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A FE APPROACH FOR THE COMPUTATION OF STRONG AND WEAK DISCONTINUITIES AT FINITE STRAINS

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Summary. *A discontinuous finite element method for the computational modelling of strong and weak discontinuities at finite strains is introduced. The location of the interface is independent of the mesh structure and therefore discontinuous elements are introduced, to capture the jump in the deformation map and its gradient, respectively.*

1 INTRODUCTION

In the present contribution a discontinuous finite element method for the computational modelling of strong and weak discontinuities in geometrically nonlinear elasticity is introduced. Thereby we denote with 'strong discontinuities' jumps in the deformation map, for example cracks, and with 'weak discontinuities' jumps in the deformation gradient, which occur e.g. at material interfaces. The discontinuity shall not be limited to interelement boundaries. Therefore we construct discontinuous elements. The suggested method is closely related to the approach suggested by Hansbo and Hansbo[1,2], where an unfitted finite element method was introduced to simulate strong and weak discontinuities, by means of an extended version of Nitsche's method.
A variational formulation based on the principle of stationary potential energy is derived for both, the modelling of strong and weak discontinuities. To model strong discontinuities the cohesive crack concept is adopted. The inelastic material behaviour is covered by a cohesive constitutive law, which associates the cohesive tractions, acting on the crack surfaces, with the jump in the deformation map. The formulation extends the approach[3] to finite strains.
If weak discontinuities, for example material interfaces or inclusions, are considered, the deformation map shall be continuous but its gradient can posses a jump along the interface. Since the same discontinuous elements are used, the continuity of the deformation map has to be ensured. Therefore a finite element method, based on Nitsche's method[4], for geometrically nonlinear elasticity is formulated. By means of Nitsche's method the continuity of the deformation map is ensured in a weak sense, but the discontinuous element formulation allows for jumps of its gradient.

2 GOVERNING EQUATIONS

We consider a body $\mathcal{B}$ which is divided by a discontinuity Γ into the parts $\mathcal{B}^1$ and $\mathcal{B}^2$. The associated normal vector $\boldsymbol{N}$ points from $\mathcal{B}^2$ to $\mathcal{B}^1$. We consider a nonlinear and non-continuous deformation map $\boldsymbol{\varphi}$, which maps the body from the reference configuration to its spatial configuration. The deformation map as well as its gradient and the related strain measures are defined separately for each continuous part of the body

$$\boldsymbol{\varphi}(\boldsymbol{X}) = \left\{ \begin{array}{lcl} \boldsymbol{\varphi}^1(\boldsymbol{X}) & : & \mathcal{B}^1 \to \mathcal{S}^1 \\ \boldsymbol{\varphi}^2(\boldsymbol{X}) & : & \mathcal{B}^2 \to \mathcal{S}^2 \end{array} \right. \qquad \boldsymbol{F} = \left\{ \begin{array}{lcl} \boldsymbol{F}^1 & = & \nabla_{\boldsymbol{X}} \boldsymbol{\varphi}^1 \\ \boldsymbol{F}^2 & = & \nabla_{\boldsymbol{X}} \boldsymbol{\varphi}^2. \end{array} \right. \tag{1}$$

The variational formulation, concerning strong discontinuities, is given by

$$\delta\Pi(\boldsymbol{\varphi}, \delta\boldsymbol{\varphi}) = \int_{\mathcal{B}} \delta\boldsymbol{F} : \boldsymbol{P}\, dV + \int_{\Gamma} [\![\delta\boldsymbol{\varphi}]\!] \cdot \bar{\boldsymbol{t}}_0([\![\boldsymbol{\varphi}]\!])\, d\bar{A} - \int_{\partial\mathcal{B}_N} \delta\boldsymbol{\varphi} \cdot \boldsymbol{t}_0\, dA = 0, \tag{2}$$

whereby $\boldsymbol{P}$ denotes the Piola stress tensor, which is derived from the strain energy function by $\boldsymbol{P} = \partial\Psi(\boldsymbol{F})/\partial\boldsymbol{F}$. The additional interfacial contribution is due to the cohesive traction vector $\bar{\boldsymbol{t}}_0 = \partial\bar{\Psi}([\![\boldsymbol{\varphi}]\!])/\partial[\![\boldsymbol{\varphi}]\!]$, which is calculated as the derivative of the cohesive potential with respect to the jump in the deformation map.
For the bulk material we assume hyperelastic material behaviour of a compressible Neo-Hooke type. Since we want the cohesive potential to depend only on the jump in the deformation map, we introduce the following cohesive potential which leads to the traction-separation law and results in a symmetric formulation

$$\bar{\Psi}([\![\boldsymbol{\varphi}]\!]) = \frac{\alpha}{\beta}\left[1 - \exp\left(-\beta\,|[\![\boldsymbol{\varphi}]\!]|\right)\right] \qquad \bar{\boldsymbol{t}}_0 = \alpha \exp\left(-\beta\,|[\![\boldsymbol{\varphi}]\!]|\right) \frac{[\![\boldsymbol{\varphi}]\!]}{|[\![\boldsymbol{\varphi}]\!]|}. \tag{3}$$

Thereby α and β are material parameter.
The variational formulation for the case of weak discontinuities contains additional interfacial contributions due to Nitsche's method and is introduced as

$$\begin{aligned} \delta\Pi(\boldsymbol{\varphi}, \delta\boldsymbol{\varphi}) = & \int_{\mathcal{B}} \delta\boldsymbol{F} : \boldsymbol{P} dV + \int_{\Gamma} [\![\delta\boldsymbol{\varphi}]\!] \cdot \{\boldsymbol{P}\} \cdot \boldsymbol{N} d\bar{A} + \int_{\Gamma} [\![\boldsymbol{\varphi}]\!] \cdot \{\mathbf{A} : \delta\boldsymbol{F}\} \cdot \boldsymbol{N} d\bar{A} \\ & + \int_{\Gamma} \theta\, [\![\delta\boldsymbol{\varphi}]\!] \cdot [\![\boldsymbol{\varphi}]\!] d\bar{A} - \int_{\partial\mathcal{B}_N} \delta\boldsymbol{\varphi} \cdot \boldsymbol{t}_0\, dA = 0, \end{aligned} \tag{4}$$

whereby the tangent operator $\mathbf{A}$ is calculated as the second derivative of Ψ with respect to $\boldsymbol{F}$. The scalar θ is a penalty parameter, which depends on the discretization and has to be sufficiently large to assure the stability of the method.

3 FINITE ELEMENT FORMULATION AND NUMERICAL EXAMPLES

The weak governing equations are solved using finite elements which allow for a jump in the deformation map and the deformation gradient. In the discontinuous elements

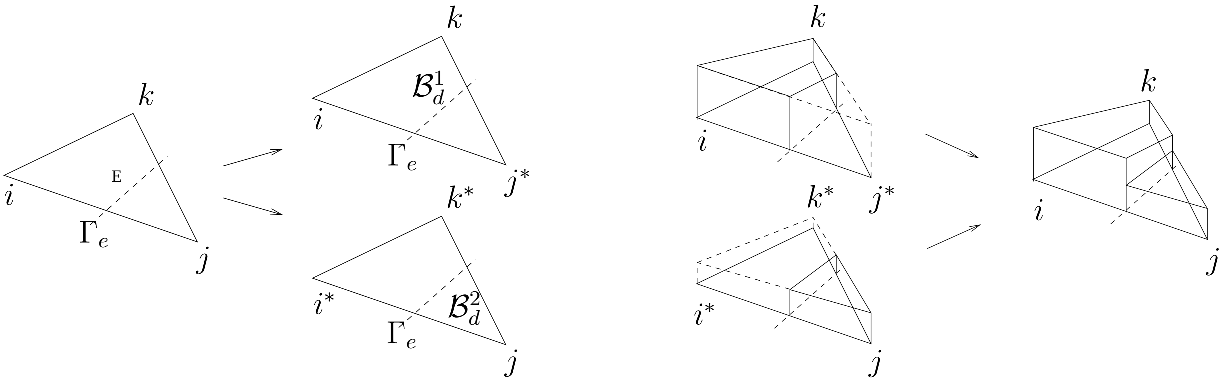

Figure 1: Split of linear triangular element

additional displacement degrees of freedom are introduced at the existing nodes. Two independent copies of the standard basis functions are used, one set is put to zero on one side of the discontinuity, while it takes its usual values on the opposite side, and vice versa for the other set. Figure 1 highlights the construction of a discontinuous linear triangular element.

Finally the applicability of the method for the mesh-independent modelling of strong and weak discontinuities is highlighted by means of numerical examples. For the simulation of strong discontinuities a stress-based crack propagation criterion is adopted. As representative examples a symmetric peel test is considered, see figure 2 and a plate with a soft circular inclusion, compare figure 3.

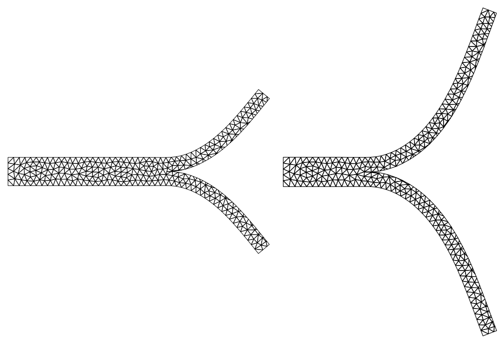

Figure 2: symmetric peel test

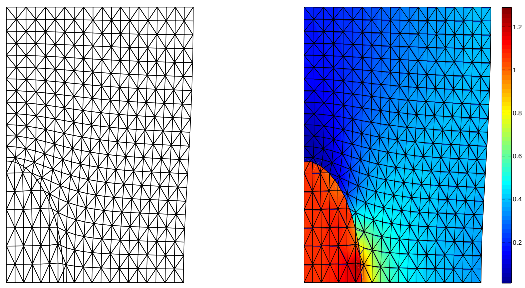

Figure 3: deformation and strain in a plate with soft circular inclusion

4 CONCLUSIONS

The present approach can be considered as a methodically unified framework for the modelling of strong and weak discontinuities, since the same discretization is used, which implies the formulation of the discontinuous elements, and the variational formulations differ only in the additional interface contributions due to the cohesive crack concept and Nitsche's method respectively. A detailed representation of the suggested approach may be found in Mergheim and Steinmann[5].

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