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ON THE ADDITIVE DECOMPOSITION OF STRAIN MEASURES IN LOGARITHMIC FRAME FOR FINITE ELASTOPLASTICITY

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Summary. *In the present paper, from the newly discovered logarithmic rate* [1]*, we study the additive decomposition* $\tilde{\boldsymbol{D}} = \tilde{\boldsymbol{D}}^{e} + \tilde{\boldsymbol{D}}^{p}$ *for elastoplastic laws. We show that the foregoing decomposition of* $\tilde{\boldsymbol{D}}$ *could only hold for certain restrictive cases of deformations, such as small elastic strain or in the case of no cyclic loading. To show that, in one numerical example, we work independently of material behavior considerations and we use the notion of cumulated tensorial strain proposed by Gilormini and Rougée* [2]*.*

1 INTRODUCTION

The development of the elastoplastic model deals with a number of basic problems in the presence of finite strain. The definition of plastic strain is one of them. Solutions have been, and still are, proposed (see for instance [3,4,5]), but none of them is entirely satisfactory as pointed out by Naghdi [6].

Among these solutions, some have become popular such as the additive decomposition of strain rate tensor $\tilde{\boldsymbol{D}} = \tilde{\boldsymbol{D}}^{\mathrm{e}} + \tilde{\boldsymbol{D}}^{\mathrm{p}}$. In this case, the hypoelastic formulation of the reversible part of the behavior poses the usual problems of this kind of law: the choice of the objective stress rate which can produce spurious phenomena such as the so-called shear oscillation, the time integration of the stress rate, and the inconsistency with the notion of elasticity.

2 TIME INTEGRATION OF THE STRAIN RATE TENSOR

The time integration of the strain rate tensor $\tilde{\boldsymbol{D}}$ is an alternative to the time integration of the stress rate. The difficulty in this case is to choose the objective reference frame in which $\tilde{\boldsymbol{D}}$ should be integrated. The obtained integrated quantity, noted $\tilde{\boldsymbol{e}}_{()}$, is called cumulated tensorial strain.

An objective reference frame is defined locally (i.e. near a material point) usually by its spin $\tilde{\boldsymbol{\Omega}}$ relative to an arbitrary fixed reference frame. It has been demonstrated by Gilormini [2], that quantities $\tilde{\boldsymbol{e}}_{(c)}$ and $\tilde{\boldsymbol{e}}_{(r)}$ obtained by the time integration of $\tilde{\boldsymbol{D}}$ in the reference frame defined by the vorticity tensor $\tilde{\boldsymbol{\Omega}}_{(c)} = \tilde{\boldsymbol{W}}$ (skew-symmetric part of the velocity gradient) or by $\tilde{\boldsymbol{\Omega}}_{(r)} = \dot{\tilde{\boldsymbol{R}}}.\tilde{\boldsymbol{R}}^T$ ($\tilde{\boldsymbol{R}}$: orthogonal tensor in the polar decomposition of the transformation gradient) depend on the deformation path. Therefore, the cumulated tensorial strains $\tilde{\boldsymbol{e}}_{(c)}$ and $\tilde{\boldsymbol{e}}_{(r)}$ are not strain tensors.

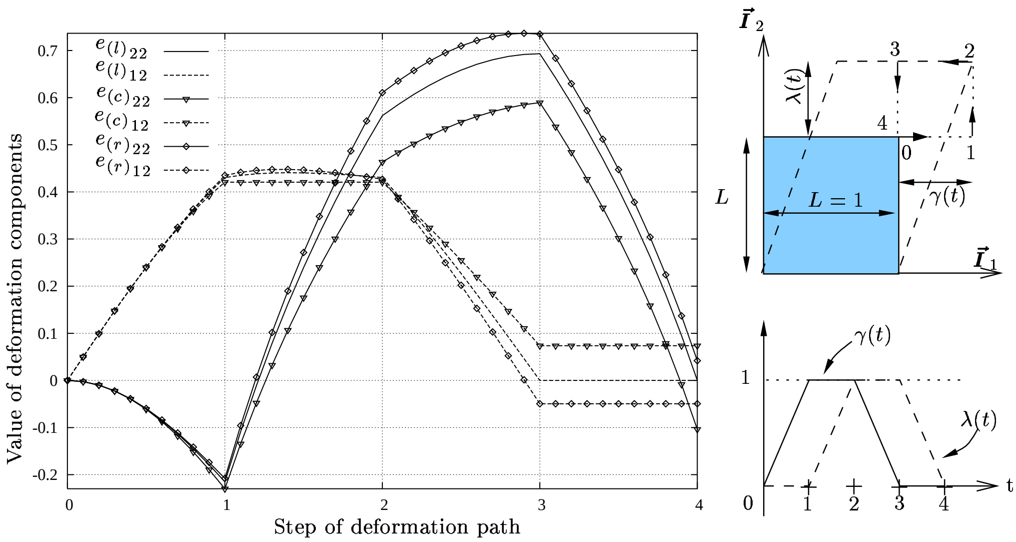

Figure 1: Components in $\left(\vec{\boldsymbol{I}}_1, \vec{\boldsymbol{I}}_2\right)$ of different cumulated tensorial strain measures for a closed deformation path. $e_{(l)}$: logarithmic rotating frame, $e_{(c)}$: corotational frame, $e_{(r)}$: rotational frame.

This results is also presented on figure 1 on a closed deformation path. This deformation path is defined by four phases characterized by the time evolution of transformation gradient under : $\tilde{\boldsymbol{F}}(t) = \tilde{\boldsymbol{1}} + \gamma(t)\vec{\boldsymbol{I}}_1 \otimes \vec{\boldsymbol{I}}_2 + \lambda(t)\vec{\boldsymbol{I}}_2 \otimes \vec{\boldsymbol{I}}_2$ where $\gamma(t)$ and $\lambda(t)$ follow course of figure 1 and $\vec{\boldsymbol{I}}_1$, $\vec{\boldsymbol{I}}_2$ are base vectors of the fixed Cartesian reference frame.

At the end of deformation path, cumulated tensorial strains $\tilde{\boldsymbol{e}}_{(c)}$ and $\tilde{\boldsymbol{e}}_{(r)}$ are not zero. In contrast to the results usually presented in term of stress components and therefore dependent on a constitutive model (see for instance[7]), the results on figure 1 are independent of any constitutive model. The results only depend on the objective reference frame in which the time integration of $\tilde{\boldsymbol{D}}$ takes place. This underlines the geometric nature of the problems classically faced by hypoelastic models.

3 LOGARITHMIC SPIN $\tilde{\boldsymbol{\Omega}}_{(l)}$

To solve those problems, Lehmann [8] and Xiao [1] have introduced logarithmic spin $\tilde{\boldsymbol{\Omega}}_{(l)}$ defined by :

$$\tilde{\boldsymbol{\Omega}}_{(l)} = \tilde{\boldsymbol{W}} + \sum_{\alpha=1,\beta=1,\alpha\neq\beta}^{n} \left[\left(\frac{1+\chi_\alpha/\chi_\beta}{1-\chi_\alpha/\chi_\beta} + \frac{2}{\ln(\chi_\alpha/\chi_\beta)} \right) D_{\alpha\beta}\, \vec{\boldsymbol{p}}_\alpha \otimes \vec{\boldsymbol{p}}_\beta \right] \tag{1}$$

This spin is such that the rotational derivative of spatial Hencky strain tensor $\tilde{\boldsymbol{h}}$ is equal to the strain rate tensor :

$$\dot{\tilde{\boldsymbol{h}}} - \tilde{\boldsymbol{\Omega}}_{(l)}.\tilde{\boldsymbol{h}} + \tilde{\boldsymbol{h}}.\tilde{\boldsymbol{\Omega}}_{(l)} = \tilde{\boldsymbol{D}} \tag{2}$$

where $\dot{\tilde{\boldsymbol{h}}}$ is the time derivation of strain tensor $\tilde{\boldsymbol{h}}$ in the fixed Cartesian frame.

It is therefore obvious that the time integration of $\tilde{\boldsymbol{D}}$ in the objective reference frame defined by spin $\tilde{\boldsymbol{\Omega}}_{(l)}$, let us say the logarithmic rotating frame, leads to the spatial Hencky strain tensor, i.e. $\tilde{\boldsymbol{e}}_{(l)} = \tilde{\boldsymbol{h}}$. As a consequence, this cumulated tensorial strain does not depend on the strain path. For example, at the end of the closed deformation path (see figure 1), we have $\tilde{\boldsymbol{e}}_{(l)} = \tilde{\boldsymbol{0}}$.

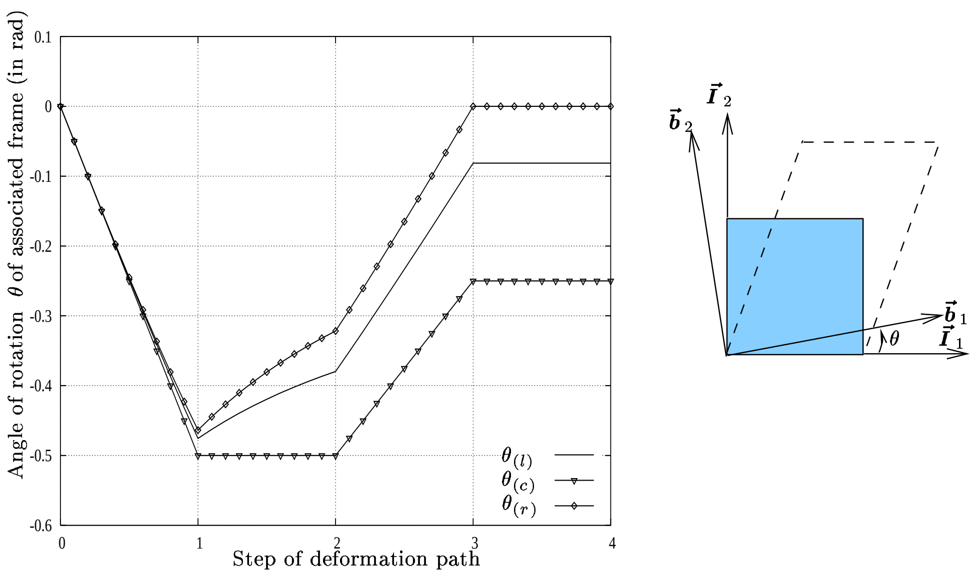

Figure 2: Angle of rotation θ of the local objective frames of cumulated tensorial strain for the closed deformation path of figure 1. $\theta_{(l)}$: rotation in logarithmic rotating frame, $\theta_{(c)}$: rotation in corotational frame, $\theta_{(r)}$: rotation in rotational frame.

Nevertheless, the interest of $\tilde{\boldsymbol{\Omega}}_{(l)}$ should be moderated by the results presented on figure 2. Indeed, during the closed deformation path, the logarithmic rotating frame has turned an angle of 0.08 rad. Therefore, the integration of additive decomposition $\tilde{\boldsymbol{D}} = \tilde{\boldsymbol{D}}^{\mathrm{e}} + \tilde{\boldsymbol{D}}^{\mathrm{p}}$ can lead to aberrant results if the elastic strains are not small (i.e. shear strain superior to 100%) or in the case of cyclic loading (see also [9]). For metallic materials, this inauspicious behavior of this rotation of the logarithmic rotating frame is insignificant for elastic strain of about 0.2%. But, for polymer materials, which can undergo large elastic strain or for materials subjected to a great number of cycles, this rotation can be lead to bad results.

4 CONCLUSION

The time integration of strain rate tensor $\tilde{\boldsymbol{D}}$ is a central problem in large transformations even if it is often an underlying one. The cumulated tensorial strains, obtained by the time integration of strain rate tensor $\tilde{\boldsymbol{D}}$, allow tackling this problem from a geometrical point of view and independently of material behavior considerations. The time integration here takes place in the local objective frame defined by the logarithmic spin. The numerical results obtained in a closed deformation path have been presented in this paper. Advantages and drawbacks of this novel integration for the development of behavior laws have also been described.

REFERENCES

[1] H. Xiao, O. T. Bruhns, A. Meyers. Logarithmic strain, logarithmic spin and logarithmic rate. *Acta Mechanica* **124** 89–105 1997.

[2] P. Gilormini, P. Roudier, P. Rougée. Les déformations cumulées tensorielles. *C. R. Mecanique* **316** 1499–1504 1993.

[3] V. A. Lubarda. *Elastoplasticity theory*, CRC Press, 2002.

[4] J.C. Simo, T.J.R. Hughes. *Computational Inelasticity*, Springer, 1998.

[5] A. Ibrahimbegovic, F. Gharzeddine. Finite deformation plasticity in principal axes: from a manifold to the Euclidean setting. *Comput. Methods. Appl. Mech. Engrg.* **171** 341–369 1999.

[6] P. M. Naghdi. A critical review of the state of finite plasticity, *Journal of Applied Mathematics and Physics* **41** 314–394 1990.

[7] R. Lin, Numerical study of consistency of rate constitutive equations with elasticity at finite deformation. *Int. J. Num. Meth. Eng.* **55** 1053–1077 2002.

[8] T. Lehmann, Z. Guo, H. Liang. The conjugacy between Cauchy stress and logarithm of the left stretch tensor. *Eur. J. Mech. A/Solids* **10** 4 395–4004 1991.

[9] V. Mora and H. Laurent and G. Rio. Déformation cumulée tensorielle dans le référentiel en rotation logarithmique. *C. R. Mécanique* **332** 11 921–926 2004.