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MULTIPHYSICAL LINK (BEAM) ELEMENT WITH CONTINUOUS VARIATION OF MATERIAL PROPERTIES

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Summary. *The aim of this contribution is to outline of a new link (beam) element suitable for electro-thermo-structural analysis. The continuous polynomial variation of cross-sectional area, electrical and thermal conductivity, elasticity modulus and thermal expansion coefficient will be considered. The proposed finite element could be used in multiphysical analysis of current paths and actuators, and in analysis of another structures, which are built from composite or functionally graded material.*

1 INTRODUCTION

The new materials (composite or functionally graded - FGM) have been produced in the present. The commercial FEM - programs contain the link and solid elements with constant or average material properties (e.g. [1]). These elements can be also used for analysis of functionally graded materials, but the solution accuracy depends very strong on the mesh fineness and the input data preparing is very time consuming. Therefor new finite elements are developed, for example [2]. The aim of this contribution is to establish the expressions of the new link finite element for solution of weak coupled electro-thermo-structural problem. This element can have longitudinal varying cross-sectional area and uniaxially graded material properties (electrical and thermal conductivity, elasticity modulus and thermal expansion coefficient). The element matrix contains the transfer constants, which depend on the material properties and cross-sectional area variation.

2 BASIC FEM EQUATIONS OF THE MULTIPHYSICAL LINK ELEMENT

2.1 Definition of the cross-sectional area and material properties variation

We assume the link element with nodes i and j (Figure 1). The variation of cross-sectional area is defined by the polynomial

$$A(x) = A_i \eta_A(x) = A_i \left(1 + \sum_{k=1}^{s} \eta_{Ak} x^k\right) \tag{1}$$

where A_i is the cross-sectional area at node i, and the polynomial $\eta_A(x)$ expresses the variation of cross-section along the element length. Its constants η_{Ak} and the order of polynomial s depend on cross-sectional area variation. By similar way we can defined the variation of material properties – thermal and electrical conductivity $\lambda(x)$ and $\sigma(x)$, and the thermal expansion coefficient $\alpha_T(x)$ and elasticity modulus $E(x)$.

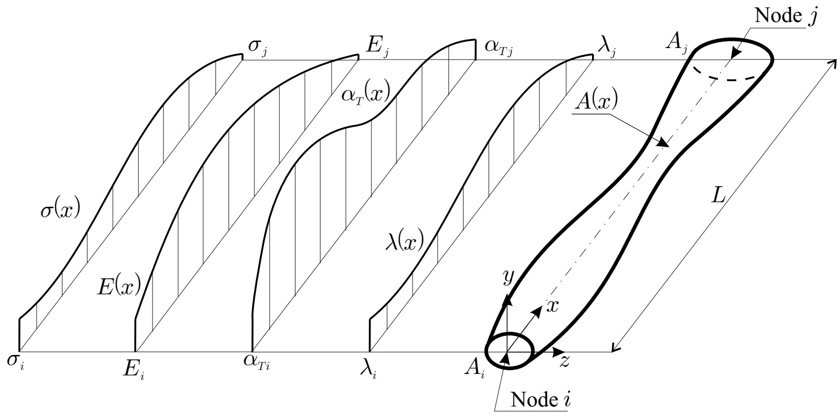

Figure 1: Link element with variation of geometry and material properties

Nodal quantities are as follow: electrical: V_i, V_j – unknown nodal electric potential and I_i, I_j – electric nodal currents; thermal: T_i, T_j – unknown nodal temperatures and Q_i, Q_j – nodal heat flows; structural: u_i, u_j – unknown nodal displacements and F_i, F_j – nodal forces.

2.2 FEM equations of weak coupled electro-thermo-structural problem

The basic FEM equations of the multiphysical finite element according the sequence solution method of weak coupled electro-thermo-structural problem have form

$$\begin{bmatrix} \mathbf{K^v} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{K^t} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{K^u} \end{bmatrix} \begin{bmatrix} \mathbf{V} \\ \mathbf{T} \\ \mathbf{u} \end{bmatrix} = \begin{bmatrix} \mathbf{I} \\ \mathbf{Q}(I) \\ \mathbf{F}(T) \end{bmatrix} \tag{2}$$

where $\mathbf{K^v}$ is electric conductance matrix, $\mathbf{K^t}$ is thermal conductance matrix, $\mathbf{K^u}$ is stiffness matrix, $\mathbf{V}$ is vector of nodal potentials, $\mathbf{T}$ is vector of nodal temperatures, $\mathbf{u}$ is vector of nodal displacements, $\mathbf{I}$ is vector of nodal currents, $\mathbf{Q}(I)$ is vector of heat flows produced by current I (which is constant along the element length L) and $\mathbf{F}(T)$ is vector of nodal forces produced by temperature $T(x)$ (which varies along the element length L).
Matrices $\mathbf{K^v}$, $\mathbf{K^t}$ and $\mathbf{K^u}$ (matrix $\mathbf{K^u}$ is part of stiffness matrix of 3D beam element with varying stiffness (1st and 2nd order theory) [3]) can be written as

$$\mathbf{K^v} = \frac{\sigma_i A_i}{b'_{2A\sigma}} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}; \ \mathbf{K^t} = \frac{\lambda_i A_i}{b'_{2A\lambda}} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}; \ \mathbf{K^u} = \frac{E_i A_i}{b'_{2AE}} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \tag{3}$$

where $b'_{2A\sigma}$, $b'_{2A\lambda}$ and b'_{2AE} is the transfer constant for electric, thermal and structural analysis, respectively. Solution of transfer constants is thoroughly described in [3].
A_i is the cross-sectional area at node i and σ_i, λ_i and E_i are material properties at node i.
Coupling is realized through the right site of the equation (2). For example, vector of nodal forces produced by temperature can be expressed as

$$\mathbf{F}(T) = \begin{bmatrix} 1 \\ -1 \end{bmatrix} \frac{E_i A_i \alpha_{Ti} T_i}{b'_{2AE}} \int_{(L)} \eta_{\alpha_T}(x)\eta_T(x)dx \tag{4}$$

where $T(x) = T_i \eta_T(x)$ is polynomial form of distributed temperature along the element length.

3 NUMERICAL EXAMPLE

Figure 2 shows bar (actuator) with variation of cross-section. The bar contains three parts with different geometry and material properties.

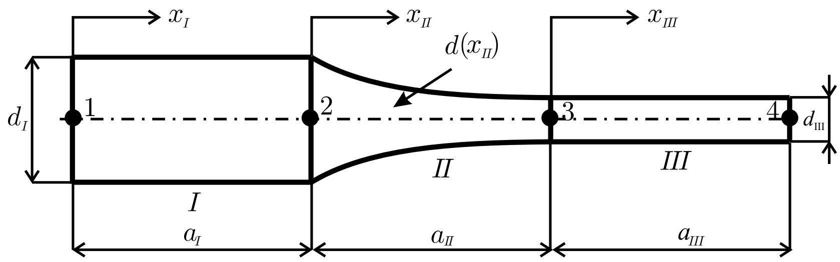

Figure 2: Example – bar with variation of geometry and material properties

Geometry (diameter d and length a) and material properties (electric conductivity σ, thermal conductivity λ, elasticity modulus E and thermal expansion coefficient α_T) for each part:

- Part I: $d_I = 0.02$m, $a_I = 0.1$m, $\sigma_I = 10000\text{Sm}^{-1}$, $\lambda_I = 40$W/mK, $E_I = 2 \times 10^{11}$Pa, $\alpha_{TI} = 1 \times 10^{-5}\text{K}^{-1}$
- Part II: $d(x_{II}) = d_I\,(1 - 9x + 30x^2 + 100x^3)$, $a_{II} = 0.1$m,
  $\sigma(x_{II}) = \sigma_I\,(1 + 20x^2 + 200x^3)$, $\lambda(x_{II}) = \lambda_I\,(1 + 20x^2 + 200x^3)$,
  $E(x_{II}) = E_I\,(1 + 50x^2 + 500x^3)$, $\alpha(x_{TII}) = \alpha_{TI}\,(1 + 200x^2 + 2000x^3)$
- Part III: $d_{III} = 0.01$m, $a_{III} = 0.1$m, $\sigma_{III} = 14000\text{Sm}^{-1}$, $\lambda_{III} = 56$W/mK, $E_{III} = 4 \times 10^{11}$Pa, $\alpha_{TIII} = 5 \times 10^{-5}\text{K}^{-1}$

Electrical and thermal constrains are $I_4 = 1$A, $V_1 = 20$V and $Q_1 = 1$W, $T_4 = 50$°C, respectively. Two modifications of structural constrains have been considered. Firstly,

only point 1 was fixed ($u_1 = 0$) – statically determinate system (SDS), secondly, also point 4 was fixed ($u_1 = 0$ and $u_4 = 0$) – statically indeterminate system (SIS). Our goal is to determine electric potential, temperature, stress and displacements of nodal points for SDS and SIS case.
The problem was solved using only one our new link element per each part with variation of cross-section and material properties along the length. The results are shown in Table 1. The stresses in part I and III in SIS case are $\sigma_I = -243.3$MPa and $\sigma_{III} = -977.4$MPa. The same problem has been solved using the multiphysical element LINK68 of code ANSYS [1]. To show the effectiveness and accuracy of our element, number of elements (NOE) LINK68 was gradually increased from 1 to 150 per each part of the bar. The results of solution are shown in Table 1. Obtained stresses in part I and III for SIS are (NOE=1) $\sigma_I = -222.9$MPa and $\sigma_{III} = -891.6$MPa; (NOE=10) $\sigma_I = -244.6$MPa and $\sigma_{III} = -978.3$MPa; (NOE=150) $\sigma_I = -244.8$MPa and $\sigma_{III} = -979.2$MPa.
The results comparison shows that the LINK68 results converge anyway to our element results obtained using very coarse mesh, i.e. new link element solution accuracy does not depend on mesh density.

	Potential [V]	Temperature [°C]			SIS – Displ. $\times 10^{-5}$[m]		SDS – Displ. $\times 10^{-4}$[m]		
NOE	V_4	T_1	T_2	T_3	u_2	u_3	u_2	u_3	u_4
ANSYS LINK68 solution									
1	20.195	103.693	95.600	76.151	-1.180	-9.246	0.965	2.499	5.653
10	20.192	102.777	94.682	76.081	-2.354	-7.146	0.987	2.808	5.969
150	20.193	102.760	94.677	76.080	-2.365	-7.127	0.987	2.811	5.972
New link element solution									
1	20.192	102.762	94.677	76.079	-2.345	-7.085	0.9872	2.810	5.960

Table 1: Example solution results

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REFERENCES

[1] ANSYS 8.1. A general purpose FEM program, 2004.

[2] Chakraborty A., Gopalakrishnan S., Reddy J. N.: A new beam finite element for the analysis of functionally graded materials, International Journal of Mechanical Sciences, Vol.45, 2003, pp.519-539.

[3] Murín J., Kutiš V.: 3D-beam element with continuous variation of the cross-sectional area, Computers and Structures, Vol. 80, 2002, pp.329-338.