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THREE-DIMENSIONAL SIMULATION OF CONCRETE FRACTURE USING EMBEDDED CRACK ELEMENTS WITHOUT ENFORCING CRACK PATH CONTINUITY

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Summary. *This paper presents a numerical implementation of the cohesive crack model for the three-dimensional analysis of concrete fracture based on finite elements with embedded strong discontinuity. The need for a tracking algorithm is avoided by using a consistent procedure for the selection of the separated nodes, and by letting the crack to adapt itself to the stress field while the crack opening does not exceed a small threshold value.*

1 INTRODUCTION

In the last years the strong discontinuity approach (SDA) has proven to be a sound, elegant and useful framework for the numerical simulations of localization phenomena. Two basic families of strategies based on the SDA exist: (1) those in which the displacement discontinuities are handled as global degrees of freedom, and (2) those in which the discontinuities are local to the finite elements and can be be solved at the element level. This last one is, in principle, the most simple way to implement the SDA in standard finite element programs and this strategy has been used in the development of finite elements with cracks modeled as embedded discontinuities.

However, this approach is not free from problems. It is known that if a straight implementation is carried out, the cracks tends to lock. Therefore strategies have been designed to avoid locking consisting in the so called *crack tracking* and *exclusion zones*. Crack tracking enforces crack path continuity, which is extraneous to the SDA itself since the internal strong discontinuity kinematics of the finite element is a kinematical enrichment with *incompatible modes* in the spirit of the EAS methods of Simo and Rifai and this does not by itself require any kind of inter-elemental path compatibility. Exclusion zones are devised to avoid secondary cracking that would (and should actually) occur. It is therefore of the greater interest, not only practical but also theoretical, to study formulations that circumvent the need for enforcing crack-path continuity. The objective of this presentation is to show how, by means of simple considerations, it is possible in many cases to avoid the enforcement of crack-path continuity in such a way that the

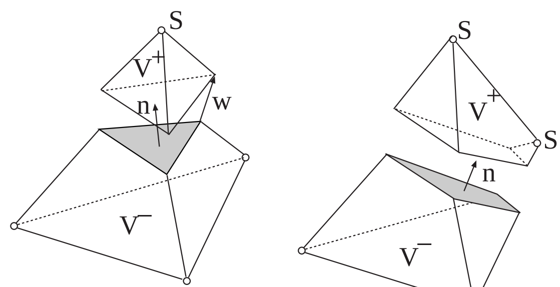

Figure 1: Tetrahedral finite element. S: Solitary node

macroscopic crack arises from purely local assumptions. As an application to the numerical three dimensional simulation of concrete fracture, the details of an element with a cohesive embedded crack that self-propagates will be discussed.

2 Finite element kinematics

Figure 1 shows a tetrahedral finite element after cracking. The element is separated in two zones V^- and V^+. The basic geometrical parameters are: the crack normal $\mathbf{n}$, and the vector $\mathbf{b}^+$ defined as the sum of the gradient of the linear shape functions associated to the separated nodes. The only kinematical variable of the crack is the crack opening vector $\mathbf{w}$.

For the constant strain tetrahedral element the continuum strain can be expressed as:

$$\boldsymbol{\epsilon}^c = \boldsymbol{\epsilon}^a - \left[\mathbf{b}^+(\boldsymbol{x}) \otimes \mathbf{w}\right]^S \qquad (1)$$

where $\boldsymbol{\epsilon}^a$ is the apparent strain tensor computed from the nodal displacements and the superscript ^{S} indicates the symmetric part of a tensor.

3 Cohesive crack model

A simple generalization of the cohesive crack to mixed mode is used and is assumed that the traction vector $\mathbf{t}$ transmitted across the crack faces is parallel to crack displacement vector $\mathbf{w}$. It is further assumed that the unloading-reloading is linear through the origin:

$$\mathbf{t} = \frac{f(\tilde{w})}{\tilde{w}}\mathbf{w} \quad \text{with } \tilde{w} = \max\left(|\mathbf{w}|\right) \qquad (2)$$

where $f(\tilde{w})$ is the softening function for pure opening mode.

4 Equilibrium equation

If we assume that the material outside the crack remains elastic and force local equilibrium as $\mathbf{t} = \boldsymbol{\sigma}\mathbf{n}$, the crack displacement for a given nodal displacement state, can be

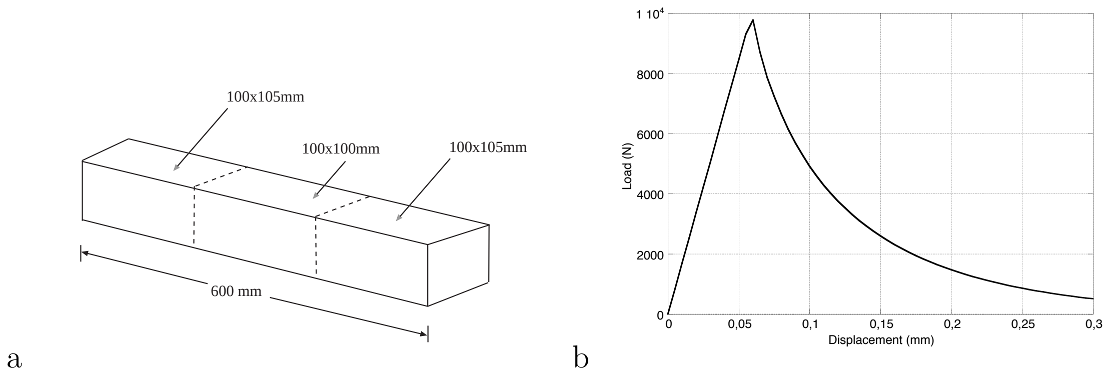

Figure 2: a) Simple tension test specimen. Material properties: $E = 10000$ MPa, $f_t = 1$ MPa, $G_f = 0.1$ N/mm. Exponential softening. b) Load-displacement curve.

obtained by solving the following nonlinear equation:

$$\mathbf{t} = \frac{f(\tilde{w})}{\tilde{w}}\mathbf{w} = \mathbf{n} \cdot \mathbf{E} : \boldsymbol{\epsilon}^a - (\mathbf{n} \cdot \mathbf{E} \cdot \mathbf{b}^+) \cdot \mathbf{w} \tag{3}$$

5 Crack initiation

Initially, $\mathbf{w} = \mathbf{0}$ in the uncracked element, and $\mathbf{n}$ and $\mathbf{b}^+$ are undefined. Thus, the element loads elastically and $\boldsymbol{\sigma} = \mathbf{E}\boldsymbol{\epsilon}^a$ until the maximum principal stress exceeds the tensile strength. Then a crack is introduced perpendicular to the direction of the maximum principal stress, and $\mathbf{n}$ is computed as a unit eigenvector of $\boldsymbol{\sigma}$

By observing that the location of the crack in the element does not enter the equilibrium equation (3), there is no need to resort to a tracking algorithm to select the solitary node or nodes. These are determined by requiring that the angle between $\mathbf{n}$ and $\mathbf{b}^+$ be the smallest possible, ie:

$$\frac{\mathbf{n} \cdot \mathbf{b}^+}{|\mathbf{b}^+|} = \max \tag{4}$$

The foregoing procedure is strictly local: no crack continuity is enforced or crack exclusion zone defined. This leads in some circumstances to locking after a certain crack growth. Such locking seems to be due to a bad prediction of the cracking direction in the element ahead of the pre-existing crack. To overcome this problem without introducing global algorithms we just introduce a certain amount of crack adaptability within each element. Therefore we allow the crack to adapt itself to the later variations in principal stress direction while its opening is small. Threshold values must be related to the softening properties of the material and values of the order of 0.1-0.2 G_F/f_t are usually satisfactory.

6 Numerical experiments

Figures 2 and 3 show a prismatic specimen with a uniform central section loaded axially. The purpose of the experiment was twofold: 1) to check the ability of the proposed model

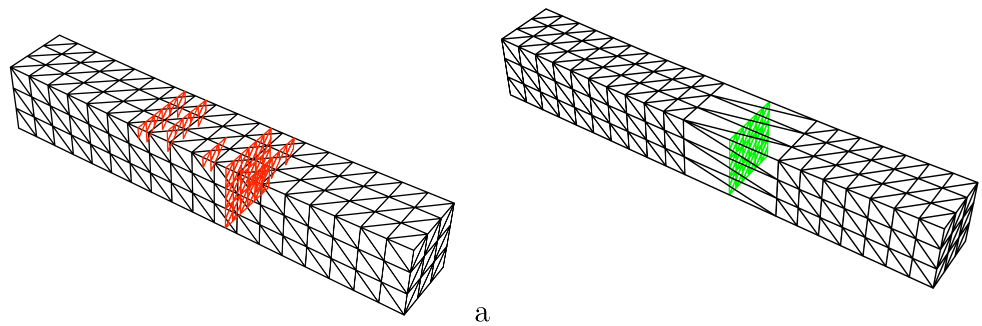

Figure 3: Deformed mesh: a) displacement= 0.06 mm, no consolidated cracks, b) displacement = 0.3 mm single consolidated crack

to predict a single plane of fracture without forcing the crack continuity and 2): to check the possibility of localization in a uniform stress field without artificial perturbations in the mechanical or geometrical characteristics of any element. Other succesful tests will be reported elsewhere

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