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FORMING SIMULATIONS – MATERIAL MODELLING AND FINITE ELEMENT TECHNOLOGY

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Summary. *In this paper a new eight-node solid-shell finite element formulation based on the concept of reduced integration with hourglass stabilization is presented. Exploiting a Taylor expansion of the first Piola-Kirchhoff stress tensor with respect to the normal through the centre of the element the stress becomes a linear function of the shell surface coordinates whereas the dependence on the thickness coordinate remains non-linear. This leads to a shell formulation which requires only two Gauss points. In the context of forming simulations the following advantages are especially important: (1) computational efficiency, (2) correct representation of the geometry in thickness direction (crucial for contact modelling), (3) no modifications with regard to kinematics and material modelling.*

1 INTRODUCTION

The numerical simulation of deep drawing processes demands high standards of finite element technology, because the workpieces undergo extreme bending, whereas the material is plastically incompressible.

To overcome the well-known problem of locking, recently, several authors (see e.g. Puso [1], Cardoso et al. [2] and Reese [3]) have proposed to transfer the enhanced strain method into finite element formulations based on reduced integration with hourglass stabilization. The present method is special in the regard that a Taylor expansion of the first Piola-Kirchhoff stress tensor with respect to a point on the shell normal is carried out. We arrive at a formulation with only two Gauss points located on the shell normal to capture the non-linearity of the stress function over the thickness. For more details see Reese [4]. Alternative large strain solid-shell formulations based on reduced integration can be found in Hauptmann et al. [5] and Legay & Combescure [6].

Besides its robustness and efficiency the formulation offers the following advantages. The element possesses eight nodes and only displacement degrees-of-freedom. As such it is very suitable to be used in contact problems. There are no additional assumptions or restrictions with respect to the kinematics or the material law, i.e. arbitrary continuum mechanical material laws can be implemented. In the present contribution we investigate in particular the performance of the new finite element technology in the context of deep drawing simulations, partially with regard to the use of the element in a commercial code. The results show that the new element formulation has the potential to develop into an excellent tool for practical forming simulations.

2 VARIATIONAL FUNCTIONAL AND INTERPOLATION

The starting point of the present paper is the same two-field functional as the one suggested by Simo et al. [7]. The total strain $\mathbf{H}^h = \operatorname{Grad} \mathbf{u}^h + \mathbf{H}^h_{\text{enh}}$ is additively decomposed into a compatible part $\operatorname{Grad} \mathbf{u}^h$ ($\mathbf{u}^h$ displacement vector) and an incompatible (or enhanced) part $\mathbf{H}^h_{\text{enh}}$. The index h indicates that the superscripted quantities have already been discretized by a suitable spatial interpolation. The interpolation of $\boldsymbol{H}^h$ (bold italic letters are used for matrix notation) is chosen as

$$\boldsymbol{H}^h = \underbrace{(\boldsymbol{B}_{\text{lin}} + (\boldsymbol{j}_0^1\,\boldsymbol{L}^1_{\text{hg}} + \boldsymbol{j}_0^2\,\boldsymbol{L}^2_{\text{hg}})\,\boldsymbol{M}_{\text{hg}})\,\boldsymbol{U}_e}_{:=\,\boldsymbol{H}^h_{\text{comp}}\,=\,\boldsymbol{H}^h_{\text{lin}}+\boldsymbol{H}^h_{\text{hg}}} + \underbrace{\boldsymbol{j}_0^1\,\boldsymbol{L}_{\text{enh}}\,\boldsymbol{W}_e}_{:=\,\boldsymbol{H}^h_{\text{enh}}} \tag{1}$$

The 24x1 vector $\boldsymbol{U}_e$ ($\boldsymbol{U}_e^T = \{\boldsymbol{U}^T_{1e}, ..., \boldsymbol{U}^T_{Ie}, ..., \boldsymbol{U}^T_{8e}\}$, $I = 1, ..., 8$) and the 9x1 vector $\boldsymbol{W}_e$ include the nodal displacement degrees-of-freedom and the internal degrees-of-freedom, respectively. It should be mentioned that $\boldsymbol{B}_{\text{lin}}$ represents the constant part of the classical $\boldsymbol{B}$-operator and is as such independent of the local (convective) element coordinates ξ, η and ζ (defined on a cube Ω_e with the side length 2). The matrices $\boldsymbol{j}_0^i$ ($i = 1, 2$) include the coefficients of the inverse Jacobian matrix $\boldsymbol{J}^{-1}$ evaluated in the centre of the element $\boldsymbol{\xi} := \{\xi, \eta, \zeta\} = \mathbf{0}$. The quantity $\boldsymbol{M}_{\text{hg}}$ incorporates the so-called hourglass stabilization vectors which are also constant within the element. The dependence of $\boldsymbol{H}^h$ on the local coordinates is described by the matrices $\boldsymbol{L}^i_{\text{hg}}$ ($i = 1, 2$) and $\boldsymbol{L}_{\text{enh}}$ which either depend linearly ($\boldsymbol{L}^1_{\text{hg}}$, $\boldsymbol{L}_{\text{enh}}$) or bilinearly ($\boldsymbol{L}^2_{\text{hg}}$) on ξ, η and ζ.

3 TAYLOR EXPANSION OF THE FIRST PIOLA-KIRCHHOFF STRESS TENSOR

At the present state of the derivation the formulation does not differ majorly from the 3D enhanced strain concept by Simo et al. [7] or the 3D reduced integration technique suggested by Reese [3]. The latter classical 3D elements are known to exhibit shear locking in the case of bending of very thin structures. In the present paper we aim to overcome this deficiency. The key question is how the present 3D approach can be transferred into a powerful solid-shell concept.

The Taylor expansion of the first Piola-Kirchhoff stress tensor $\boldsymbol{P}^h$ with respect to the element centre $\boldsymbol{\xi} = \mathbf{0}$, as it has been carried out in Reese [3], leads to a stress-strain relation which is *linear* in the local coordinates ξ, η and ζ. For thick-walled geometries this is not a problem because a realistic finite element modelling of such structures usually requires several element layers in each direction. In the case of thin-walled structures, however, established shell formulations work with only one element over the thickness. A new solid-shell concept can only be competitive if it also shows the just mentioned advantage.

The linear dependence on ζ (if ζ is the thickness direction) is therefore here not useful because the solid-shell concept should be able to capture as well as possible any *non-linear* stress-strain behaviour *over the thickness* by means of one element. For this reason the Taylor expansion is carried out with respect to the point $\boldsymbol{\xi}_\star$ ($\boldsymbol{\xi}_\star^T = \{0, 0, \zeta\}$) so that the non-linear dependence on ζ is retained in the constitutive quantities. Inserting the Taylor expansion into the two equations of weak form yields two contributions. The first one has a structure which is very similar to the internal residual force vector known from standard displacement formulations. The second one is the so-called hourglass contribution.

4 NUMERICAL EXAMPLE: PINCHED CYLINDRE WITH END DIAPHRAGM

A typical shell example is the so-called "cola can". Geometry and boundary conditions are given in Figure 1a. For more details, also about the chosen material parameters, see Reese [4]. The curves plotted in Figure 1b show that the reaction force in node A does not monotonically increase with increasing displacement w_A. This well-known effect is due to local buckling phenomena. It contributes to the fact that the present computation reacts rather sensitively to the choice of the time step and the discretization. Convergence is nevertheless obtained (Figure 1b). The results are comparable to the ones of Eberlein & Wriggers [8] (indicated by the legend EW in Figure 1b) who use a four-node shell element. The visible kink at a reaction force of about 1000 N is also described in this paper. In Figure 2 the deformed state of the structure at $w_A = 200\,\text{mm}$ is plotted for different discretizations. The contours refer to the yield criterion Φ.

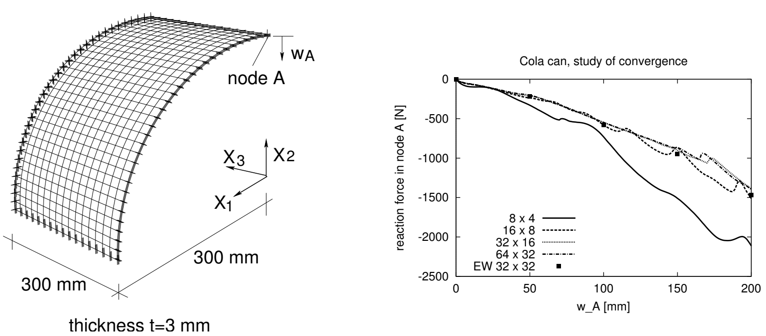

Figure 1: (a) Pinched cylindre. Geometry, boundary conditions and discretization, (b) Study of convergence

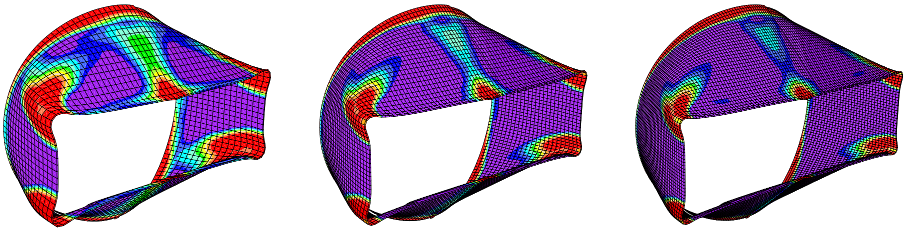

Figure 2: Contours of the yield criterion Φ plotted on the deformed configuration at $w_A = 200\,\mathrm{mm}$ (meshes: 32x16, 48x24, 64x32)

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