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FINITE ELEMENT MODELLING OF HYDRO-FRACTURE FLOW IN POROUS MEDIA

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Summary. *In this paper a computational framework for the analysis of coupled hydro-fracture flow in porous media, using a Finite/Discrete Element Method, is presented. It encompasses the description of the basic assumptions underlying the proposed strategy, as well as the details of the numerical approach. The effectiveness of the overall strategy is demonstrated and the key features of the model are emphasised. Finally, some concluding remarks are presented.*

1 INTRODUCTION

Over recent years, there has been an increasing industrial awareness of the potential benefits that derive from employing scientific approaches in the exploitation of natural resources. This is particularly true in the extraction of oil, gas and water and is also of paramount importance in preventing the leakage of hazardous materials, such as toxic and radioactive waste. This type of application is characterized by the presence of soil or rock-like materials, in which the pores of the solid phase are filled with one or more fluids. An additional degree of complexity is introduced by the coalescence and growth of voids, which results in the appearance of macro-cracks and subsequent fragmentation.

In order to understand the complex interplay between different phenomena, several approaches and theories have been proposed. One strategy aims to model the flow through cracks [1] without taking into account the flow within the material itself. This simplification is reasonable for soils or rock-like materials with low permeabilities, since the flow throughout the fractures is dominant. However, an accurate analysis of the *in situ* stress field can only be obtained if the soil or rock-like material is treated as a porous medium. This is particularly relevant for materials with high permeabilities where the seepage behaviour becomes prominent.

2 CONSTITUTIVE EQUATIONS

In porous materials, the definition of an *effective stress* is certainly not an easy task. Several expressions have been proposed in the literature and it is accepted that, the effective stress should include the deformation of the grains within the skeleton. In addition, in a porous material the pores can be filled with two or more fluids. Therefore, the determination of an *effective pressure* has also received special consideration. A widespread definition for the effective stress, $\sigma^{"}_{ij}$, is given by [2]:

$$\sigma^{"}_{ij} = \sigma_{ij} + \alpha\, \delta_{ij}\, S_w\, p \tag{1}$$

where, α is the Biot number, which is introduced to take into account the volumetric deformability of the particles. It is related with the bulk modulus of the skeleton (K_T) and the bulk modulus of the grains (K_S). The effective stress, $\sigma^{"}_{ij}$, in this work, is related to the incremental strain, $d\epsilon_{ij}$, and rotation, $d\Omega_{kl}$, by means of an incremental constitutive relationship:

$$d\sigma^{"}_{ij} = D_{ijkl}(d\epsilon_{kl} - d\epsilon^0_{kl}) + d\sigma^{"}_{ik} d\Omega_{kj} + d\sigma^{"}_{jk} d\Omega_{ki} \tag{2}$$

where, the last two terms account for the rotational stress in the Green-Naghdi rate and D_{ijkl} is a fourth order tensor defined by state variables and the direction of the increment. Finally, $d\epsilon^0_{kl}$ represents the incremental initial strain tensor due to thermal or autogeneous strain of the grain compression.

3 GOVERNING EQUATIONS

The first equation is the total momentum equilibrium equation for the partially saturated soil-fluid mixture, where, the acceleration of the fluid relatively to the solid and the convective terms have been neglected. This assumption is valid for medium speed and dynamics of lower frequencies phenomena [3]. The second equation is derived by ensuring mass conservation of the fluid flow in the seepage. These assumptions allow the final form of the governing equations, which is known as $\mathbf{u}-p$ formulation, to be written as:

$$\sigma_{ij,j} - \rho\, \ddot{u}_i + \rho\, b_i = 0 \tag{3}$$

$$\frac{\dot{p}_s}{Q_s} + \alpha\, \dot{\epsilon}_{ii} + k_{ij}(-p_{s\prime_j} + \rho_s\, S_w\, b_j - \rho_s\, S_w\, \ddot{u}_j)_{\prime i} = 0 \tag{4}$$

In this work, an individual fracture is treated as a single confined aquifer, where the mass conservation is based on a cubic law for the flow within the fracture:

$$\frac{e^3}{12\mu}(-p_{n\prime_j} + \rho_n\, b_j)_{\prime i} + \frac{\dot{p_n}}{Q_n} = 0 \tag{5}$$

where, e is the aperture of the fracture (with unit of $[m]$) and μ is the viscosity (with unit of $[Pa.s]$). The Q_n term represents the storativity of a single fracture, and like a porous medium, reflects the compressibilities of the fluid and rock. However, the rock compressibility term does not reflect the intergranular skeleton, but rather the pressure dependence of the fracture volume, which is simply the normal stiffness of the fracture, k_f, (with unit of $[m/Pa]$).

The system given by the three governing equations (3), (4) and (5) and the boundary conditions describes a well-defined problem which can then be discretized and solved.

4 THE DISCRETE APPROXIMATION AND ITS SOLUTION

Firstly, the Galerkin (or weighted residual) method is used to obtain the weak form of the system of equations, presented in the previous section. Then, a discretization in space is undertaken by a standard finite element procedure. Finally, a central difference approximation is employed for the time discretization.

The governing equations (3, 4 and 5) have been transformed into a set of algebraic equations in space with only time derivatives remaining. The system can then be written as

$$\begin{aligned} &\int_{\Omega_s} (\nabla \mathbf{N}^u)^T \sigma^{"} \, d\Omega_s - \mathbf{Q}\bar{\mathbf{p}}_\mathbf{s} + \mathbf{M}\ddot{\bar{\mathbf{u}}} = \bar{\mathbf{f}}_\mathbf{1} \\ &\mathbf{H_s}\bar{\mathbf{p}}_\mathbf{s} + \mathbf{Q^T}\dot{\bar{\mathbf{u}}} + \mathbf{S_s}\dot{\bar{\mathbf{p}}}_\mathbf{s} = \bar{\mathbf{f}}_\mathbf{2} \\ &\mathbf{H_n}\bar{\mathbf{p}}_\mathbf{n} + \mathbf{S_n}\dot{\bar{\mathbf{p}}}_\mathbf{n} = \bar{\mathbf{f}}_\mathbf{3} \end{aligned} \tag{6}$$

with the constitutive equation supplying the increments of $\sigma^{"}_{ij}$.

5 COUPLING PROCEDURE

From the analysis of the previous system, it is not easy to recognize that the mass conservation of the fluid network, Equation $(6)_3$, is coupled with the other two equations. However, it can be observed that:

- The displacements computed by Equation $(6)_1$ are affected by an external force caused by the pressure of the fluid network. This load is applied as a traction boundary condition at the solid/fracture interface. On the other hand, the displacements calculated in Equation $(6)_1$ define the coordinates of the fluid network. Therefore, it controls the apertures in the fluid network and triggers new ones.

- The coupling between the fluid flow in the seepage/fracture interface is made using a master/slave procedure. In this procedure the nodes in the network field are considered as master and the nodes in the seepage field, which are in the seepage/fracture interface are slaves. As a consequence, the mass matrices and the internal and external force vectors of the slaves are added into the master. Also, the equations of the slaves nodes are modified by the ones related with the specific master nodes. Finally, the equations are then solved and the balance flow in the interface is guaranteed.

The inter-dependence of the different phenomena is depicted in Figure 1.

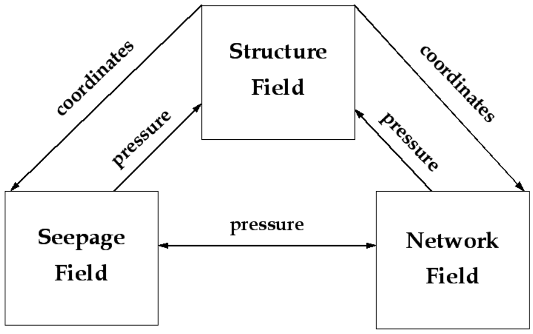

Figure 1: Coupling procedure

6 DISCRETE CRACK INSERTION

The transition of a body from a continuum description into discrete is developed from dispersed micro cracks coalescing into macroscopic fractures. The appearance of a discrete fracture within the material results in the global realisation of inelastic strains and the associated unloading of the surrounding material. The process of inserting a discrete fracture into a continuum based finite element mesh follows three key steps: (i) the creation of a non local failure map, that is based upon the weighted nodal averages of the damage within the finite element system; (ii) the failure map is used to determine the likelihood of fracture within the domain; and (iii) a numerical code to perform the topological update whereby a fracture is inserted in the domain, and any additional nodes are inserted and necessary elemental connectivities are updated.

The so-called *failure factor* is typically defined as the ratio of the inelastic fracturing strain ϵ^f to the critical fracturing strain ϵ_c^f or the ratio of damage and the critical damage. The elemental or local failure factor F_k that is associated with the Gauss point of an element k is given by

$$F_k = \left(\frac{\epsilon^f}{\epsilon_c^f}\right)_k \qquad \text{or} \qquad F_k = \left(\frac{D}{D_{\mathrm{cr}}}\right)_k \tag{7}$$

where F_k is associated with the elemental local fracture direction θ_k which is defined as being normal to the direction of the local failure softening direction. Discrete fracture is realised through the failure factor reaching unity. In addition, the nodal basis of a finite element system leads to a simpler and more efficient approach for the insertion and creation of discrete fractures. The associated failure factor $\bar{F}_p$ and the corresponding direction of failure $\bar{\theta}_p$ for the nodal point p are given by,

$$\bar{F}_p = \frac{\sum_{k=1}^{ngauss_{adj}} F_k w_k}{\sum_{k=1}^{ngauss_{adj}} w_k}, \qquad \bar{\theta}_p = \frac{\sum_{k=1}^{ngauss_{adj}} \theta_k w_k}{\sum_{k=1}^{ngauss_{adj}} w_k}, \tag{8}$$

where the summation is calculated over the number of element Gauss integration points that are immediately adjacent and w_k is the elemental weighting factor, which is normally taken as the elemental volume or failure factor. When the associated failure factor $\bar{F}_p$ and the direction of failure for the considered node p have been determined, a discrete fracture of the given orientation will be inserted into the finite element mesh, passing through the associated nodal point.

Generally, there are one of two choices to be made at this stage. Firstly, the fracture plane can be aligned in the exact orientation of the weighted average nodal failure direction, thereby following a process known as *intra-element* fracturing. In the second approach, the discrete fracture orientation is aligned with the best orientated element boundary attached to the node considered , thereby following a process known as *inter-element* fracturing where a series of new nodal points are systematically created but no new elements are generated.

7 CONCLUSIONS

The applicability of the concepts described in the above will be illustrated in the presentation through several practical examples. These comprise the prediction of material failure in slope-stability, oil and water confinement. A more detailed description of the methodology will be given in a forthcoming publication where the performance of the overall strategy is demonstrated.

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