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PROPAGATION OF INTERACTING MICROCRACKS RESULTS IN TENSILE STRAIN-SOFTENING OF BRITTLE MATERIALS – A COMBINED FRACTURE AND MICROMECHANICS APPROACH

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Summary. *The experimentally observed stress decline in displacement-controlled experiments is referred to as strain-softening. Although microcracking is a commonly accepted reason for strain softening, the majority of theoretical developments is purely macroscopic. To overcome inherent shortcomings of this approach, we propose a micromechanics-based damage propagation law by combining (i) the propagation law for a single penny-shaped crack embedded in an infinite matrix subjected to remote stress (taken from linear-elastic fracture mechanics) and (ii) stiffness estimates for RVEs of a material damaged by interacting microcracks (based on continuum micromechanics). This combination provides theoretical evidence that propagation of interacting microcracks results in tensile strain-softening, as is observed in macroscopic laboratory tests on brittle materials. The initial degree of damage, i.e. the initial crack size and the number of cracks per unit volume, implies two different types of tensile strain-softening: (i) continuous strain-softening in case of initial damage beyond a critical value, and (ii) instantaneous stress drop at the peak load ("snap-back") in case of initial damage below a critical value.*

1 INTRODUCTION

Fracture processes are the most common reason for degradation and failure of brittle materials. The majority of related material models, developed over the last decades, are based on the mathematical description of observations from macroscopic laboratory tests, such as, e.g., uniaxial compression tests, uniaxial tensile tests, tensile splitting tests, biaxial compression tests, triaxial tests, confined compression tests, and hydrostatic compression tests. Dating back to the late 1960s, constitutive modeling of brittle materials was significantly influenced both by damage theory and plasticity theory. Accordingly, loading surfaces in the principal stress space were derived for brittle materials from experimental data. They include limit surfaces for elastic stress states and stress states at failure. Cracking phenomena at observation scales below macroscopic testing were not considered explicitly, but rather in the form of flow rules or damage evolution rules. Therefore, traditional constitutive models involve a lot of model parameters which are not directly associated with microscopic cracking phenomena. Hence, determination of model parameters that are valid for different loading conditions is an awkward task.

Some progress was made by considering different crack orientations and formulating constitutive laws on the respective "microplanes". Most of such microplane models are based on the "kinematic constraint", i.e., on the assumption that strains acting on the microplanes can be derived by projection of the macroscopic strain tensor. These local strains are the input for stress-strain relations defined separately for each plane. Depending on the sophistication of the respective model, these local constitutive laws account for elasticity, plasticity, and damage. Based on the principle of virtual work, and, more recently, embedded in the framework of thermodynamics, the macroscopic stresses are derived by integration over the stresses acting on the microplanes [1]. Accordingly, microplane models still focus on the effects of cracks, rather than modeling cracks as components of the morphology of brittle materials.

2 CONTINUUM MICROMECHANICS OF CRACKED (DAMAGED) MATERIALS

To consider cracks as components of the morphology of brittle materials is the aim of recent developments in 3D continuum micromechanics of damaged brittle materials [2, 3, 4]: Penny-shaped cracks are introduced as a part of the microstructure of brittle materials, i.e., on a characteristic length scale of several microns to several millimeters, which is at least one order of magnitude smaller than the characteristic length-scale of laboratory specimens (ℓ = 5-10 cm). Cracks of various orientations are considered, whereby cracks of similar orientation are collected to so-called crack families. In general, each crack family comprises a different number of cracks of various sizes, which is taken into account by the introduction of Budiansky's crack-density parameter (see [5]), separately for each crack family. A distinction between open and closed cracks is made, and the respective unilateral constraints are taken into account [2]. Moreover, different types of friction between the crack faces (free sliding, von Mises friction, Coulomb friction) can be considered [3].

To end up with a macroscopic description of a brittle material, a homogenization technique is used. For this purpose, cracks are modeled as ellipsoidal inclusions embedded in an elastic matrix. Averaging over the solutions of Eshelby's inclusion problem (see [6]), solved for each crack family, leads to the well-known Mori-Tanaka homogenization scheme (see e.g. [7]). It delivers the effective stiffness-tensor of a representative volume element of a brittle material of characteristic length ℓ, taking into account interactions between the microcracks.

3 CRACK PROPAGATION BASED ON A LINK BETWEEN FRACTURE-MECHANICS AND MICROMECHANICS

Recently, it was shown by the authors that modeling the propagation of microcracks can be based on the energy approach of linear elastic fracture mechanics [8]. This way, a combined fracture-micromechanics damage model is obtained, which requires a relatively small number of physically reasonable input parameters: Young's modulus (E), Poisson's ratio (ν) and the critical mode I fracture toughness of the material (K_{Ic}), as well as the radius (a) and the number (N) of the penny-shaped cracks per unit volume.

4 UNIAXIAL TENSILE STRESS-STRAIN BEHAVIOR OF IDEALLY BRITTLE MATERIALS – INVESTIGATED BY A COMBINED FRACTURE-MICROMECHANICS DAMAGE MODEL

Herein, we focus on the behavior of brittle materials in direct tension tests, as predicted by the proposed combined fracture-micromechanics based damage model. Remarkably, macroscopic strain softening is obtained as a result (Fig. 1). Moreover, the initial degree of damage, i.e. the initial crack size and the number of cracks per unit volume, implies two different types of tensile strain-softening: (i) continuous strain-softening in case of initial damage beyond a critical value, and (ii) instantaneous stress drop at the peak load ("snap-back") in case of initial damage below a critical value.

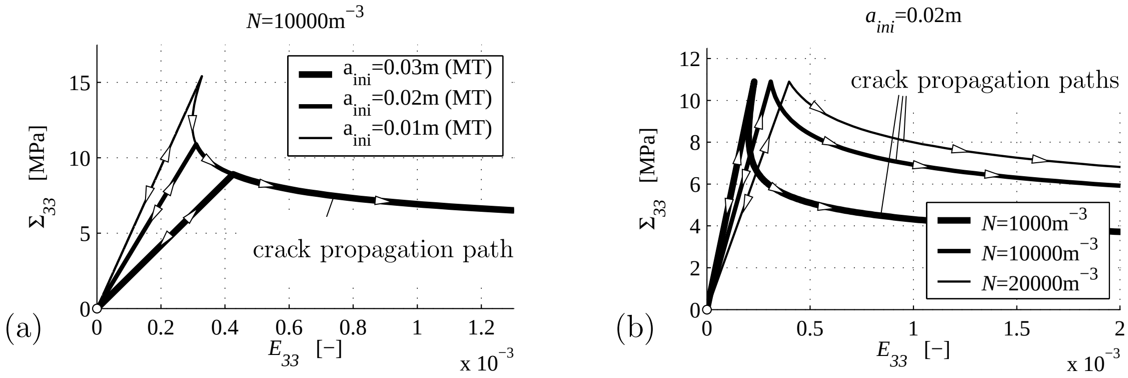

Figure 1: Stress-strain behavior of brittle Silurian sedimentary rock in a uniaxial tensile test considering interacting cracks (a) effect of initial crack radius and (b) effect of crack density; a_{ini}... initial crack radius, N... number of cracks per unit volume; $E = 49.4\,\mathrm{GPa}$, $\nu = 0.24$ $K_{Ic} = 1.74\,\mathrm{MPa}\sqrt{m}$

5 CONCLUSIONS

In this paper, a combined fracture-micromechanics damage model was addressed. Thereby cracks were introduced as a morphological unit of the materials (rather than considering "smeared" effects of cracks). Tensile strain softening was identified as the consequence of the propagation of interacting microcracks.

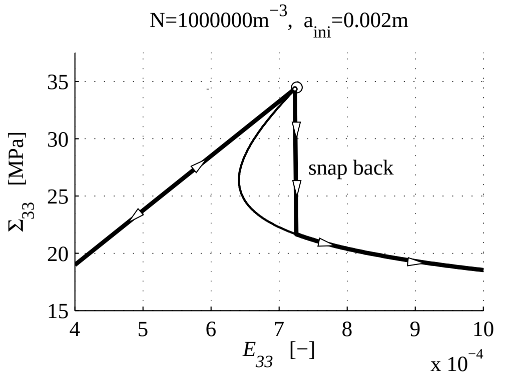

Figure 2: Stress-strain diagrams for uniaxial tensile tests: results of a crack-radius controlled experiment (thin line) and a strain-driven experiment (thick line) exhibiting a discontinuous (snap-back) strain-softening behavior; $a_{ini}\ldots$ initial crack radius, $N\ldots$ number of cracks per unit volume; $E = 49.4\,\mathrm{GPa}$, $\nu = 0.24$ $K_{Ic} = 1.74\,\mathrm{MPa}\,\sqrt{m}$

Recently, the proposed approach was successfully extended to describe axial splitting under uniaxial compression [9]. The extended model, e. g., accounts for a realistic ratio between the uniaxial tensile strength and the uniaxial compressive strength, and for the transition from axial splitting to faulting in confined uniaxial compression tests. Because the combined fracture-micromechanics damage model so far gave satisfactory results [8,9], one may well expect that the described approach can significantly enhance the insight into cracking of brittle materials and pave the way towards more realistic constitutive modeling of such materials subjected to mechanical loads.

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