VIII International Conference on Computational Plasticity
COMPLAS VIII
E. Oñate and D.R.J. Owen (Eds)

COVARIANT DESCRIPTION OF ANISOTROPIC CONTACT SURFACES

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Key words: Contact problems, covariant description, anisotropic friction and adhesion.

Summary. *Considering contact between bodies whose surfaces can be described by algorithmically organized asperities, e.g. after the machining processes, leads to a general problem of anisotropic frictional contact. A covariant contact description allows to generalize all contact characteristics of these surfaces into an anisotropic domain for both adhesion and sliding behavior in a straightforward form as well as to construct an effective numerical algorithm within an iterative solution scheme of a Newton's type. Numerical examples show the possibility to describe the average behavior of machined surfaces.*

1 INTRODUCTION

A generalization of the isotropic macro characteristics for contact surfaces is described in the literature only for the sliding region mostly by means of the anisotropic friction tensor. A first general derivation of sliding characteristics based on mechanics of a rigid block on an inclined plane was presented by Michalowski and Mroz [1]. Zmitrowicz [2] used a similar model to develop the structure of the friction tensor for sliding forces and described its properties based on symmetry groups for the tensor. He and Curnier [3] used the theory of tensor function representations to obtain the structure of the friction tensor for an arbitrary nonlinear case according to the relative sliding velocity and derived also thermodynamical restrictions for the friction tensor components.

Despite the extensive literature on finite element solutions for contact problems, there are only few publications on finite element models for anisotropic friction, e.g. Buczkowski and Kleiber [4] created an interface element containing the orthotropic sliding law applicable only for small displacements. Review of other non-Coulomb models e.g. statistical models used in finite element applications can be found in Wriggers [5].

A covariant contact description, developed by Konyukhov and Schweizerhof [6], allows to construct a numerical algorithm based on a Newton's type solution within the finite element method for arbitrary approximations of contact surfaces. The main advantage is that all contact characteristics as well as all necessary for solution operations are derived in a covariant form in a spatial coordinate system defined according to the closest point procedure on the contact surface. The covariance of all relations allows to generalize them into anisotropy for both friction and adhesion. It is shown that the latter can be used to define properties of machined surfaces.

2 COVARIANT DESCRIPTION AND ITS GENERALIZATION

A spatial local coordinate system defined on the master surface is obtained at the projection point **C** based on the closest projection of the slave point **S**:

$$\mathbf{r}_s(\xi^1, \xi^2, \xi^3) = \boldsymbol{\rho}(\xi^1, \xi^2) + \mathbf{n}\xi^3. \quad (1)$$

The first two convective coordinates ξ^1, ξ^2 define the surface point **C** and, therefore, are responsible for the tangential contact interaction. The third coordinate ξ^3 is the value of the penetration and is used to define the properties of the normal interaction. The full contact traction vector **R** is defined in contravariant basis vectors $\boldsymbol{\rho}^i$ and $\mathbf{n}$ as follows:

$$\mathbf{R} = \mathbf{T} + \mathbf{N} = T_i\boldsymbol{\rho}^i + N\mathbf{n}. \quad (2)$$

The penalty method is applied for the regularization of the contact conditions leading to an equation for the normal traction in a closed form: $N = \epsilon_N \xi^3$, if $\xi^3 \leq 0$; while the tangent traction **T** is written in a rate form:

$$\frac{d\mathbf{T}}{dt} = -\epsilon_T \dot{\xi}^i \boldsymbol{\rho}_i, \quad (3)$$

where a full time derivative $\dfrac{d\mathbf{T}}{dt}$ is taken in covariant form in the spatial coordinate system on the tangent plane with $\xi^3 = 0$. A generalization for adhesion is obtained after the introduction of the adhesion tensor **B** in the evolution equation (3):

$$\frac{d\mathbf{T}}{dt} = \mathbf{B}(\mathbf{v}_s - \mathbf{v}), \quad (4)$$

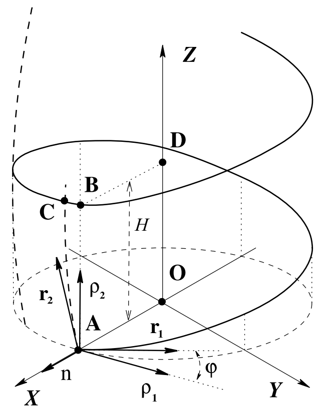
where $\mathbf{v}_s - \mathbf{v} = \dot{\xi}^i\boldsymbol{\rho}_i$ is a relative velocity vector of the contact point **C**. The generalization of the isotropic Coulomb friction law is obtained via the friction tensor **F** defined in the surface metrics:

$$\Phi = \sqrt{f^{ij}T_iT_j} - N = \sqrt{\mathbf{T}\cdot\mathbf{FT}} - N. \quad (5)$$

The sliding criteria are written then as follows

$$if \quad \Phi \leq 0 \rightarrow sticking \quad (adhesion), \qquad if \quad \Phi > 0 \rightarrow sliding. \quad (6)$$

The adhesion tensor **B** and the friction tensor **F** are chosen to fulfill some thermodynamical restrictions, as e.g. discussed in [3] and [2] concerning the friction tensor. In particular, the anisotropy can be inherited from the arbitrary coordinate system on the contact surface. In this case the tensor is defined via the unit vectors $\mathbf{e}_i = \frac{\mathbf{r}_i}{|\mathbf{r}_i|}, \;\; i = 1, 2$ of this coordinate system as $\mathbf{B} = \lambda_i \mathbf{e}_i \otimes \mathbf{e}_i$. Fig. 1 shows the so-called spiral orthotropy on a cylinder, which is defined via the orthogonal spiral net with tangent vectors $\mathbf{r}_1, \mathbf{r}_2$. The adhesion tensor with stiffnesses along the coordinate lines $\varepsilon_1, \varepsilon_2$ is also given in the picture after the transformation to the contravariant basis $\boldsymbol{\rho}^1$, $\boldsymbol{\rho}^2$ of the cylindrical coordinate system.



$$\mathbf{B} = b_{ij}\boldsymbol{\rho}^i\boldsymbol{\rho}^j =$$

$$= -\frac{1}{R^2 + \left(\frac{H}{2\pi}\right)^2} \begin{bmatrix} g_\varepsilon R^2 & (\varepsilon_1 - \varepsilon_2)\frac{R^2 H}{2\pi} \\ (\varepsilon_1 - \varepsilon_2)\frac{R^2 H}{2\pi} & \varepsilon_1 \left(\frac{H}{2\pi}\right)^2 + \varepsilon_2 R^2 \end{bmatrix}$$

$$\text{with} \quad g_\varepsilon = \varepsilon_1 R^2 + \varepsilon_2 \left(\frac{H}{2\pi}\right)^2$$

Figure 1: Spiral orthotropy on a cylinder. Definition of a spiral net and the adhesion tensor $\mathbf{B}$.

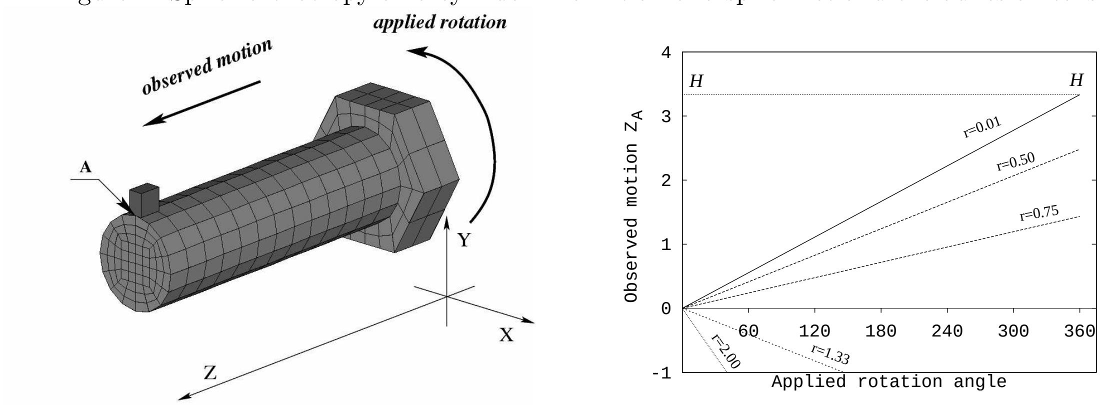


Figure 2: The variation of the parameters $r = \frac{\varepsilon_1}{\varepsilon_2}$ allows to describe different kinematics of the screw connection.

2.1 Derivation of the sliding forces and displacements

The anisotropic friction problem is formulated as an optimization problem via the principle of maximum dissipation in a continuous form. In order to construct the numerical algorithm, the return-mapping algorithm with regard to inequalities (6) within the Euler backward scheme for the evolution equation (4) is applied to compute all characteristics for sliding, such as a sliding force $\mathbf{T}^{sl}$ and a sliding displacement vector $\boldsymbol{\xi}^{sl}$ at each load step. The sliding force $\mathbf{T}^{sl}$ e.g. is computed as follows:

$$\mathbf{T}^{sl} = -\frac{\hat{\mathbf{T}}}{\sqrt{\hat{\mathbf{T}} \cdot \mathbf{F}\hat{\mathbf{T}}}}|N| \quad \text{with} \quad \hat{\mathbf{T}} = \mathbf{B}\mathbf{F}\mathbf{T}^{tr}, \tag{7}$$

where the trial force $\mathbf{T}^{tr}$ is computed via the Euler backward scheme for the evolution equation (4). The linearization procedure of the weak form, formulated also in the spatial coordinate system, is consistently performed as covariant differential operations on the tangent plane. As an advantage, all tangent matrices contain geometrical parameters of the contact surface and, therefore, can be implemented into a finite element program independent of the order of the corresponding surface approximation.

3 NUMERICAL EXAMPLE

A possibility to model a curved machined surface within the proposed approach is illustrated with a model of a screw connection, see Fig. 2. A screw, meshed with a relatively coarse finite element mesh, is contacting with a thread inside a rigid cylinder (the cylinder is schematically depicted with a single upper element in Fig. 2). Contact is modeled with a "point-to-analytical surface" approach. These elements are specified on the cylindrical part of the screw. As a result, the parameters of both, the adhesion and the friction tensors, can be calibrated in order to prescribe the resulting kinematics exactly by keeping the necessary sliding force at a certain level.

4 CONCLUSIONS

- The fully covariant description for the anisotropic contact surfaces was developed. The description includes the anisotropy for both sliding and adhesion and is realized as numerical approach within a Newton type iterative solution scheme for problems discretized by finite elements.

- Parameters of the model such as adhesion and friction tensors can be calibrated in order to describe e.g. the properties of the machined surfaces.

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