VIII International Conference on Computational Plasticity
COMPLAS VIII
E. Oñate and D.R.J. Owen (Eds)

A PRIMAL-DUAL ACTIVE SET STRATEGY FOR ELASTOPLASTIC CONTACT PROBLEMS IN THE CONTEXT OF METAL FORMING PROCESSES

S. Brunssen*, M. Bambach†, G. Hirt†, B. Wohlmuth*

† Institut fuer Bildsame Formgebung (IBF)
RWTH Aachen University of Technology
Intzestr. 10, 52056 Aachen, Germany
e-mail: bambach@ibf.rwth-aachen.de, web page: http://ibf.rwth-aachen.de
* Institut für Angewandte Analysis und Numerische Simulation
Universität Stuttgart, 70569 Stuttgart, Germany
e-mail: brunssen@ians.uni-stuttgart.de
web page: http://www.ians.uni-stuttgart.de/

Key words: Computational Plasticity, Contact Algorithm, Forming Process

Summary. *The work detailed in this paper describes the implementation of a primal-dual active set formulation for contact problems involving elastic-plastic material behaviour. Since a contact algorithm based on an active set strategy does not deteriorate the condition number, it seems to be particularly useful in the context of metal forming simulations in combination with iterative solvers. The method has been implemented into the finite element code LARSTRAN/SHAPE. 3D benchmark examples are given.*

1 INTRODUCTION

The efficient treatment of contact problems is crucial to the performance of FE codes in the context of metal forming. Efficient contact algorithms have been developed in recent years [1, 5]. In contrast to the commonly used penalty-method, the primal-dual active-set strategy, which is in the following referred to as active-set strategy, allows one to adjust the geometric constraints of the tool exactly in a weak integral sense. The contact stress can be easily recovered from the displacements in a variationally consistent way and does not depend on a tuning parameter. Furthermore, the deterioration of the condition number of the stiffness matrix which arises in penalty formulations can be avoided by the active set strategy. Therefore, this strategy is very attractive for use in metal forming, especially in combination with iterative solvers, as their convergence behaviour strongly depends on the condition number of the system matrix. In the present paper, we adapt the active set strategy for non-linear material-behaviour. The strategy was implemented in LARSTRAN/SHAPE.

2 ACTIVE SET STRATEGY

As an example of a forming operation, we consider a rectangular workpiece which is indented by a rigid, hemispherical punch (see Fig. 1(a)).

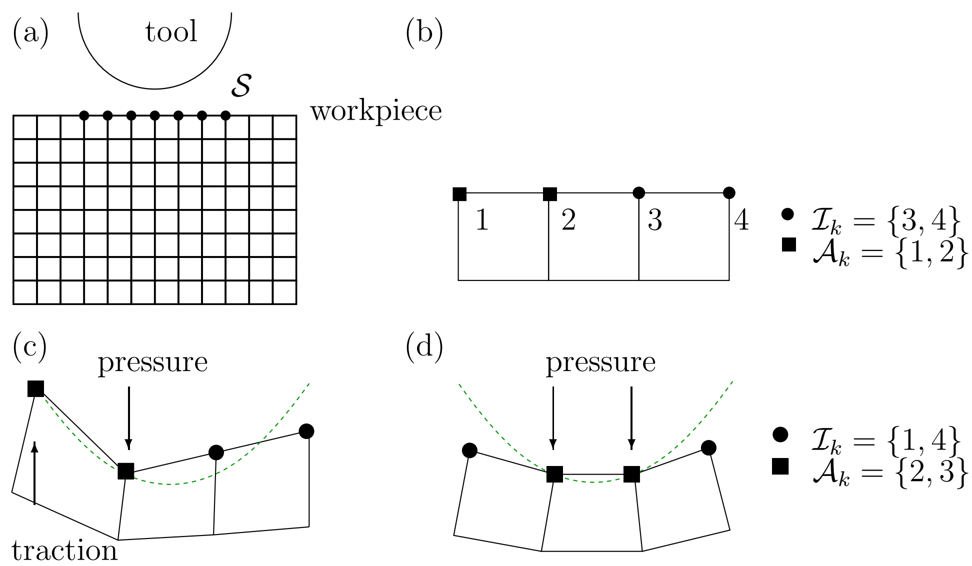

Figure 1: Active set strategy

The basic idea behind the active set formulation presented here is to variationally project the displacements from the tool onto the workpiece using certain Lagrange multipliers, which define a dual, orthogonal base to the standard interpolation functions. For details see [3].

We denote with $\mathcal{S}$ the set of potential contact nodes (see Fig. 1(a)). Let us consider the load step where the tool comes in contact with the workpiece for the first time. The iteration index for the active set loop is denoted by k. $\mathcal{A}_k \subset \mathcal{S}$ is the active set, for which the body is in contact with the obstacle and the complementary set $\mathcal{I}_k = \mathcal{S} - \mathcal{A}_k$ is called the inactive set for all steps $k \geq 1$ until convergence of the active set iteration. We consider a patch of three elements to illustrate how the sets $\mathcal{I}_{k+1}$ and $\mathcal{A}_{k+1}$ are determined. The situation in the k-th step is given in Fig. 1(b): Nodes {1,2} are active, nodes {3,4} are inactive. The tool geometry is then adjusted to the active nodes, no matter if the active set is already correct or not (see Fig. 1(c)). Now the nodal forces are used to update the active and inactive node sets:

$$\begin{aligned}
\mathcal{A}_{k+1} &= \{p \in \mathcal{I}_k : p \text{ penetrates}\} \cup \{p \in \mathcal{A}_k : p \text{ under compression}\} \\
\mathcal{I}_{k+1} &= \{p \in \mathcal{I}_k : \text{no penetration}\} \cup \{p \in \mathcal{A}_k : p \text{ under tension}\}
\end{aligned}$$

Then the restrictions from the tool geometry are again adjusted to the correct active nodes and the inactive nodes are released (see Fig. 1(d)). The step from (c) to (d) is one step of the active set iteration which can be run parallel to the Newton iteration for the nonlinear material law. This is possible because the search for the correct active set can also be interpreted as a Newton iteration [2].

3 NUMERICAL EXAMPLES

To demonstrate some of the difficulties which arise from using a penalty approach in the contact, we consider a 2D-example, where a rigid punch is pressed into a linear elastic rectangle.

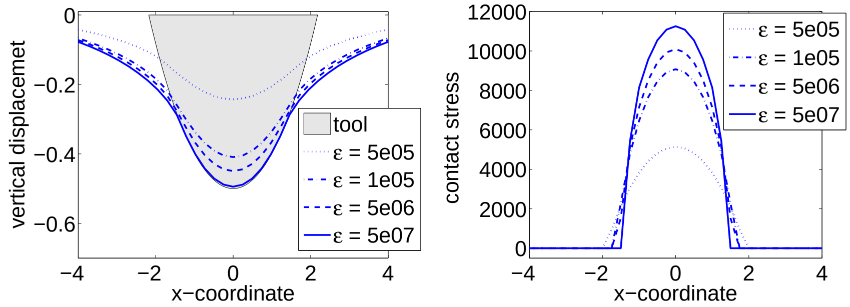

Figure 2: Comparison of the solution of the penalty method for different penalty parameters: Vertical displacement (left) and contact stresses (right)

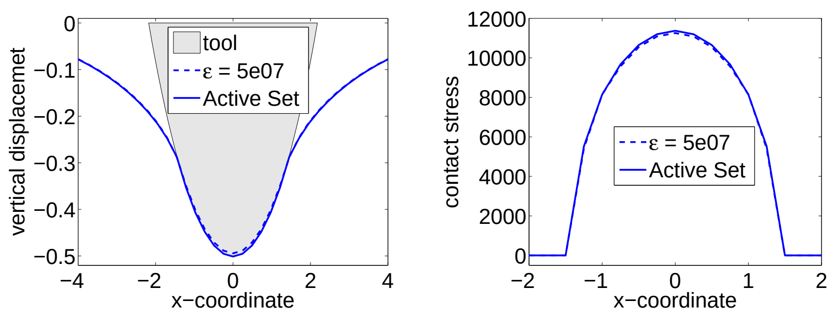

Figure 3: Comparison of the solution of the penalty method with the solution of the active set strategy: Vertical displacement (left) and contact stresses (right)

The bigger the penalty-parameter ε, the smaller the penetration and the more accurately the contact stresses can be approximated. The quality of the active-set strategy solution is only reached for very high penalty-values, which have the previously mentioned drawback of deterioration of the condition number.

Initial 3D-tests have been performed using a model problem which is motivated by surface rolling: A block of dimensions $1 \times 1 \times 0.5$ is indented by a spherical tool which moves along a closed loop. Two steps of the process are depicted. The block is discretized with 4000 hexaedral volume elements. The sphere has a radius of 0.25 and penetrates the block by 0.05.

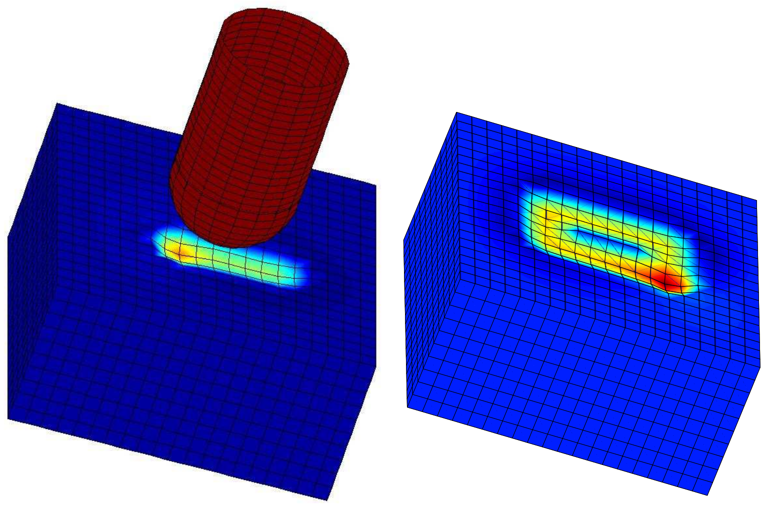

Figure 4: 3D contact example with elasto plastic material law

4 ACKNOWLEDGEMENTS

The work in this paper was funded by the German Research Foundation (DFG) through the projects HI790/5-1 and WO671/4-1 in the framework of SPP1146: Modelling of incremental forming operations.

REFERENCES

[1] J.M. Allwood. Survey and Performance Assessment of Solution Methods for Elastic Rough Contact Problems. Journal of Tribology, Vol. 127-1, 10–23, 2005.

[2] M. Hintermüller, K. Ito and K. Kunisch, *The primal-dual active set strategy as semi-smooth Newton method*, Technical Report 214, Karl-Franzens-Universität Graz and TU Graz, January 2001, Sonderforschungsbereich F 003.

[3] S. Hüeber and B. Wohlmuth, *A primal-dual active set strategy for non-linear multi-body contact problems*, Universitaet Stuttgart, preprint SFB404 2003-40. To appear in Computer Methods in Applied Mechanics and Engineering.

[4] P. Wriggers, *Computational Contact Mechanics*, Wiley, 2002.

[5] P. Wriggers and K. Fischer, *Recent New Developments in Contact Mechanics*, Proceedings of the 4th European LS-DYNA Users Conference, Ulm, 2003.