VIII International Conference on Computational Plasticity
COMPLAS VIII
E. Oñate and D. R. J. Owen (Eds)

NONLINEAR FINITE ELEMENT ANALYSIS OF REINFORCED CONCRETE DEEP BEAMS

Samir M. O. Hassan Dirar[*] **and Chris T. Morley**[†]

[*] Department of Engineering
University of Cambridge
Cambridge, CB2 1PZ, England
e-mail: sh394@eng.cam.ac.uk, web page: http://www-civ.eng.cam.ac.uk/abstract/shabst.html

[†] Department of Engineering
University of Cambridge
Cambridge, CB2 1PZ, England
e-mail: ctm1@eng.cam.ac.uk, web page: http://www-civ.eng.cam.ac.uk/ctm/ctm_web.htm

Key words: Deep Beam, Finite Element, Reinforced Concrete.

Summary. *This paper describes a series of nonlinear finite element analyses carried out using the commercial package, DIANA7, to predict the ultimate load and mode of failure for three different types of reinforced concrete continuous two-span deep beams. Only one parameter, the shear retention factor, was varied during the analyses. The predicted results were in good agreement with the experimental results.*

1 INTRODUCTION

In spite of the sophisticated state-of-the-art attained in nonlinear finite element (FE) analysis of reinforced concrete (RC) structures in terms of material models and solution techniques, the dependency of the method on too many parameters has always been a shortcoming.

For the analyses described hereinafter, the FE package DIANA7 has been applied to estimate the ultimate load and the mode of failure for a series of RC two-span continuous deep beams. Most of the nonlinear analytical parameters were fixed according to experimental values or logical reasoning and the only variable tuned was the shear retention factor, which was given a value about 0.1 in most of the analyses.

2 EXPERIMENTAL PROGRAM

The experimental program consisted of testing three different types of deep beams, S1, S2 and S3. All beams, which were identical in geometry and longitudinal reinforcement, had a thickness of 90 mm and dimensions as shown in Figure 1. The Figure also depicts the test setup and gives the reinforcement yield strength (Y) and Young's modulus (E).

The differences between the three series were in the vertical reinforcement patterns and the concrete properties. Beams of series S1 had no vertical stirrups while those of series S2 and

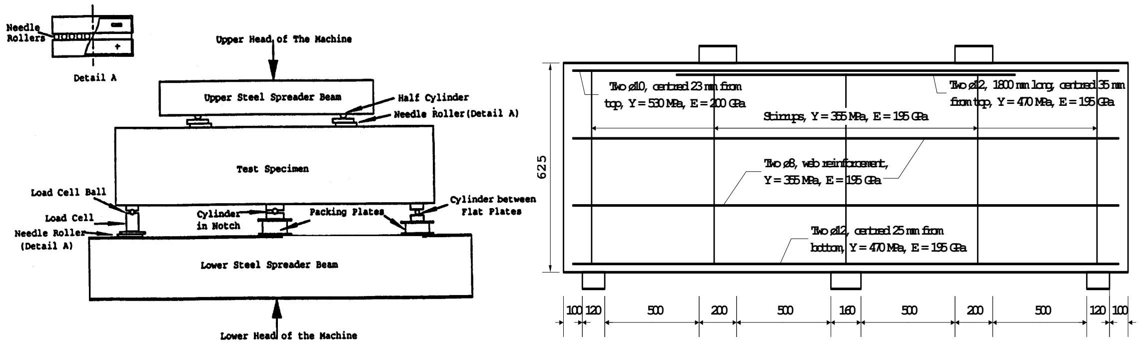

Figure 1: Test setup and specimen dimensions (Not to scale. All dimensions in mm.)

S3 had 6 mm and 8 mm diameter stirrups at 150 mm and 130 mm spacing respectively.

Table 1 gives the basic concrete properties for the analyzed beams. The first row in the table refers to the serial number and the year of testing while the second and third rows gives the concrete cube compressive strength and split tensile strength respectively in MPa.

S1/95	S1/96	S1/97	S1/98	S2/95	S2/96	S3/95	S3/98	S3/99	S3/03
40	39	36	42	26	37	34	31	45	37
N/A	3.10	2.75	3.60	2.40	2.70	2.70	2.60	3.40	2.90

Table 1: Concrete properties of modeled specimens

Further information about the experimental setup, testing apparatus and instrumentations can be found elsewhere[1].

3 FINITE ELEMENT MODELING

A 3-D FE model capable of predicting the ultimate loads and modes of failure of the three series was built. The model was kept as simple as possible in terms of element selection and usage of default integration schemes, without detracting from the accuracy of the results. FE modeling of these beams has been reported elsewhere[2]. This paper, however, models the support plates better and gives better agreement over the whole range of behaviour.

The concrete body and the steel loading plates were modeled by 8-noded brick elements. The supports were modeled by 6-noded wedge elements, which were connected by their base to the 8-noded elements while their sharp edges acted as rotating edges, modeling the behaviour of roller supports. Reinforcement is modeled by 3-noded truss elements. For more detailed description of these elements, the reader is referred to DIANA Element Library[3].

Figure 2 illustrates the typical concrete FE mesh used together with the reinforcement FE meshes and their boundary conditions for the three series.

Because of its computational convenience as well as its resemblance to reality, the smeared crack approach was used to model the cracking of concrete. The initiation of cracks was governed by a linear tension cut-off criterion. After cracking, Hordijk softening curve was incorporated to capture the effects of tension stiffening and softening.

Two constitutive models only could be incorporated with the brick elements to model the

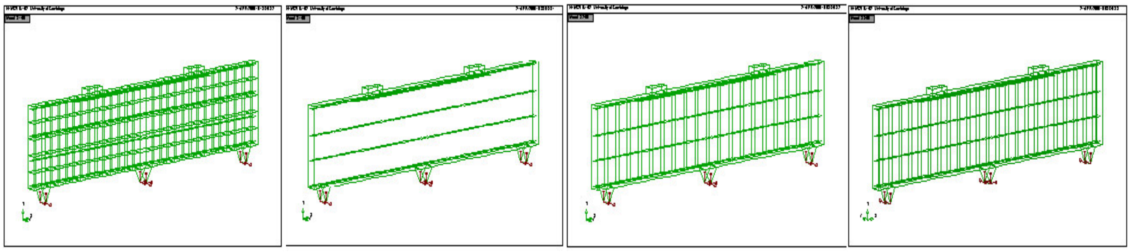

Figure 2: Typical concrete FE mesh and reinforcement FE meshes for the three series

compressive behaviour of concrete under loading. Those were: (1) the constitutive models of plasticity, where Von-Mises criterion was chosen to specify yield, and (2) crack models based on total strain, where compressive behaviour was in general described by a nonlinear function between stress and strain in a certain direction.

Shear was embodied by a constant shear retention model incorporating a value of about 0.1 for the shear retention factor (β), after cracking, in most of the analyses.

The loading plates, supporting plates, and reinforcing steel were modeled as elastic-perfectly plastic materials. Von-Mises yield criterion was used again to model plasticity.

4 RESULTS AND DISCUSSION

A comparison between the experimental and predicted ultimate loads for the ten analyzed specimens is given in Figure 3. The peak load predictions were generally very close to the experimental results. The plasticity and total strain constitutive models gave approximately similar results with mean predicted to experimental ratios of 1.04 and 1.05, and standard deviations of 0.11 and 0.10 respectively.

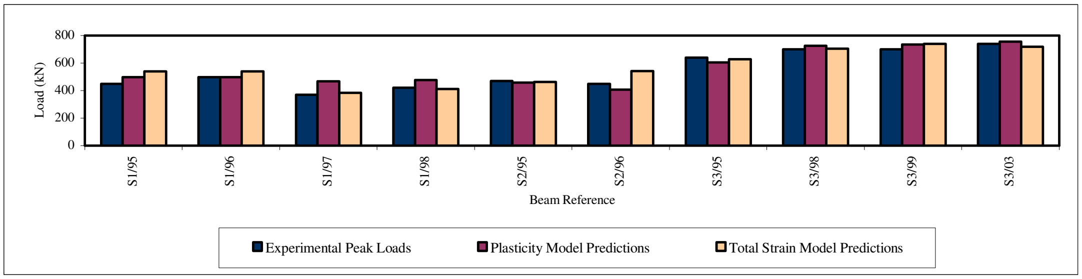

Figure 3: Comparison between FE predictions and experimental results

The experimental and analytical load-midspan deflection relationships for S1/96 and S3/98 are compared in Figure 4. These relationships could be divided into four stages. Throughout the first stage, the relationship was linear and the deflections developed at a relatively low level. After this initial stage, the flexural cracks reduced the beams' stiffness significantly and the mid-span deflections increased markedly. The third stage marked the sudden appearance of diagonal cracks at the middle of the deep beams running between the loading plate and the central support. These diagonal cracks caused a sudden drop in the applied load, usually regained as displacement continued. During the last stages, the beams continued to lose their stiffness until they finally collapsed. Figure 5 shows the various stages of crack initiation and propagation in terms of crack strains, using a coloured scale of strain magnitude.

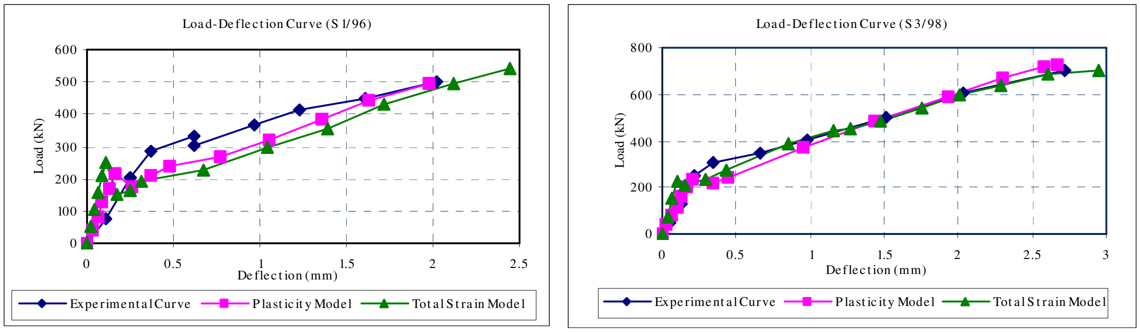

Figure 4: Load-Deflection Curves

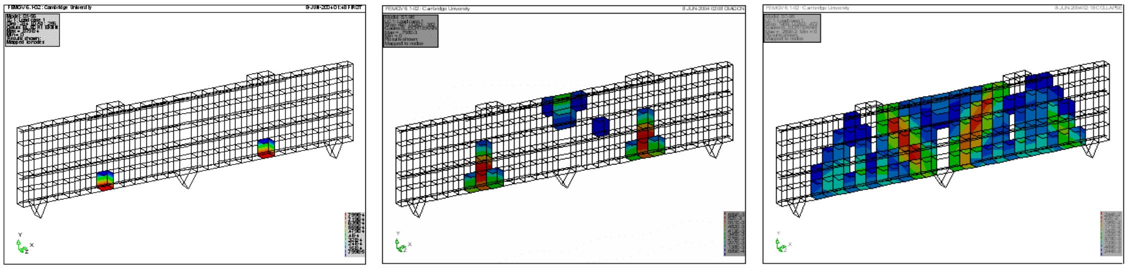

Figure 5: Cracks initiation and propagation

5 CONCLUSIONS

- The FE method was capable of modeling the behaviour of the RC deep beams. Shear retention factor was the only parameter varied during the analyses.
- Predictions of the ultimate load were within an accuracy region of 5%. The method also picked the mode of failure characterized by a diagonal shear crack.

REFERENCES

[1] S.M.O. Hasan, *Nonlinear Finite Element Analysis of Reinforced Concrete Deep Beams*, MPhil Thesis, University of Cambridge, Department of Engineering, (2004).

[2] I.M. May, C.T. Morley and C.T. Chen, “Benchmarking of nonlinear finite element analyses for reinforced concrete deep beams”, *Comp. Modelg. of Conc. Struc.*, 751-758, (2003).

[3] TNO Building and Construction Research, *DIANA User’s Manual, Element Library*, Delft, The Netherlands, (1998).