VIII International Conference on Computational Plasticity
COMPLAS VIII
E. Oñate and D.R.J. Owen (Eds)

INTERACTION OF LOCKING AND ELEMENT STABILITY AT LARGE STRAINS

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Key words: Finite Element Technology, Hourglass Instabilities, Large Strains, Stabilization Method.

Summary. *The phenomenon of hourglass instabilities of EAS elements at large strains is addressed. It is demonstrated that this problem has its origin in reduction of the in-plane bending stiffness to avoid locking. A stabilized formulation is presented which establishes a compromise between removing locking and obtaining a stable formulation. Performance of the element is demonstrated in a numerical experiment.*

1 INTRODUCTION

Efficiency of standard displacement finite elements is often severely limited due to locking phenomena. For 2-d and 3-d solids these are predominantly shear locking and volumetric locking. It is therefore customary to avoid these locking phenomena by advanced finite element technology, for instance methods based on multifield variational principles. These concepts allow efficient application of low-order displacement elements also for thin-walled structures or nearly incompressible material behavior. Among these concepts the Enhanced Assumed Strain (EAS) method by Simo and Rifai [4] enjoys particular popularity.

It has been found that EAS elements tend to exhibit spurious instabilities ("hourglassing") in the presence of large compressive strains, see for instance Wriggers and Reese [6]. Since then, a couple of remedies and alternative formulations have been proposed (de Souza Neto et al. [1], Wall et al. [5], Reese and Wriggers [3], among others), aspiring to remove artificial instabilities while retaining locking-free behavior.

2 LOCKING AND INSTABILITIES

In a simple uniform compression test (Figure 1) the point of instability can be predicted by investigating the eigenvalues of an individual element stiffness matrix. As soon as the eigenvalue which belongs to one of the in-plane bending modes becomes zero, the instability is initiated. The hourglass pattern schematically shown on the right in Figure 1 is at the same time the explanation for the origin of the artificial instability: Incompatible displacement modes, indicated as dashed lines, which belong to the enhanced strain

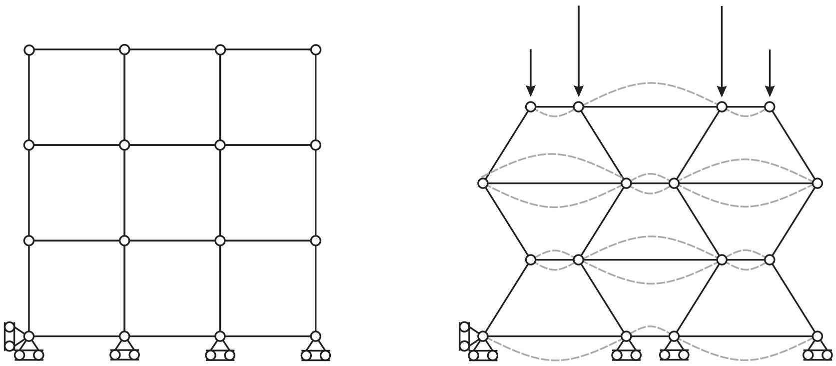

Figure 1: Hourglass instabilities and incompatible modes

parameters, severely violate inter-element continuity conditions. As these incompatible in-plane bending modes are necessary to avoid locking we are in a dilemma: Removing (or stiffening) those modes re-introduces locking; retaining them inevitably leads to instable behavior in non-linear problems. Apparently, some compromise has to be accepted—completely avoiding locking effects *and* ensuring element stability at large strains seem to be mutually exclusive.

A further fundamental difficulty in this context is the circumstance that, to the authors' best knowledge, there is neither a proper definition nor—consequently—a mathematical analysis of stability of non-linear finite element formulations. As a result, methods to avoid *numerical* instabilities may tend to ignore *physical* instabilities as well, because they are undistinguishable.

3 A STABILIZED FORMULATION

In the case of four-node elements element technology focuses on designing the stiffness for the two in-plane bending modes. Rigid body and constant strain modes are usually not touched in order to obtain a consistent formulation which satisfies the patch test. The key problem is now to find a feasible compromise between making these modes flexible enough not to exhibit locking, but stiff enough not to suffer from artificial instabilities.

In geometrically non-linear problems the stiffness matrix

$$\boldsymbol{K} = \boldsymbol{K}_{e+u} + \boldsymbol{K}_g \,. \tag{1}$$

consists of two parts: $\boldsymbol{K}_{e+u}$ contains the so-called elastic and initial displacement stiffnesses, $\boldsymbol{K}_g$ is the geometric (or initial stress) stiffness. Applying the EAS method changes the former but leaves the latter untouched. This means that there is a certain unbalance in the sense that a "weak" elastic stiffness is confronted with a "strong" geometric stiffness (the latter one being identical to that of a standard displacement element). In the case of compressive stresses the geometric stiffness is negative and an instability occurs as soon as it has "eaten up" the elastic stiffness. Because of the aforementioned unbalance the point of instability on the element level occurs too early in EAS elements.

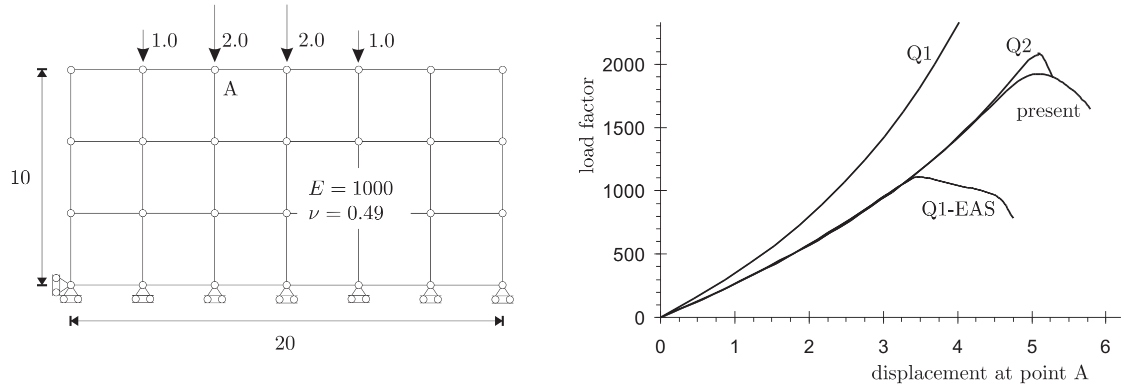

Figure 2: Compression of rubber block, problem data and numerical results

The simple and straightforward idea pursued in the present work is therefore to reduce both $\boldsymbol{K}_{e+u}$ and $\boldsymbol{K}_g$ simultaneously within the context of a stabilized finite element formulation. Technically, we follow the concept proposed by Kosloff and Frazier [2] (later on adopted and further developed by numerous authors), splitting the stiffness matrix into a constant part and a high order part

$$\boldsymbol{K} = \boldsymbol{K}_o + \boldsymbol{K}_h \,, \tag{2}$$

or, in other words, using reduced integration plus hourglass control.

$\boldsymbol{K}_h^{STAB}$ can be designed such that an element performance is achieved which is comparable to that of EAS elements for linear problems (see [2] for details). The key idea of the finite element formulation presented herein is to apply the same procedure to the geometric stiffness matrix. This means that both elastic and geometric stiffness are reduced, thus achieving the balance which is not present in EAS elements.

4 NUMERICAL EXPERIMENT AND CONCLUSION

To demonstrate the behavior of the new element we investigate the problem documented in Figure 2. We compare results obtained with Q1, Q1-EAS and the present formulation to a reference solution with quadratic elements (Q2). While Q1-EAS exhibits an artificial instability at a load level of approximately 1100, Q1 behaves too stiff. Both computations do not reflect the actual physical behavior. The present formulation matches the results of Q1-EAS until the point of instability. Later it runs into an instability as well. The same behavior is observed for Q2 which is generally accepted to be stable. We conclude that this instability is actually a physical one.

The deformation plots in Figure 3 confirm this proposition (the plots in the last row represent the state of deformation at the last step that has been computed). Observe the qualitative resemblance of the results obtained with Q2 and the present formulation, including the "buckling mode".

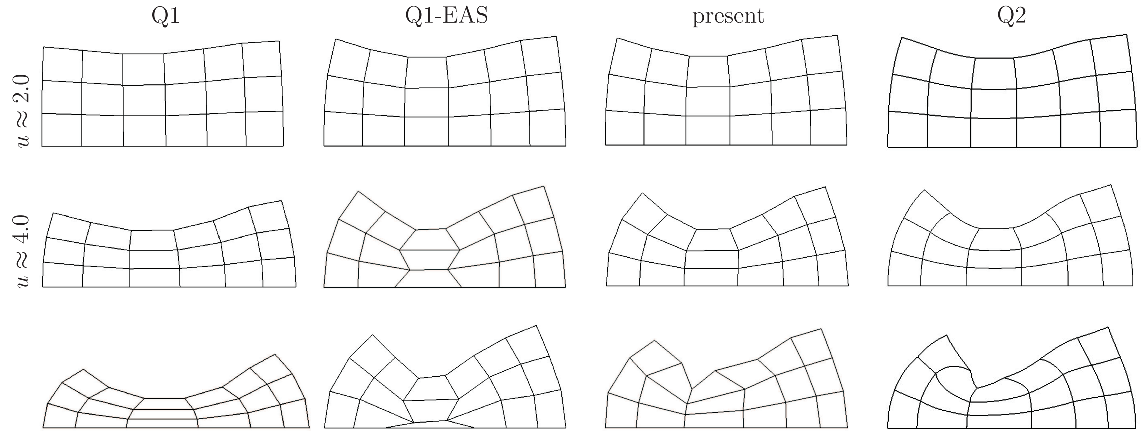

Figure 3: Comparison of deformed configurations (not to scale)

In the investigated example the presented formulation is locking-free and avoids artificial instabilities without ignoring physical ones. The principle idea of "balancing" geometric and elastic stiffness seems to be a step into the right direction. The authors believe that it is of no major importance whether this is accomplished with a stabilized formulation, as described here, or on some different basis. In the future it will be crucial to perform a more rigorous stability analysis of this (and other) non-linear finite elements.

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