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A SECOND ORDER HOMOGENIZATION PROCEDURE FOR MULTI–SCALE ANALYSIS BASED ON NONLINEAR MICRO–POLAR KINEMATICS

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1 INTRODUCTION

A possible drawback of the first order homogenization technique is that it does not account for the absolute size of the microstructure, thus failing to represent geometrical size effects. Moreover, an ambiguity concerns the assumption of uniform macroscopic fields across the microstructure, which works well in regions where the macroscopic fields vary smoothly, but it does, in fact, not work well in regions where steep gradients occur at e.g. corners, boundary layers, cracks, inclusions etc. The proper consideration of these issues requires (at least) a second order homogenization technique along with a matching higher order continuum formulation. As to the exploitation of generalized continuum formulations (such as micro–polar, strain–gradient), which all have the effect of introducing a material length scale into the constitutive model, quite much work has been presented in the literature, cf. [1] and references therein. In the present paper we present a higher order homogenization scheme based on non–linear micro–polar kinematics representing the macroscopic variation within a Representative Volume Element (RVE) of the material. This approach is similar the to the second gradient continuum formulations presented e.g. in ref. [1].

2 SECOND ORDER HOMOGENIZATION – MICROPOLAR THEORY

Let us consider the situation where a solid component occupies the undeformed configuration $\mathcal{B}_0$. During deformation the component enters it deformed configuration $\mathcal{B}$, in accordance with the (macroscopic) non–linear deformation map $\boldsymbol{\varphi}_M[\mathbf{X}_M]$. Associated with the deformation of the material point $\mathbf{X}_M$, we thereby consider an RVE with the width W and height H (in 2D) resolving the miniscale.

The averaging within the RVE is carried out by considering the heterogeneous material within the RVE as a first order continuum, obtained from the mapping, cf.Kuosnetzova et al. [1], written as

$$\mathbf{x} = \mathbf{F}_M \cdot \mathbf{X} + \frac{1}{2}\mathbf{G}_M : (\mathbf{X} \otimes \mathbf{X}) + \boldsymbol{\varphi}[\mathbf{X}] \text{ with } \mathbf{G}_M = (\boldsymbol{\varphi}_M \otimes \nabla_X) \otimes \nabla_X = \mathbf{F}_M \otimes \nabla_X \quad (1)$$

where the actual macro–micro transition is considered by imposing conditions on the micro–problem based on the macroscopic deformation tensor $\mathbf{F}_M$ and its gradient $\mathbf{G}_M$. Moreover, $\boldsymbol{\varphi}[\mathbf{X}]$ is the microstructural fluctuation field.

A basic requirement in the homogenization procedure is to conserve momentum balance within the RVE, which may be stated for the static case and in the absence of body forces within the RVE as

$$\mathbf{S}_1^t \cdot \nabla_X = \mathbf{0} \;\; \forall \mathbf{X} \text{ in } V_0 \Leftrightarrow \int_{V_0} \mathbf{S}_1^t : \delta\mathbf{F} dV_0 = \int_{\Gamma_0} \mathbf{t}_1 \cdot \delta\mathbf{x} \; d\Gamma_0 \quad (2)$$

where $\mathbf{S}_1^t$ is the microfirst Piola Kirchhoff stress, V_0, Γ_0 are the volume and boundary of the RVE. On the basis of the micro–polar kinematics, let us first recall the macroscopic deformation gradient described in terms of the independent micro–polar rotation $\overline{\mathbf{R}}_M$ and a micro–polar right stretch tensor $\overline{\mathbf{U}}_M$ written as $\mathbf{F}_M = \overline{\mathbf{R}}_M \cdot \overline{\mathbf{U}}_M$.

As compared to the Taylor series expansion above, we now make the restriction that $\mathbf{F}_M \approx \overline{\mathbf{R}}_M$ in the second order term to obtain the matching kinematics of the RVE with the micro–polar theory. Hence, we obtain the following formulation of the kinematics of the RVE:

$$\mathbf{x} = \mathbf{F}_M \cdot \mathbf{X} + \frac{1}{2}\left(\left(\overline{\mathbf{R}}_M \otimes \nabla_X\right) \cdot \mathbf{X}\right) \cdot \mathbf{X} + \boldsymbol{\varphi}[\mathbf{X}] \quad (3)$$

It appears in view of (2) (after some derivations) that this condition may be formulated in the present context of micro–polar theory as

$$\frac{1}{V_0}\int_{V_0} \mathbf{S}_1^t : \delta\mathbf{F} dV_0 = \boldsymbol{\tau}_M^t : \left(\delta\mathbf{l}_{\varphi M} - \text{spn}[\delta\boldsymbol{\theta}_M]\right) + \overline{\mathbf{M}}_{1M}^t : \delta\mathbf{K}_M \quad (4)$$

where $\boldsymbol{\tau}_M^t$ is the non–symmetric homogenized Kirchhoff stress tensor, $\overline{\mathbf{M}}_{1M}^t$ is second order homogenized couple stress tensor. Due to the energy conjugation between stress and deformation variables in the micro–polar theory, we identify the following homogenized stress measures:

- The homogenized Kirchhoff stress tensor $\boldsymbol{\tau}_M^t$

$$\boldsymbol{\tau}_M^t = \frac{1}{V_0}\int_{\gamma_0} \mathbf{t}_1 \otimes \mathbf{x}^{1st} d\Gamma_0 \text{ with } \mathbf{x}^{1st} = \mathbf{F}_M \cdot \mathbf{X} \quad (5)$$

- The homogenized couple stress tensor $\overline{\mathbf{M}}_{1M}^t$

$$\overline{\mathbf{M}}_{1M}^t = \frac{1}{V_0}\int_{\Gamma_0} \mathbf{Q} \otimes \mathbf{X} \mathbf{d}\Gamma_0 \text{ with } \mathbf{Q} = \text{axl}[\mathbf{t}_{1R} \otimes \mathbf{X}] \text{ and } \mathbf{t}_{1R} = \overline{\mathbf{R}}_M^t \cdot \mathbf{t}_1 \quad (6)$$

EXAMPLE: MIXED 1st ORDER AND MICROPOLAR DEFORMATION

In the numerical example we consider the influence of the combination first order deformation and second order micropolar curvature on the response of the periodic heterogeneous micro-structure given in Fig. 1, with a void of spherical shape. The material model of the micro-constituent is governed by the stored energy function of Neo-Hookean type written as

$$W[\hat{\mathbf{C}}, J] = G(\mathbf{1} : \hat{\mathbf{C}} - 3) + \frac{1}{2}K(J - 1)^2 \text{ with } \hat{\mathbf{C}} = J^{-\frac{2}{3}}\mathbf{F}^t \cdot \mathbf{F} \tag{7}$$

where $\hat{\mathbf{C}}$ is the isochoric right Cauchy-Green deformation tensor. Moreover, G and K are the shear and bulk moduli of the micro-material. The soft inclusion is modeled by assuming that it obeys the same material law but has has 20 times lower stiffness, cf. Fig. 1. It is also assumed that the solid is planar from the micro-polar viewpoint, whereby the axial vector of the micro-polar rotation tensor $\overline{\mathbf{R}}_M$ is a priori associated with the out-of-plane $\mathbf{E}_3$-vector, i.e. $\boldsymbol{\theta}_M = \theta_M \mathbf{E}_3$. As to the applied macroscopic deformation, we shall consider the first order deformation represented by *simple shear*, which may be written with the plane strain assumption for the solid as

$$\mathbf{F}_M = \begin{bmatrix} 1 & \tan[\gamma_M] & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad \overset{3}{\mathbf{K}}_M = \mathbf{K}_{\theta M} \otimes \nabla_X \theta_M \text{ with } \mathbf{K}_{\theta M} = \text{spn}[\mathbf{E}_3] \tag{8}$$

where γ_M is the measure of the amount of shear. Moreover, Finite element discretization is used to resolve the fluctuation field $\boldsymbol{\varphi}[\mathbf{X}]$ with Dirichlet boundary conditions along the external boundaries of the RVE.

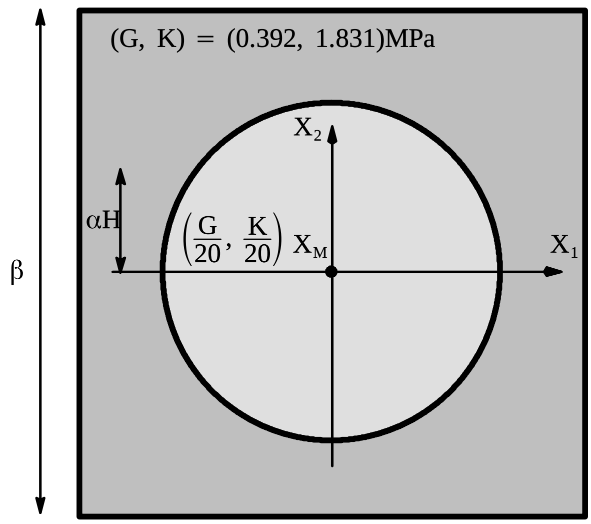

Figure 1 Schematic of periodic RVE with void of specical shape

In this example the situation of pure micro-polar deformation is considered. The loading is thereby defined by the scenario: $\gamma_M = 0$, $\theta_M = 0$ and $\theta_{M,1} = \theta_{M,2} = 0 - 2$, applied proportionally in 20 load steps. In the present case we consider the RVE sizes $\beta = \{0.1, 0.5, 0.7, 1\}$. The result is shown in Fig. 2a for the homogenized couple stress, and in Fig. 2b the deformed configuration in the final load step $\beta = 1$ is shown. It noted that a size dependent stiffening response obtained.

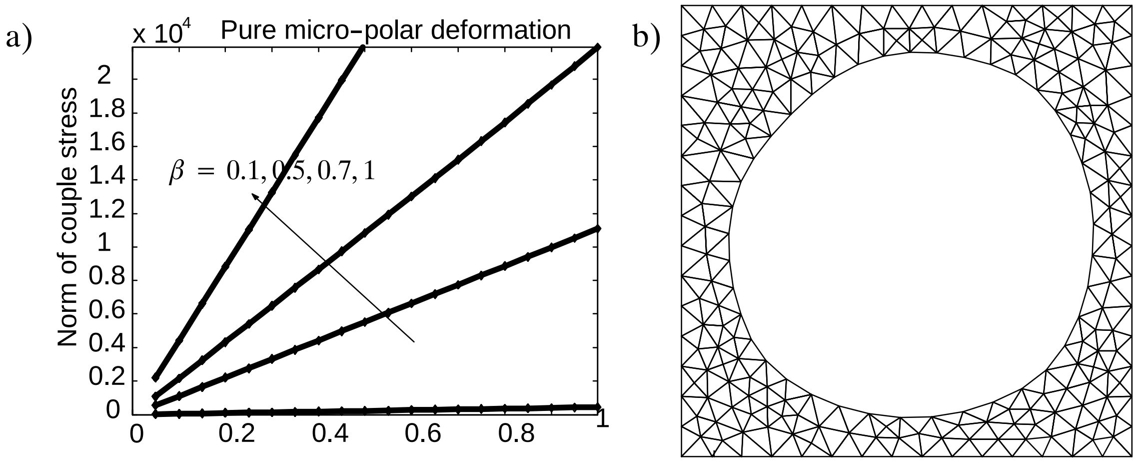

Figure 2 a) Couple stress response versus prescribed deformation for pure micro-polar deformation, b) Fluctuation field $\boldsymbol{\varphi}[\mathbf{X}]$ within RVE at final load step.

3 CONCLUDING REMARKS

In the present paper we developed a homogenization procedure in context of micro-polar continuum, considered as a restriction of the second gradient theory. It appears that the actual averaging is obtained with respect to the ordinary homogeneous modes and in addition the higher order curvature mode represented as a second order tensor. We emphasize the close relation between the micro-polar and second gradient continuum descriptions.

4 REFERENCES

[1] V. Kouznetsova, M. G. D. Geers, and W. A. M. Brekelmans, ”Multi-scale constitutive modelling of heterogeneous materials with a gradient-enhanced computational homogenization scheme”, Int. J. Numer. Meth. Engrg., 54:1235-1260 (2002).