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# MODELING OF LARGE INELASTIC DEFORMATIONS OF STRUCTURAL FOAM AT HIGH DEFORMATION RATES

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## 1 INTRODUCTION

In chase of weight reduction, structural foam can be used in vehicle body components. The primary goal is then to maintain crashworthiness, i.e. stiffness and energy absorption of the load-bearing structures, while saving weight. Computer crash simulation is used as a natural part of the vehicle design process and for foam-type material to be considered for utilization, a proper constitutive model is needed. In the effort to provide such model, a large deformation isotropic hyperelasto-viscoplastic constitutive model for foam materials has been developed. With foam being a mixture of a solid and a gas phase, the Theory of Porous Media is used to model foam phenomenologically at the macroscopic level. Foam materials in compression show a characteristic increase in stiffness, why hardening is modeled connected to the logarithmic compaction strain. This tends to infinity as the point of compaction is approached. Since the response of many foamed polymers is deformation-rate dependent, a viscoplastic flow rule is utilized. Furthermore, the yield surface is initially pressure sensitive, while as compaction develops, it hardens towards pure shear dependency. The constitutive model has been implemented into the finite element software LS-DYNA and verification simulations have been made[1].

## 2 CONTINUUM MECHANICAL RELATIONS

The Theory of Porous Media (TPM)[2,3] is used, describing the foam as a mixture of solid and fluid constituents denoted by $\alpha = \{\mathrm{s}, \mathrm{f}\}$. The volume fractions $n^\alpha(\mathbf{x}, t)$ denote the ratio of local constituent volume to mixture volume. Moreover, each spatial point $\mathbf{x}$ of the current configuration is simultaneously occupied by the material particles $\mathbf{X}^\alpha$ where the two constituents relate to different reference configurations, i.e. $\mathbf{X}^\mathrm{s} \neq \mathbf{X}^\mathrm{g}$. The deformation gradient and the Jacobian associated with the solid skeleton material are then $\mathbf{F} = \boldsymbol{\varphi}(\mathbf{X}^\mathrm{s}) \otimes \boldsymbol{\nabla}$ and $J = \det(\mathbf{F})$ respectively.

### 2.1 Limitations

The first limitation is the assumption that the solid material is incompressible. The other limitation regards how the phase interaction is handled. Foam material can either have closed cells, open cells or a combination of both. For the case of *closed cells*, whereby the air cannot escape from the pores, the velocity of the gas relative to the solid is zero. Then the gas pressure can be described via the ideal gas law. *Open cells* lead to gas flow through the skeleton structure. This interaction may become significant for very high deformation rates. However, in the present paper it is assumed that the foam has closed cells and the boundary conditions will result in little gas pressure so that the drag force will be insignificant. It is therefore disregarded.

## 3 MODELING THE EFFECTIVE STRESS

The kinematics are specified in terms of the solid phase material, for which the deformation gradient is split multiplicatively into elastic and inelastic (plastic) portions $\bar{\mathbf{F}}$ and $\mathbf{F}_\mathrm{p}$, respectively, as $\mathbf{F} = \bar{\mathbf{F}} \cdot \mathbf{F}_\mathrm{p}$. Consequently, the right Cauchy-Green deformation tensor is obtained as $\mathbf{C} = \mathbf{F}^\mathrm{t} \cdot \mathbf{F} = \mathbf{F}_\mathrm{p}^\mathrm{t} \cdot \bar{\mathbf{C}} \cdot \mathbf{F}_\mathrm{p}$, with the elastic component $\bar{\mathbf{C}} = \bar{\mathbf{F}}^\mathrm{t} \cdot \bar{\mathbf{F}}$. The dissipation $\mathcal{D}$ in the cellular solid becomes:

$$\mathcal{D} = \frac{1}{2}\bar{\mathbf{S}} : \dot{\bar{\mathbf{C}}} + \bar{\mathbf{T}} : \bar{\mathbf{L}}_\mathrm{p} - n_0^\mathrm{s}\rho^\mathrm{s}\dot{\Psi} \geq 0 \quad \text{with } \bar{\mathbf{T}} = \bar{\mathbf{C}} \cdot \bar{\mathbf{S}} \text{ and } \bar{\mathbf{L}}_\mathrm{p} = \dot{\mathbf{F}}^\mathrm{p} \cdot \mathbf{F}_\mathrm{p}^{-1} \tag{1}$$

where $\bar{\mathbf{S}}$ is the "intermediate" 2:nd Piola Kirchhoff stress, $\bar{\mathbf{T}}$ the Mandel stress and $\bar{\mathbf{L}}_\mathrm{p}$ the plastic "velocity gradient".

In modeling the Helmholts free energy of the solid material $\Psi$, it is assumed to be a function of the elastic deformation $\bar{\mathbf{C}}$ and the hardening variable $\kappa$. This leads to the constitutive state equations and the reduced dissipation:

$$\bar{\mathbf{S}} = 2\hat{\rho}_0^\mathrm{s}\frac{\partial \Psi}{\partial \bar{\mathbf{C}}}, \quad K = -\hat{\rho}_0^\mathrm{s}\frac{\partial \Psi}{\partial \kappa} \quad \text{and} \quad \mathcal{D} = \bar{\mathbf{T}} : \bar{\mathbf{L}}_\mathrm{p} + K\dot{\kappa} \geq 0. \tag{2}$$

According to the principle of maximum dissipation, the associated visco-plastic flow rule of Perzyna type[4] is written as:

$$\bar{\mathbf{L}}_\mathrm{p} = \frac{\eta}{t_*}\bar{\mathbf{N}} \text{ with } \bar{\mathbf{N}} = \frac{\partial \Phi}{\partial \bar{\mathbf{T}}} \quad \text{and} \quad \dot{\kappa} = \frac{\eta}{t_*}\bar{N} \text{ with } \bar{N} = \frac{\partial \Phi}{\partial K}, \tag{3}$$

where $\eta = (< \Phi > /\sigma_\mathrm{c})^n$ is the overstress function. $\sigma_\mathrm{c}$ and $n$ are the Norton creep parameters.

## 4 PROTOTYPE MODEL

### 4.1 Solid phase compaction

The solid phase compaction is represented in terms of the logarithmic compaction strain $\beta = -\log(n^\mathrm{g}/n_0^\mathrm{g})$, where $n^\mathrm{g}$ and $n_0^\mathrm{g}$ are current and initial gas content, respectively.

$\beta$ is split into recoverable and irrecoverable parts:

$$\beta = \beta^{\mathrm{e}} + \beta^{\mathrm{p}} = -\log(\frac{1}{\bar{J}}\frac{\bar{J} - n_{\mathrm{p}}^{\mathrm{s}}}{1 - n_{\mathrm{p}}^{\mathrm{s}}}) - \log(\frac{1 - n_{\mathrm{p}}^{\mathrm{s}}}{1 - n_{0}^{\mathrm{s}}}) \tag{4}$$

It is noted that $\beta = 0$ in the initial stage before deformation, whereas $\beta \to \infty$ when complete compaction occurs, i.e. at the point of compaction. Dilation from the initial configuration is obtained whenever $\beta < 0$.

## 4.2 Isotropic elasticity

In order to formulate the volumetric stiffening effect of the foam as the material compaction is increased, we propose the isochoric/volumetric split of the free energy $\Psi = \Psi^{\mathrm{iso}} + \Psi^{\mathrm{vol}}$, where we introduce the argumentation:

$$\hat{\rho}_0^{\mathrm{s}}\Psi^{\mathrm{iso}}(\bar{\mathbf{C}}) = \frac{1}{2}G(\mathbf{1} : \hat{\bar{\mathbf{C}}} - 3) \quad \text{and} \quad \hat{\rho}_0^{\mathrm{s}}\Psi^{\mathrm{vol}}(\beta^{\mathrm{e}}, \beta^{\mathrm{p}}) = \frac{1}{2}K_{\mathrm{b}}\beta^{\mathrm{e}2} + \frac{1}{2}H_{\mathrm{b}}\beta^{\mathrm{p}2} \tag{5}$$

where $G$ and $K_{\mathrm{b}}$ are the shear and bulk parameters of the solid cellular material and $H_{\mathrm{b}} > 0$ represents hardening associated with the inelastic compaction $\beta^{\mathrm{p}}$. The first part in (5) is just the standard isochoric energy contribution, whereas the second volumetric part describes in particular the elastic stiffening effect of the material as $\bar{J}$ approaches the *inelastic* portion of the solid phase volume fraction $n_{\mathrm{p}}^{\mathrm{s}}$. A major motivation for the expression in (5) is the compaction condition $n^{\mathrm{s}} \leq 1$, leading to $\beta^{\mathrm{e}} > 0$ or $n_{\mathrm{p}}^{\mathrm{s}} \leq \bar{J}$. Indeed the situation that $\bar{J} \to n_{\mathrm{p}}^{\mathrm{s}}$ is penalized in the expression for the volumetric free energy $\Psi^{\mathrm{vol}}$. We also note that $\Psi^{\mathrm{vol}}(0,0) = 0$ and that $\Psi^{\mathrm{vol}}$ is convex. Next, the evolution of the hardening variable $\kappa$ is modeled by $\kappa = \beta^{\mathrm{p}}$.

## 4.3 Elastically admissible stresses

The elastically admissible region is defined in terms of the $p$ and $q$ invariants, i.e. the pressure and the von Mises stress respectively:

$$\mathcal{B} = \{\bar{\mathbf{T}}^{\mathrm{eff}} : \Phi(\bar{\mathbf{T}}^{\mathrm{eff}}, \beta^{\mathrm{p}}) \leq 0\} \text{ with } \Phi(\bar{\mathbf{T}}^{\mathrm{eff}}, \beta^{\mathrm{p}}) = (q^2 + \alpha p^2)^{\frac{1}{2}} - 3\beta p - c \leq 0, \tag{6}$$

where $\Phi$ is the quasi-static yield function. In order to describe the hardening of the yield surface, we shall consider the parameters $\alpha = \alpha_0 f(\beta^{\mathrm{p}})$ and $\beta = \beta_0 f(\beta^{\mathrm{p}})$ as a function of a hardening factor $f = 1/(1 + \beta^{\mathrm{p}})^{\gamma}$. It is noted that $f(\beta^{\mathrm{p}})$ (as desired) significantly reduces the values of $\alpha$ and $\beta$ towards the end of the compaction. The shape of the yield function is initially parabolic, i.e. pressure dependent. As compaction develops, the shape changes towards purely isochoric response as the point of compaction is approached (Fig. 1).

# 5 VERIFICATION EXAMPLE

To illustrate the model behavior in compression, Figure 2 shows the reaction force curve for a foam cube in a compression test simulated in LS-DYNA. Constant vertical velocity,

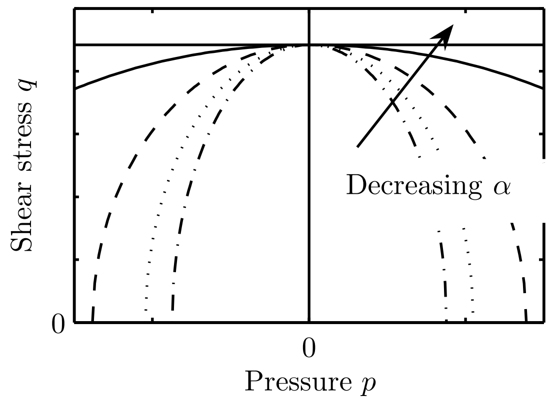


Figure 1: Yield surface in $p$-$q$ space for decreasing $\alpha$

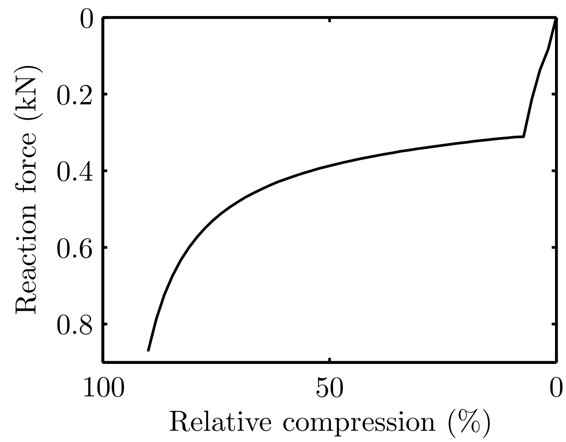


Figure 2: Compression test response curve

towards the bottom, was prescribed to the nodes on the top surface while the vertical displacement of the nodes on the bottom surface were restrained. One can clearly see the three different stages of deformation which are typical for a foam material. The first stage is the practically linear *elastic phase*, at the beginning of the deformation (upper-right corner in Fig. 2). This ends abruptly with the beginning of the *plateau phase*, during which only little increase in stiffness occurs. Then, the deformation turns gradually into the *densification phase*, where the stiffness increases rapidly (lower-left corner in Fig. 2).

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