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ELASTIC-PLASTIC FRACTURE ANALYSIS FOR SURFACE CRACKS USING X-FEM

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Summary. *The present paper describes the application of X-FEM to stress analyses considering materially nonlinear behavior. Although the two-dimensional near-tip asymptotic displacement function has been used in X-FEM analyses of linear elastic problems with small deformation, it is not clear whether the near-tip function is valid for elastic-plastic problems. Therefore, the near-tip functions for a homogeneous isotropic crack are examined in nonlinear problems. As numerical examples, surface crack problems in elastic-plastic materials are solved. The J-integrals are evaluated in post processing of the results by elastic-plastic stress analysis based on X-FEM. The obtained results are compared with those obtained using conventional finite element analysis.*

1 INTRODUCTION

The extended finite element method (X-FEM) [1][2], which can model the domain without explicitly meshing the crack surface, can be used to perform stress analyses for efficiently solving fracture mechanics problems. X-FEM has been applied to the evaluation of fracture parameters, such as stress intensity factor (SIF), in the field of linear elastic fracture mechanics (LEFM). Moreover crack propagation simulations have been performed efficiently using X-FEM in conjunction with the level set method [2]. On the other hand, consideration of nonlinearity in structural analyses is often required for practical engineering problems. For example, an evaluation of the J-integral of a crack in the structure of an elastic-plastic material requires materially nonlinear analysis. In X-FEM analysis, the interpolation function is enriched with the *Heaviside* function to model the discontinuity of the displacement field along the crack surface. However, when the crack tip is located inside an element, the interpolation function enriched with the *Heaviside* function cannot yield the appropriate displacement field perfectly. In such a case, a near-tip function including a branch function should be introduced in order to enrich the element containing the crack tip. Thus far, the near-tip asymptotic function for a homogeneous isotropic crack has been used in X-FEM for solving linear elastic problems. However, these near-tip functions are obtained for isotropic elastic problems under small deformation conditions. Therefore, it remains unclear as to whether these functions can be applied to solve materially nonlinear problems.

In the present study, in order to determine the possibility of applying nonlinear X-FEM analysis to materially nonlinear problems, crack modeling using enrichment functions is examined. The formulations for X-FEM analysis considering materially nonlinear behavior with small deformation are shown herein. Elastic-plastic X-FEM analyses for a three-dimensional body with a surface crack are performed. The numerical results are compared with results obtained using conventional FEM, and the distribution of enriched nodes, which is used for enrichment near the crack tip, is examined.

2 NUMERICAL METHODS

2.1 Finite element discretization [3]

In the present study, the small deformation problem considering the elastic-plastic constitutive equation is treated. The elastic-plastic analysis by X-FEM conducted in the present study uses the framework of the incremental method within the limitation of small deformation. Namely, the principle of virtual work described by the Cauchy stress tensor and the linear strain tensor is discretized using finite elements, and the solution is obtained iteratively using the Newton-Raphson method. In the iterative process, the backward Euler scheme is utilized to determine the incremental stress from the incremental displacement by considering the elastic-plastic constitutive equation.

The incremental displacement $\Delta\mathbf{U}^{(i)}$ at iteration i is obtained by solving the following equations:

$$ {}^{t+\Delta t}\mathbf{K}_{EP}{}^{(i)}\Delta\mathbf{U}^{(i)} = {}^{t+\Delta t}\mathbf{F} - {}^{t+\Delta t}\mathbf{Q}^{(i-1)} \tag{1.1} $$

$$ {}^{t+\Delta t}\mathbf{U}^{(i)} = {}^{t+\Delta t}\mathbf{U}^{(i-1)} + \Delta\mathbf{U}^{(i)} \tag{1.2} $$

and

$$ {}^{t}\mathbf{K}_{EP} = \sum_{e}\iiint_{Ve}\mathbf{B}_L{}^{T}\,{}^{t}\hat{\mathbf{C}}_{EP}\mathbf{B}_L dV_e \tag{2.1} $$

$$ {}^{t}\mathbf{Q} = \sum_{e}\iiint_{Ve}\mathbf{B}_L{}^{T}\,{}^{t}\hat{\boldsymbol{\sigma}} dV_e \tag{2.2} $$

where

${}^{t}\mathbf{K}_{EP}$ is the linear strain incremental stiffness matrix at time t,

${}^{t}\mathbf{U}$ is the vector of the dislacement at time t,

${}^{t}\mathbf{F}$ is the vector of the external forces at time ,t

$\mathbf{B}_L$ is the linear strain-displacement transformation matrix,

${}^{t}\mathbf{Q}$ is the vector of the nodal forces equivalent to the element stresses at time *t*,

${}^{t}\hat{\boldsymbol{\sigma}}$ is the vector of Cauchy stresses at time *t*, and

$\hat{\mathbf{C}}_{EP}$ is the stress-strain elastic-plastic material property matrix at time *t*.

2.2 Interpolation function

In the present study, a planar surface crack in an elastic-plastic material is assumed. The analyzed domain is defined in Cartesian coordinates (x, y, z), and the planar surface crack is assumed to be on the x-y plane. In X-FEM, the approximate displacement function $\boldsymbol{u}^{h}$ of the distributed displacement $\boldsymbol{u}$ near a delamination is expressed as:

$$\mathbf{u}^{h}(\mathbf{x}) = \sum_{I=1}^{8} N_I(\mathbf{x})\mathbf{u}_I + \sum_{I\in\mathbf{C}} N_I(\mathbf{x}) \sum_{k=1}^{4} \gamma_k(\bar{f}(\mathbf{x}), z)\mathbf{a}_I^k + \sum_{I\in\mathbf{J}} N_I(\mathbf{x})H(z)\mathbf{b}_I \tag{3.1}$$

$$\bar{f}(\mathbf{x}) = \sum_{I=1}^{8} N_I(\mathbf{x}) f(\mathbf{x}_I) \tag{3.2}$$

where N_I is the interpolation function of an eight-node linear element used in the formulation of the conventional FEM, $\boldsymbol{C}$ and $\boldsymbol{J}$ denote the node set considering the asymptotic solution and the discontinuity of displacement near a crack, respectively, and $\mathbf{u}_I$, $\mathbf{a}_I^k$, and $\mathbf{b}_I$ denote the vectors of freedoms assigned to each node. Here, $\boldsymbol{C} \cap \boldsymbol{J} = \phi$ is satisfied. In addition, γ_i (i =1,*4*) are the near-tip functions, which consider the discontinuity near the crack tip, H(x) is the *Heaviside* function used to express the discontinuity of the displacement on a crack, and $f(\mathbf{x})$ is a level set function that is introduced in order to express the shape of the front tip of the crack using nodal information. The level set function f(x) is described below.

In the present study, the level set function $f(\mathbf{x})$ used in Eq. (3.2) is defined as follows:

$$f(\mathbf{x}) = \min_{\bar{\mathbf{x}}\in\Gamma} \|\mathbf{x} - \bar{\mathbf{x}}\| sign(\mathbf{n}(\bar{\mathbf{x}})^T(\mathbf{x} - \bar{\mathbf{x}})) \tag{4}$$

where Γ represents the curved front line of the crack, $\bar{\mathbf{x}}$ is a point on the curved line Γ, and $\mathbf{n}(\bar{\mathbf{x}})$ denotes the vector orthogonal to the curved line Γ at point $\bar{\mathbf{x}}$.

This function is called the signed distance function, and the absolute value of f is the distance between the point and Γ.

In the present study, the near-tip function γ_i, which is determined from the asymptotic solution of a crack in a homogeneous isotropic material, is defined as follows:

$$\gamma_1 = \sqrt{r}\cos\left(\frac{\theta}{2}\right), \gamma_2 = \sqrt{r}\sin\left(\frac{\theta}{2}\right), \gamma_3 = \sqrt{r}\sin\left(\frac{\theta}{2}\right)\sin\theta, \gamma_4 = \sqrt{r}\cos\left(\frac{\theta}{2}\right)\sin\theta \tag{5}$$

where r and θ are the polar coordinates in a plane defined near the crack tip, and

$$r = \sqrt{f^2 + z^2} \tag{6.1}$$

$$\theta = \arctan(z/f). \tag{6.2}$$

3 NUMERICAL EXAMPLES

An elastic-plastic body with a planar surface crack under tensile loading, as shown in **Fig. 1**, was analyzed using X-FEM. The structured finite element mesh (40 x 40 x 40) shown in **Fig. 2** was employed. A crack with an arbitrary front tip shape can be modeled by various enrichment nodes. In the calculation, the three types of models shown in **Fig. 3** were used. The crack opening displacement and the *J*-integral at the center of the crack were evaluated. The *J*-integrals calculated for various loads are shown in **Fig. 4**.

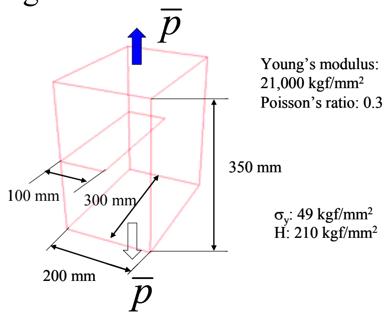

Figure 1: Single-edge crack problem

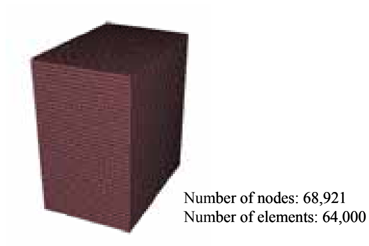

Figure 2: Finite element mesh

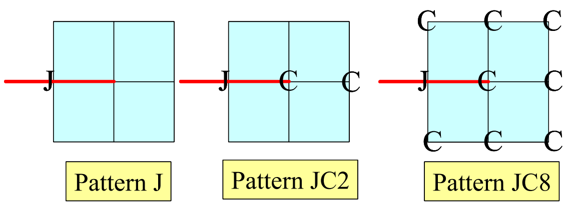

Figure 3: Crack modeling

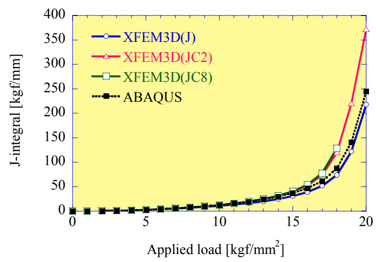

Figure 4: J-integral vs. applied load

4 CONCLUDING REMARKS

The present paper presents a basic study of the application of X-FEM to elastic-plastic problems and describes the results of a three-dimensional elastic-plastic analysis using X-FEM. The effect of the distribution of enrichment nodes near the crack on the numerical results was examined.

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