VIII International Conference on Computational Plasticity
COMPLAS VIII
E. Oñate and D. R. J. Owen (Eds)


# COMPUTATION OF THE YIELD SURFACE OF DOWELLED TIMBER JOINTS BY USING A FEASIBLE DIRECTION METHOD

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**Key words:** Limit analysis, Yield criterion, Timber, Joint.

**Summary.** *This article presents recent works in relation with the strength of dowelled timber connections. When numerous dowels are involved, computing the global strength of a joint can be quite tedious. The proposed method consists in using the individual embedment strength of each dowel, computed according to EC5*[1]*, and finding a lower bound of the global strength of the joint by using the static theorem of limit analysis*[2]*. The resulting optimization problem is solved with a feasible direction method*[3].

## 1 INTRODUCTION

Let us consider two timber members 1 and 2 connected by a set of steel dowels of diameter *d*. Hence, the joint is in double shear, which means that member 1 is made of twin beams of individual thickness $t_1$ on each side of member 2 which is a single beam of thickness $t_2$.

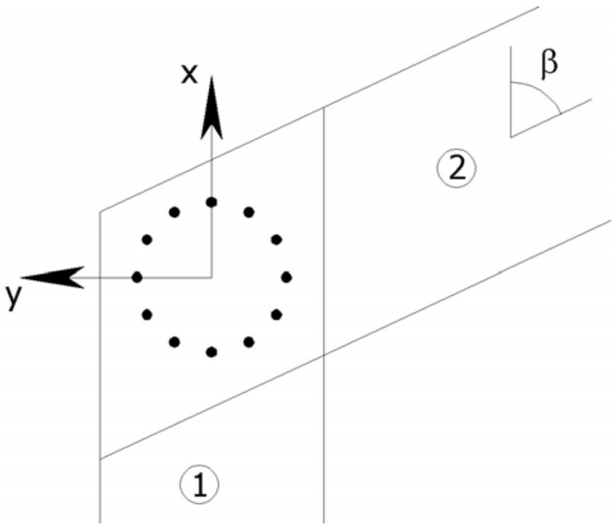


Figure 1: dowelled joint

As shown on figure 1, those members are subjected to several actions: bending moments $M_{Sd1}$ and $M_{Sd2}$, shear forces $V_{Sd1}$ and $V_{Sd2}$ and normal forces $N_{Sd1}$ and $N_{Sd2}$. Note that the magnitudes of these forces are computed for every considered ultimate limit state (ULS) load combination. The aim of the study consists in generating the yield surface of the joint in a 3-dimension space, taking into account the interaction of bending moment, shear force and normal force.

## 2 NON-DIMENSIONAL FORCES

$M_{Rd1}$ and $M_{Rd2}$ denote the strength of members 1 and 2 in pure bending. $N_{Rd1}$ and $N_{Rd2}$ denote their strength in compression while $V_{Rd1}$ and $V_{Rd2}$ are the respective shear strengths. If the same symbols are used with a * to denote the respective joint strengths, it is convenient to define the following variables:

$$\mu_1 = \frac{M^*_{Rd1}}{M_{Rd1}} \qquad \mu_2 = \frac{M^*_{Rd2}}{M_{Rd2}}$$
$$\eta_1 = \frac{N^*_{Rd1}}{N_{Rd1}} \qquad \eta_2 = \frac{N^*_{Rd2}}{N_{Rd2}} \tag{1}$$
$$\nu_1 = \frac{V^*_{Rd1}}{V_{Rd1}} \qquad \nu_2 = \frac{V^*_{Rd2}}{V_{Rd2}}$$

This way, the yield criterion of the joint may be represented as a surface of equation:

$$y(\mu_1, \eta_1, \nu_1) = 0 \tag{2}$$

This surface can be computed point by point, by setting for example:

$$\mu_1 = \theta\eta_1 \tag{3}$$
$$\nu_1 = \varphi\eta_1$$

## 3 MATHEMATICAL FORMULATION

Let us consider a local coordinate system *0xy* attached to member 1, and a set of *n* dowels of index *i*, each of them located at point ($x_i$, $y_i$) and subjected to unknown forces $F_{xi}$ and $F_{yi}$. In order to find a point of the yield surface of the joint, we have to maximize:

$$\sqrt{\mu_1^2 + \eta_1^2 + \nu_1^2} \tag{4}$$

Two kinds of constraints must be taken into account. First of all, to ensure that the set of forces $F_{xi}$ and $F_{yi}$ is plastically admissible, the force applied at each dowel may not exceed its embedment force $F_{Vj}$ (with j=1..4) given by the equations of Johansen used in EC5[1]. The resulting constraints are:

$$F_{xi}^2 + F_{yi}^2 \le F_{Vj}^2 \qquad \begin{matrix} i \in [1,n] \\ j \in [1,4] \end{matrix} \tag{5}$$

The set of forces $F_{xi}$ and $F_{yi}$ must also be statically admissible, thus compatible with the forces and moments acting on the members:

$$\begin{aligned} M^*_{Rd1} &= \sum_{i=1}^{n} \left( F_{xi} y_i - F_{yi} x_i \right) \\ N^*_{Rd1} &= \sum_{i=1}^{n} F_{xi} \\ V^*_{Rd1} &= \sum_{i=1}^{n} F_{yi} \end{aligned} \tag{6}$$

The forces $F_{xi}$ and $F_{yi}$ are the design variables of the optimization problem. The objective function (4) can be expressed from (6) as a function of those forces. In order to allow the use of a feasible direction method[3], constraints (3) must be converted into inequality constraints. If $\varepsilon$ denotes a small positive number acting as a tolerance, they become:

$$\begin{aligned} \mu_1 &\geq \theta\eta_1 - \varepsilon \qquad \nu_1 \geq \varphi\eta_1 - \varepsilon \\ \mu_1 &\leq \theta\eta_1 + \varepsilon \qquad \nu_1 \leq \varphi\eta_1 + \varepsilon \end{aligned} \tag{7}$$

Thus for a $n$-dowel joint, the optimization problem involves $4n+4$ constraints. For given values of $\theta$ and $\varphi$, the solution of the problem leads to a point of the yield surface. With this method, the designer has two options:

For each ULS load combination, the corresponding values of $\theta$ and $\varphi$ can easily be computed from the actions $M_{Sd1}$, $N_{Sd1}$ and $V_{Sd1}$. The related directions allow to compute one point of the yield surface for each combination. A quick verification of the joint is then possible.

On the other hand, the whole yield surface may be generated, using a point-by-point construction. In this case, the behaviour of the joint can be studied thoroughly.

## 4 CONCLUSIONS

The implementation of a feasible direction algorithm combined with the static theorem of limit analysis allows a correct assessment of the strength of dowelled timber connections, while the more widely used simplified approach tends to underestimate it.

The computation time for optimization remains reasonable, compared to the obtained gain of accuracy on the joint strength.

A further step should be taken in considering not only the strength of a given joint, but also an optimization of the joint itself under a given set of load combinations.

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