consider the viscoelastic plastic model derived by Sercombe et al [1, 2]. The model can be formulated as

$$\left\{ \begin{array}{c} d\boldsymbol{\epsilon} - d\boldsymbol{\epsilon}^p \\ dq \\ dA \end{array} \right\} = \left[\begin{array}{ccc} \mathbf{C} & 0 & \frac{\mathbf{s}}{|\mathbf{s}|} \\ 0 & \frac{\partial q}{\partial \kappa} & \frac{\partial q}{\partial \chi} \\ \frac{\mathbf{s}}{|\mathbf{s}|} & 0 & \mathcal{E} \end{array} \right] \left\{ \begin{array}{c} d\boldsymbol{\sigma} \\ d\kappa \\ d\chi \end{array} \right\}, \tag{1}$$

where the viscous strains are expressed as $\frac{\mathbf{s}}{|\mathbf{s}|}\chi$. Here χ and $\mathbf{s}$ are the averaged viscous strain and deviatoric stress tensor, respectively. Another deviation from a standard plas-

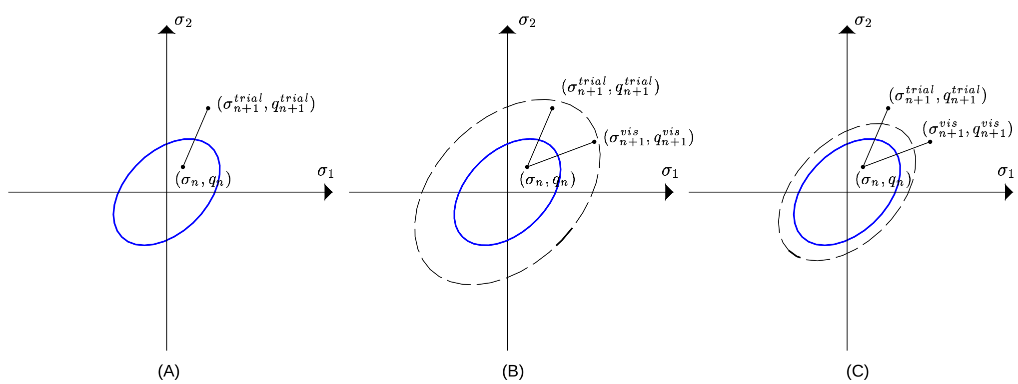

Figure 1: An extra trial stress is necessary for the viscoelastic model [1, 2]: trial stress (A) is outside the initial elastic domain (blue line) and inadmissible; however, the stress state (B) can be elastic after viscoelastic effects (black line) or it can remain outside and a return mapping scheme is activated (C).

ticity formulation is the chemical affinity A, which represents the stress associated with the viscous process, including the rigidity $\mathcal{E}$. Algorithmic aspects of the model can be found in [1, 6]. An important feature of the coupled viscoelastic plastic model is illustrated in Figure 1. In this model, an intermediate trial stress is necessary. This is due to the hardening force q being a function of the equivalent plastic strain κ and the averaged viscous strain χ. At the global level, these dependencies lead to an increase of the peak load and the initial stiffness.

3 VISCOELASTIC MODEL USING FRACTIONAL TIME DERIVATIVES

A spring-pot is a general viscoelastic rheological element, which makes use of fractional time derivatives. In what follows, we give a general overview of the model. More details can be found in [4, 7, 8].

In an elastic spring, the stress is proportional to the instantaneous strain and is independent of the strain rate. In a viscous fluid element, the stress is proportional to the strain rate and independent of the strain. However, in a viscoelastic material a combination of both phenomena is present and a spring-pot interpolates between the properties of the two elements. The relative influence of the two phenomena is defined through the order of derivative α $(0 \leq \alpha \leq 1)$ in Equation 2.

In impact dynamics a wide range of frequencies are activated and, due to inhomogeneities in concrete, relaxation times may change. Therefore, elements in rheological models are often arranged in series or in parallel and any combination of springs and dashpots can be included in this formulation. The model hinges on the concept of fractional derivative. The fractional derivative of a function f can be approximated by

$$\frac{d^\alpha f(t)}{dt^\alpha} \approx \left[\left(\frac{t}{N} \right)^{-\alpha} \sum_{j=0}^{N_l} A_{j+1} f\left(t - j\frac{t}{N} \right) \right], \tag{2}$$

where $\frac{t}{N}$ is the time step, N_l is a number of history terms and A_{j+1} are the Grünwald coefficients [4] defined as

$$A_{j+1} = \frac{\Gamma(j-\alpha)}{\Gamma(-\alpha)\Gamma(j+1)}, \tag{3}$$

in which Γ is the Gamma function. In Figure 2, results from a creep test for a Kelvin element are given to show the influence of the order of derivative, viscosity and the number of history terms. Note that $\alpha = 1.0$ results in a standard differential equation with integer order of derivatives, where the analytical solution is an exponential function.

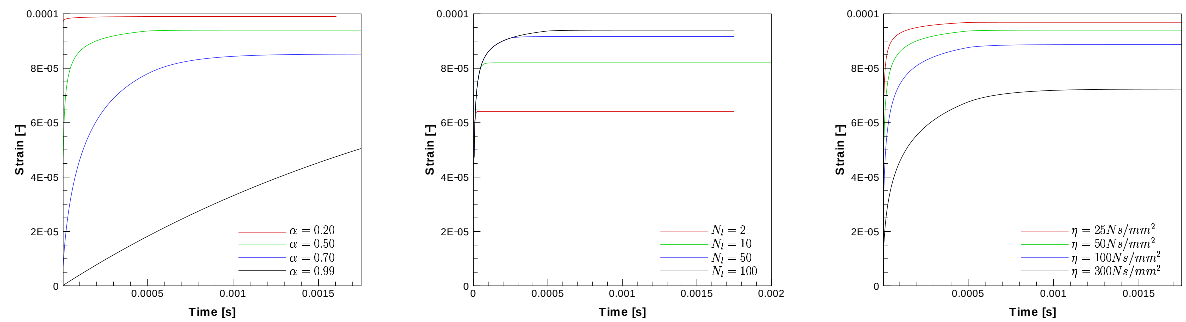

Figure 2: Creep test for a Kelvin element. Left: influence of the order of derivative ($N_l = 100$ and $\eta = 50\frac{Ns}{mm^2}$). Middle: number of history terms ($\alpha = 0.50$ and $\eta = 50\frac{Ns}{mm^2}$). Right: different viscosities ($\alpha = 0.50$ and $N_l = 100$).

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