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SHAPE OPTIMIZATION OF ELASTO-PLASTIC STRUCTURES WITH SHAKEDOWN CONSTRAINTS

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Summary. *An integrated approach for all necessary variations within direct analysis, variational design sensitivity analysis and shakedown analysis based on Melan's static shakedown theorem for linear unlimited kinematic hardening material behavior is formulated. Remarks on the computer implementation and numerical examples show the efficiency of the proposed formulation. Important effects of shakedown conditions in shape optimization with elasto-plastic deformations are highlighted in a comparison with elastic and elasto-plastic material behavior and the necessity of applying shakedown conditions when optimizing structures with elasto-plastic deformations is concluded.*

1 INTRODUCTION

When optimizing structures with elasto-plastic deformations the fundamental problem occurs that deformations and stresses depend on the load path. Thus the optimized structure depends not only on the maximal loading but also on the loading history. The resulting shapes of structures optimized for the same maximal loading but different loading paths can be very diverse. This problem becomes even more aggravated if more than one load case must be considered. All possible load-combinations form the so-called load domain. For elastic problems with multiple load cases it is permissible to consider only the corners of this load domain during optimization. The optimized structure will be safe for all possible load paths within this load domain. For elasto-plastic structures this is not the case. For the optimization of these elasto-plastic structures with multiple load cases it is necessary to consider shakedown conditions.

2 ELASTIC SHAKEDOWN ANALYSIS

Shakedown of elasto-plastic systems subjected to variable loading occurs if after initial yielding plastification subsides and the system behaves elastically. This is due to the fact that a stationary residual stress field is formed and the total dissipated energy becomes stationary. Elastic shakedown (or simply shakedown) of a system is regarded as a safe state.

Classical Prandtl-Reuß elasto-plasticity for linear unlimited hardening with von Mises yield condition is considered here. Consider a polyhedral load domain $\mathcal{M}$ defined by M load vertices. The elastic stresses corresponding to each of these vertices are denoted $\boldsymbol{\sigma}_j^{\mathrm{E}}$. The system will shakedown if there exist at least one admissible residual stress field $\bar{\boldsymbol{\rho}}(\boldsymbol{X})$ and one backstress field $\bar{\boldsymbol{\gamma}}\,(\boldsymbol{X})$, such that

$$\Phi = \|\mathrm{dev}\,[\beta\boldsymbol{\sigma}_j^{\mathrm{E}}(\boldsymbol{X}) + \bar{\boldsymbol{\rho}}(\boldsymbol{X}) - \bar{\boldsymbol{\gamma}}\,(\boldsymbol{X}))]\| - \sqrt{\tfrac{2}{3}}Y_0 \leq 0 \quad \forall\,(\boldsymbol{X}, j) \in \Omega_o \times \mathcal{M} \tag{1}$$

is satisfied for all possible load vertices j within the given load domain $\mathcal{M}$. If this condition is fulfilled exactly the multiplier β will be a lower bound to the true elastic shakedown multiplier, see[2] for more details on shakedown analysis. Thus, we seek to maximize β subject to the above constraint.

The numerical formulation of the shakedown analysis takes advantage of the local nature of the failure for the considered material. In a first step the solution of the equilibrium conditions $\mathcal{G}_j = 0$ for any load vertex j is computed. Then the vectors of elastic stresses $\underline{\sigma}_j^{\mathrm{E}}(i)$ in any Gaussian point i of the discretized structure are calculated. In a final step the solution of the global discretized shakedown optimization problem is calculated by solving local optimization problems in any Gaussian point. The global maximal load factor β is given by

$$\beta = \bar{\beta} \quad \text{with} \quad \bar{\beta} = \min_{i=1,NGP} \beta_i, \tag{2}$$

where β_i is the solution of the local sub-problem defined in the i-th Gaussian point

$$L(\beta_i, \underline{y}(i), \lambda_i) = -\beta_i + \lambda_i \Phi[\beta_i \underline{\sigma}_j^{\mathrm{E}}(i), \underline{y}(i)] \rightarrow \mathrm{stat} \tag{3}$$

where the internal stresses $\underline{y}(i)$ are unconstrained difference vectors of residual stress fields $\underline{\bar{\rho}}(i)$ and backstresses $\underline{\bar{\gamma}}(i)$.

3 SHAPE OPTIMIZATION OF SHAKEDOWN PROBLEMS

Structural optimization essentially needs an efficient strategy for performing the sensitivity analysis, i.e. for calculating the design variations of functionals modeling the objective and the constraints of the optimization problem. These demands are addressed within the so-called design sensitivity analysis.

3.1 Variational design sensitivity analysis of shakedown problems

The sensitivity analysis of the objective function and the constraints can be performed with different methods. Our approach is based upon the variational design sensitivity analysis of the investigated functionals[1]. This means the variations of the continuous formulation are calculated and then in a subsequent step they are discretized in order to get computable expressions. The main advantage of this methodology is that the

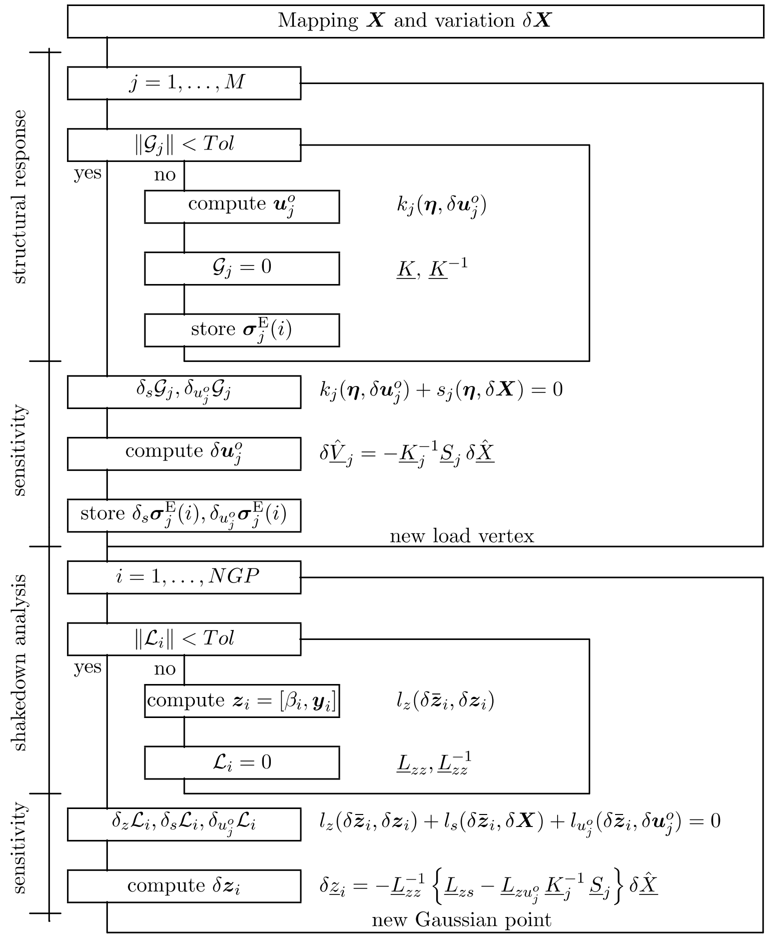

Figure 1: Flow chart of structural analysis, sensitivity analysis and shakedown analysis

sensitivity analysis can be formulated analogous to and consistent with the structural analysis and the shakedown analysis.

The sensitivity expressions are derived by linearizing the weak form of equilibrium $\mathcal{G}_j$ with respect to geometry, while, as is well known, the tangent stiffness is derived by linearizing the weak form of equilibrium with respect to displacements. The variation of the displacement mapping $\delta\boldsymbol{u}_j^o$ is implicitly defined by

$$\frac{\partial\mathcal{G}_j}{\partial\boldsymbol{u}_j^0}\delta\boldsymbol{u}_j^0 = -\frac{\partial\mathcal{G}_j}{\partial\boldsymbol{X}}\delta\boldsymbol{X}. \tag{4}$$

A subsequent discretization process ends up in the finite dimensional expressions for sensitivity analysis introducing the element sensitivity matrix obtained by the linearization process with respect to design as a natural counter part to the well known element stiffness matrix.

Additionally, the partial variations of the optimality condition $\boldsymbol{\mathcal{L}} := \delta_z L = 0$ of the shakedown analysis problem w.r.t. displacements and geometry must be calculated. The variations of the load factor and of the internal stresses $\delta \boldsymbol{z} := (\delta\beta, \delta \boldsymbol{y})^T$ are implicitly defined by

$$\frac{\partial \mathcal{L}}{\partial \boldsymbol{z}}\delta \boldsymbol{z} = -\frac{\partial \mathcal{L}}{\partial \boldsymbol{X}}\delta \boldsymbol{X} - \frac{\partial \mathcal{L}}{\partial \boldsymbol{u}_j^0}\delta \boldsymbol{u}_j^0. \tag{5}$$

A discretization of this formulation finally ends up in computable expressions for the sensitivity of the load factor w.r.t. geometry.

3.2 Numerical formulation and implementation

In Figure 1 the implemented steps for structural and shakedown analysis as well as for sensitivity analysis in one single iteration of the shape optimization are shown. See[3] for more details on the numerical formulation. First of all the solution $\underline{u}_j^0$ of the equilibrium conditions $\mathcal{G}_j = 0$ for any load vertex j is computed. Then the vectors of elastic stresses $\underline{\sigma}_j^{\mathrm{E}}(i)$ in any Gaussian point i of the discretized structure as well as their sensitivity w.r.t. displacements $\delta_{u_j^0}\underline{\sigma}_j^{\mathrm{E}}$ and geometry $\delta_s\underline{\sigma}_j^{\mathrm{E}}$ are calculated and stored. In a final step the solution of the global discretized shakedown optimization problem β and its sensitivity $\delta\beta$ is calculated by solving local optimization problems and performing their variation w.r.t. displacements and geometry in any Gaussian point.

4 SUMMARY

The proposed representation of variational design sensitivity describes an integrated treatment of all necessary linearizations in structural analysis and sensitivity analysis of shakedown problems.

The resulting geometries derived by optimization with shakedown constraints are well adapted to arbitrary load paths and an unlimited number of load cycles in the whole load domain. The comparison with results for problems with elastic and elasto-plastic deformations where only one load case is considered show that the level of structural safety as well as the savings derived in optimization with shakedown constraints are in-between these two limiting cases.

REFERENCES

[1] F. J. Barthold and K. Wiechmann. Variational design sensitivity for inelastic deformation. In *Proc. COMPLAS V* Barcelona, 792–797, 1997.

[2] J. A. König. Theory of shakedown of elastic-plastic structures. *Arch. Mech. Stos.* 18, 227–238, 1966.

[3] K. Wiechmann and E. Stein. Shape optimization for elasto-plastic deformation under shakedown conditions. *International Journal of Solids and Structures*, submitted.