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SIMPLE ELASTOPLASTICITY FOR NATURAL CLAYS

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Summary. *A simple framework is presented to model the mechanical behaviour of natural clays under general loading conditions, including non-monotonic loading. Transformation of the isotropic model to accommodate anisotropic hardening involves simple modifications to the flow rule and plastic modulus.*

1 INTRODUCTION

The constitutive modelling of anisotropic geomaterials is a particularly active area of ongoing research, and there are many different approaches even within the confines of elastoplasticity. Our objective here is to present a material-point model capable of describing the rate-independent mechanical behaviour of clays in general and of sensitive, over-consolidated, and natural clays in particular, under completely general loading conditions, including non-monotonic (e.g. seismic) loading, and cases where the principal loading axes may not coincide with the material's inherent axes of anisotropy. The modelling system is presented in simple yet general terms, motivated by the need for a clear conceptual framework within which to assess the suitability of the various options for modelling anisotropic hardening and destructuration processes. Subloading plasticity has been shown to have significant advantages, particularly in its ability to capture the softening behaviour of real soils[1]; and the failure surfaces predicted are similar to the empirically derived yield surfaces of the more conventional cap- or Hvorslev-type models. In contrast to some other approaches, subloading plasticity actually leads to simpler, more regular model equations. The decomposition of the yield function into intensity and strength functions presented here allows for a variation of the dilatancy response in sub-limit yielding. Transformation of the isotropic model to accommodate anisotropic (translational and rotational kinematic) hardening involves simple modifications to the flow rule and plastic modulus.

2 STATE

Let the state of our isothermal thermodynamic system be defined by three intensive symmetric second-order tensors: the effective stress $\boldsymbol{\sigma}$, the fabric stress $\boldsymbol{\zeta}$, and the backstress $\boldsymbol{\alpha}$. The fabric pressure p_ζ and the fabric stress ratio $\boldsymbol{\beta}$ are defined such that $p_\zeta(\boldsymbol{\beta}-\boldsymbol{\iota}) = \boldsymbol{\zeta}$ and $\boldsymbol{\beta} : \boldsymbol{\iota} = 0$, where $\boldsymbol{\iota}$ is isotropic and $\det[\boldsymbol{\iota}] = 1$. Suppose that $S = \{\boldsymbol{\sigma}, \boldsymbol{\zeta}, \boldsymbol{\alpha}\}$

describes the state of an elastoplastic system (S, F), where F is a scalar function of state identified as the normal yield function. We consider functions of the form

$$F(\boldsymbol{\sigma}, \boldsymbol{\alpha}, \boldsymbol{\zeta}) = f(\tilde{\bar{\boldsymbol{\sigma}}}, p_\zeta) \tag{1}$$

for some equivalent isotropic normal yield function f, where $\tilde{\bar{\boldsymbol{\sigma}}} = \boldsymbol{I}_\beta : \bar{\boldsymbol{\sigma}}$, $\bar{\boldsymbol{\sigma}} = \boldsymbol{\sigma} - \boldsymbol{\alpha}$, $\boldsymbol{I}_\beta = \boldsymbol{I} + \frac{1}{3}\boldsymbol{\beta} \otimes \boldsymbol{\iota}$ and $\boldsymbol{I}$ is the fourth-order identity tensor. The contours of F in principal stress space are obtained from those of f by 'rotating' the hydrostatic axis so that the point $-p_\zeta \boldsymbol{\iota}$ maps onto $\boldsymbol{\zeta}$, and then shifting the stress origin to $\boldsymbol{\alpha}$.

3 ISOTROPIC SYSTEM

Here we present the constitutive rate equations for an arbitrary isotropic elastoplastic system for which $\boldsymbol{\alpha} \equiv \boldsymbol{0} \equiv \boldsymbol{\beta}$. The normal yield function is decomposed into two parts:

$$f(\boldsymbol{\sigma}, p_\zeta) = f_1(\boldsymbol{\sigma}, p_\zeta) - f_2(\boldsymbol{\sigma}, p_\zeta) \tag{2}$$

where $f_1 \geq 0$ and $f_2 \geq 0$ are the intensity and strength functions respectively. This is a generalization of the form presented by Hashiguchi et al.[1], where $f_1 = f_1(\boldsymbol{\sigma})$ and $f_2 = f_2(p_\zeta)$. The normal yield ratio and the yield function itself are defined as

$$R(\boldsymbol{\sigma}, p_\zeta) \equiv f_1/f_2, \qquad Y(\boldsymbol{\sigma}, p_\zeta) \equiv f_1 - R f_2 \tag{3}$$

Note that the yield function is identically zero. In the limit state ($R = 1$) Y is identical to the normal yield function f. For a given strain rate $\dot{\boldsymbol{\varepsilon}}$ the evolution rules are:

$$\dot{\boldsymbol{\sigma}} = \boldsymbol{E} : (\dot{\boldsymbol{\varepsilon}} - \dot{\gamma}\boldsymbol{m}),\ \dot{p_\zeta} = \dot{\gamma} h_p,\ \dot{\gamma} \neq 0 \Rightarrow \dot{R} = \dot{\gamma} h_R \tag{4}$$

where $\boldsymbol{E}$ is the elastic modulus, $\boldsymbol{m}$ is the flow direction tensor, h_R is the subloading function, h_p is the isotropic hardening function, and $\dot{\gamma}$ is the consistency parameter. It is assumed that h_R satisfies the conditions $\text{sgn} h_R = \text{sgn}(1 - R), \quad \lim_{R \to 0} h_R = \infty$
The differential operator $\boldsymbol{n}$ and the plastic modulus H are defined such that

$$\dot{\gamma} \neq 0 \Rightarrow \dot{Y} = \boldsymbol{n} : \dot{\boldsymbol{\sigma}} - H\dot{\gamma} \tag{5}$$

It follows from the consistency condition ($\dot{Y} = 0$) and the loading criterion ($\boldsymbol{n} : \boldsymbol{E} : \dot{\boldsymbol{\varepsilon}} \leq 0 \Rightarrow \dot{\gamma} = 0$) that $\dot{\gamma} = \boldsymbol{\gamma} : \dot{\boldsymbol{\varepsilon}}$ and

$$\boldsymbol{\gamma} = \begin{cases} \boldsymbol{0} & \text{if } \boldsymbol{n} : \boldsymbol{E} : \dot{\boldsymbol{\varepsilon}} \leq 0 \\ \dfrac{\boldsymbol{n} : \boldsymbol{E}}{H + \boldsymbol{n} : \boldsymbol{E} : \boldsymbol{m}} & \text{otherwise} \end{cases} \tag{6}$$

$$\boldsymbol{n} = \boldsymbol{n}_o(\boldsymbol{\sigma}, p_\zeta) \equiv \frac{\partial f}{\partial \boldsymbol{\sigma}} + (1 - R)\frac{\partial f_2}{\partial \boldsymbol{\sigma}} \tag{7}$$

$$H = H_I(\boldsymbol{\sigma}, p_\zeta) \equiv f_2 h_R - \left[R\frac{\partial f}{\partial p_\zeta} + (1-R)\frac{\partial f_1}{\partial p_\zeta}\right] h_p \tag{8}$$

Purely elastic behaviour is obtained when $R = 0$. The stress rate may be written:

$$\dot{\boldsymbol{\sigma}} = \boldsymbol{E} : (\boldsymbol{I} - \boldsymbol{m} \otimes \boldsymbol{\gamma}) : \dot{\boldsymbol{\varepsilon}} \tag{9}$$

Our interpretation is slightly different from the traditional one since $\boldsymbol{n}$ is in general not equal to the stress-gradient of the normal yield function: equality holds either when the strength function f_2 is independent of $\boldsymbol{\sigma}$, or in the limit state ($R = 1$). This gives a flow rule that would normally be considered non-associative while preserving the symmetry of the elastoplastic modulus.

4 ANISOTROPY AND STRUCTURE

Rotational (kinematic) and (translational) kinematic hardening are incorporated by writing the yield function in terms of $\tilde{\bar{\boldsymbol{\sigma}}}$. The consistency condition then implies

$$\boldsymbol{n} = \boldsymbol{n}_o(\tilde{\bar{\boldsymbol{\sigma}}}, p_\zeta) : \boldsymbol{I}_\beta, \ \ H = H_I(\tilde{\bar{\boldsymbol{\sigma}}}, p_\zeta) + \boldsymbol{n} : (\bar{p}\boldsymbol{h}_\beta + \boldsymbol{h}_\alpha) \tag{10}$$

where $\bar{p} = -\frac{1}{3}\bar{\boldsymbol{\sigma}} : \boldsymbol{\iota}$, and $\boldsymbol{h}_\beta$ and $\boldsymbol{h}_\alpha$ are material state functions defining the rotational and kinematic hardening rules:

$$\dot{\boldsymbol{\beta}} = \dot{\gamma}\boldsymbol{h}_\beta, \ \ \boldsymbol{h}_\beta : \boldsymbol{\iota} = 0; \quad \dot{\boldsymbol{\alpha}} = \dot{\gamma}\boldsymbol{h}_\alpha \tag{11}$$

Equations (6-9) then apply as for the isotropic case. The destructuration process is incorporated most simply by introducing the sensitivity $r \geq 1$ as an additional independent internal intensive scalar variable and supposing that changes in r affect only the material strength. This is achieved via the following transformation:

$$p_\zeta \mapsto r p_\zeta, \qquad h_p \mapsto r h_p - h_r p_\zeta, \qquad \dot{r} = -\dot{\gamma} h_r \tag{12}$$

where $h_r \geq 0$ controls the degradation of r, and $\lim_{r\to 1} h_r = 0$. More realistic would be to suppose for example that r also affects the stiffness, via $\boldsymbol{E}$, and the capacity for anisotropy, via $\boldsymbol{h}_\beta$.

5 APPLICATION

The fact that the elastic centre is associated with the point of zero intensity ($R = 0$) motivates the following decomposition of the modified Cam clay yield function:

$$f_1(\boldsymbol{\sigma}, p_\zeta) = q^2/M^2 + (p-\rho)^2, \ \ f_2(\boldsymbol{\sigma}, p_\zeta) = p(p_\zeta - 2\rho) + \rho^2 \tag{13}$$

where $p = -\frac{1}{3}\boldsymbol{\sigma} : \boldsymbol{\iota}$, $q \equiv \mp\sqrt{\frac{3}{2}} \parallel \boldsymbol{\sigma}' \parallel$, $\boldsymbol{\sigma}' = \boldsymbol{\sigma} + p\boldsymbol{\iota}$, M is the critical stress ratio, and ρ is the pressure at the elastic centre. Two natural choices are $\rho = 0$ [1] and $\rho = \frac{1}{2}p_\zeta^2$. The flow rule for the anisotropic system is obtained by substituting (13) into (10) a:

$$\bar{\boldsymbol{n}} = \frac{3}{M^2}\tilde{\bar{\boldsymbol{\sigma}}}' - \left[\frac{2}{3}(\bar{p} - p_\delta) - \frac{\tilde{\bar{\boldsymbol{\sigma}}}' : \boldsymbol{\beta}}{M^2}\right]\boldsymbol{\iota} \tag{14}$$

where $p_\delta = \frac{1}{2}Rp_\zeta + \rho(1-R)$ and $M = M(\tilde{\bar{\boldsymbol{\sigma}}})$ and $R = R(\tilde{\bar{\boldsymbol{\sigma}}}, p_\zeta)$. The effect of subloading with $h_R = -u\ln(R/R_o)\|\boldsymbol{m}\|$ is shown in Fig 1 where $R_o = 0.1$ is the size of the normal yield surface (i.e. the bubble) divided by the size of the bounding surface.

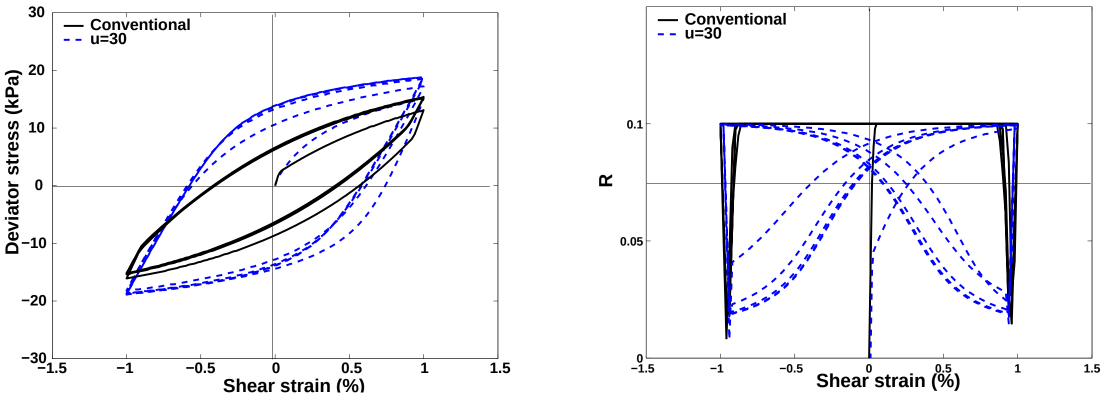

Figure 1: Effect of subloading on the kinematic hardening model.

6 CONCLUSIONS

A simple system is presented for the smooth elastoplastic modelling of natural clay, in which the effects of anisotropy and destructuration are easily incorporated by modifying the flow rule and plastic modulus of an equivalent isotropic system. A shift stress has been introduced into the decomposition of the yield function, allowing a degree of control over the dilatancy response in the sub-limit state without altering the limit-state response or the shape of the normal yield surface. The validity of this approach has been demonstrated by reproducing the results of some simple tests, which will be presented at the conference.

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