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MODEL OF HARDENING CONCRETE WITH EVOLVING PARAMETERS

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Summary. *Chen model of plasticity was originally derived for solid materials with different strengths in compression and tension, such as concrete or some types of soil. In this work, it is tried to extend the range of application of the Chen model to hardening concrete. At very early ages, the microstructure of concrete is developing very rapidly which means that once compared with already hardened concrete, the material parameters of the Chen model should be a function of the microstructure evolution. The proposed modification of the Chen model considers this effect in terms of the degree of hydration. This approach is described and all necessary equations and parameters are given. The results of the modified Chen model are compared with experimental results.*

1 INTRODUCTION

The mechanical behavior of solidifying and further hardening concrete is influenced by the rapidly progressing hydration. Chen model of plasticity was originally derived for solid materials with different strengths in compression and tension. In this paper, a combination of Chen model of plasticity and an evolutionary function of the microstructure is proposed. Therefore, the material parameters used in the Chen model are formulated as a function of microstructure evolution. Due to this modification the range of application of the Chen model can be extend to hardening concrete.

2 METHOD OF SOLUTION

2.1 Evolution of microstructure of concrete

The solidifying and hardening state is dominated by the rapid progress of hydration, which must be taken into account without exception when dealing with solidifying or hardening concrete. The evolutionary function, as its name implies, is introduced in order to describe the evolutionary changes in the microstructure of solidifying and hardening concrete and therefore to control the mechanical behavior of concrete in the modeling.

The function of microstructure evolution, which expresses the effect of aging, was identified from experiments on the evolution of penetration resistance, pullout resistance and compressive strength of concrete at the ages up to the final setting time, as shown in the paper[1], and is given by

$$h(t_n) = a_5 \cdot \left(\frac{a_3 t_n^{a_2}}{a_1 + a_3 t_n^{a_2}} \right)^{a_4}, \qquad \begin{aligned} a_1 &= 10, \\ a_2 &= 9.164 - 7.2W/C, \\ a_3 &= 0.72, \\ a_4 &= 1, \\ a_5 &= 15, \end{aligned} \tag{1}$$

where t_n is a normalized time with respect to the final setting time, W/C is the water/cement ratio (in decimal) and a_1, a_2, a_3, a_4 and a_5 are empirical parameters.

Figure 1 shows the comparison between the compressive strength evolution expressed by the evolutionary function (1) (scaling parameters are given in Figure 1) and experimental data.

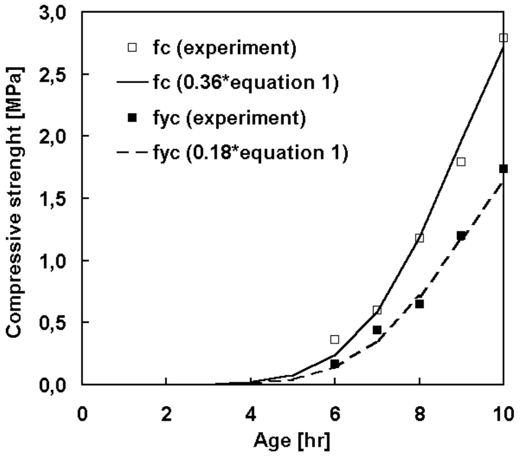

Figure 1: Compressive strength evolution

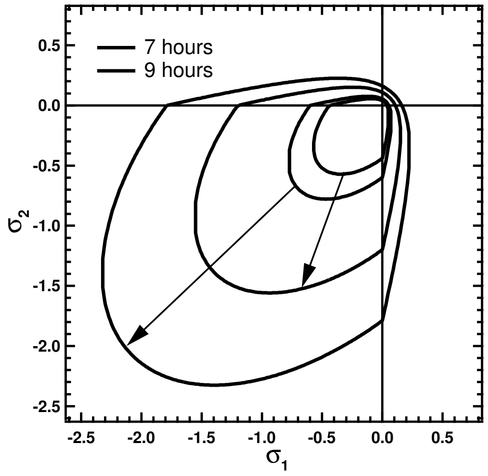

Figure 2: Evolution of initial yield and failure surfaces

2.2 Chen model of plasticity

Chen model of plasticity is a three-parameter model for concrete displaying isotropic hardening [2]. This model expresses the elastoplastic behavior of concrete. The typical behavior of concrete is varying stress-strain characteristic under tension and compression. Two different, but similar, functions were proposed for each of the loading surfaces, in the compression-compression region and in the tension-tension or tension-compression regions. In our example, compression loading is considered hence only the equations for the compression-compression region are in the paper introduced.

The failure surface is assume in the compression-compression region

$$f_u^c(\sigma, h) = J_2 + \frac{A_u(h)}{3} I_1 - \tau_u^2(h) = 0. \tag{2}$$

The initial yield surface in the compression-compression region is given by

$$f_0^c(\sigma, h) = J_2 + \frac{A_0(h)}{3} I_1 - \tau_0^2(h) = 0, \tag{3}$$

where $A_0(h)$, $\tau_0(h)$, $A_u(h)$ and $\tau_u(h)$ are material constants which can be determined from simple tests. They are determined as functions of the ultimate stresses under uniaxial compression, $f_c(h)$, and under equal biaxial compression, $f_{bc}(h)$, and of the initial yield stresses under similar conditions, $f_{yc}(h)$ and $f_{ybc}(h)$. With increasing strength of concrete the loading surfaces are expanding. This is evidenced in Figure 2 where the dotted lines denote the initial yield surface and the failure surface at the age 7 hours and the solid lines denote the initial yield surface and the failure surface at the age 9 hours.

2.3 Example of uniaxial compressive strength

Experimental data used for this example are presented in literature[3], see Table 1.

Age of concrete	f_{yc}	f_c
6 hours	0.18	0.36
7 hours	0.44	0.60
8 hours	0.65	1.18
9 hours	1.20	1.79
10 hours	1.74	2.79

Table 1: Yield stresses and ultimate stresses under uniaxial compression

In Table 1 are the yield stresses and the ultimate stresses from experimental data for concrete of the ages 6 to 10 hours listed. Chen in his paper[4] introduced a method of acquiring material parameters of his model from compressive strength. Also, the ultimate stress in compression is only known, it is possible to calculate the remaining values from

$$\begin{aligned}
f_t &= 0{,}09 f_c\ , \\
f_{bc} &= 1{,}16 f_c\ , \\
f_{yc} &= 0{,}6 f_c\ , \\
f_{yt} &= 0{,}054 f_c = 0{,}09 f_{yc}\ , \\
f_{ybc} &= 0{,}6 f_{bc} = 1{,}16 f_{yc}\ .
\end{aligned} \tag{4}$$

From Table 1 it is obvious that the yield stress is approximately equal to 60% of the ultimate stress as well as Chen[4] shows in his equations. In Figure 3 the results from experiments and the proposed model are compared.

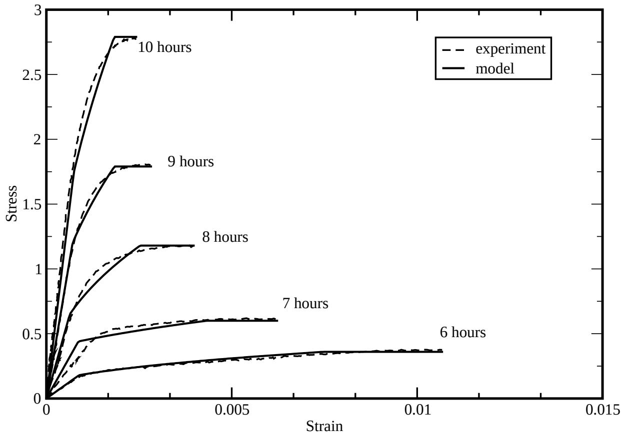

Figure 3: Comparison between experiment and model

3 CONCLUSIONS

In this paper, modification of Chen model was described, so that it is possible to use for solidifying and hardening concrete. Therefore, the parameters of Chen model are define as a function of microstructure evolution. The data for an example was obtained experimentally before.

Results obtained from this modified Chen model of plasticity was compared with results of the experiments. Very good agreement of both results verify the possibility of using Chen model of plasticity not only for hardened concrete but also for hardening concrete with yet evolving structure.

4 ACKNOWLEDGEMENT

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