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ELASTIC-PLASTIC ANALYSIS OF SPOT-WELDED THIN-WALLED STRUCTURES

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Summary. *Spot welded thin wall structures are widely used in carrying structure of vehicles. Presented finite element approach combines modeling of thin structural elements by shell elements, spot welded joints by beam (welded) elements and contact conditions between welded shits by gap elements. Improved Bathe-Dvorkin shell element with incompatible modes is used in combination with von Mises material model with mixed hardening assumption and the Ramberg-Osgood relation (incompressible plastic flow). The implicit stress integration procedure, according to the governing parameter method (GPM), is used. Gap elements are used to prevent penetration and overlapping of modeled surfaces. In this paper we analyze influence of modeling real contact conditions during process of girder deformation. Comparison to case when contact conditions are neglected is given. Presented approach is illustrated with example that demonstrate usability and efficiency.*

1 INTRODUCTION

The finite element method has notable application in optimization of carrying structure of vehicle. This paper presents a part of research in connection with modelling of sheets in contact and prevention of penetration one sheet through the other one during process of deformation. This is important in order to real modelling of spot welding joints. In the car industry, i.e. in modelling of car body, gap elements are used in order to prevent penetration one modelling surface through the other one. A welded spots were modelled by beam elements and improved shell elements were used for sheets modelling, [3], [6]-[9].

2 THEORETICAL BASIS

The gap element is nonlinear with different stiffness upon pressure K_n, tension K_{nz} and shear K_S. It is used for modelling interface of surfaces that could be separated or connected with or without sliding relative to each other. For modeling of the shells we use the Bathe-Dvorkin 4-node shell element, which contains mixed strain interpolation for transversal shear, and enhancements for the membrane strains is introduced. The implicit stress integration procedure for von Mises model with mixed hardening assumption and the Ramber-Osgood relation (incompressible plastic flow), according to the governing parameter method (GPM), is used, [2], [5]. Implicit stress integration procedures for the von Mises plasticity is given in Table 1.

Table 1 Stress implicit integration for shell element

Known quantities ${}^{t}\bar{e}^{P}, {}^{t}\mathbf{e}^{P}, {}^{t}\boldsymbol{\alpha}, {}^{t+\Delta t}\mathbf{e}$, $\sigma_3 = 0$

Elastic strain, (trial solution for $\Delta\bar{e}^{P} = 0$): Trial deviatoric strains

$${}^{t+\Delta t}e_1'' = c_1\left({}^{t+\Delta t}e_1 - {}^{t}e_1^P\right) - c_2\left({}^{t+\Delta t}e_2 - {}^{t}e_2^P\right) \qquad c_1 = \frac{2-\nu}{3(1-\nu)} \qquad c_2 = \frac{1-2\nu}{3(1-\nu)}$$

$${}^{t+\Delta t}e_2'' = -c_2\left({}^{t+\Delta t}e_1 - {}^{t}e_1^P\right) + c_1\left({}^{t+\Delta t}e_2 - {}^{t}e_2^P\right)$$

$${}^{t+\Delta t}e_3'' = -\left({}^{t+\Delta t}e_1'' + {}^{t+\Delta t}e_2''\right) \qquad {}^{t+\Delta t}e_i'' = {}^{t+\Delta t}e_i - {}^{t}e_i^P \qquad i = 4,5,6$$

Elastic stress radius ${}^{t+\Delta t}\hat{\mathbf{S}}^{E} = 2G\,{}^{t+\Delta t}\mathbf{e}'' - {}^{t}\boldsymbol{\alpha}$ $\qquad G = E/2(1+\nu)$

Yield surface radius and yield stress

$${}^{t+\Delta t}\hat{\sigma}^{E} = \sqrt{\frac{3}{2}}\left\|{}^{t+\Delta t}\hat{\mathbf{S}}^{E}\right\| \qquad {}^{t}\hat{\sigma}_y = \sigma_{yv} + C_y\left(M\,{}^{t}\bar{e}^{P}\right)^n$$

Yield check ${}^{t+\Delta t}f_y^{E} = {}^{t+\Delta t}\hat{\sigma}^{E} - {}^{t}\hat{\sigma}_y \le 0$ - for $\Delta\lambda = 0$ and ${}^{t+\Delta t}\hat{\mathbf{S}} = {}^{t+\Delta t}\hat{\mathbf{S}}^{E}$ go to 2.

Plastic strain: Find the zero of the governing function ${}^{t+\Delta t}f_y(\Delta\bar{e}^{P}) = 0$

1. Trial solution for $\Delta\bar{e}^{P}$ from ${}^{t+\Delta t}f_y = 0$

Effective plastic strain and yield stress

$${}^{t+\Delta t}\bar{e}^{P} = {}^{t}\bar{e}^{P} + \Delta\bar{e}^{P} \qquad {}^{t+\Delta t}\hat{\sigma}_y = \sigma_{yv} + C_y\left(M\,{}^{t+\Delta t}\bar{e}^{P}\right)^n$$

Stress radius and effective stress radius

$$\Delta\lambda = \frac{3}{2}\frac{\Delta\bar{e}^{P}}{{}^{t+\Delta t}\hat{\sigma}_y} \qquad \hat{C} = \frac{2}{3}(1-M)nC_y\left({}^{t+\Delta t}\bar{e}^{P}\right)^{n-1}$$

$$p_1 = 1 + \left(2Gc_1 + \hat{C}\right)\Delta\lambda \qquad p_2 = 2Gc_2\Delta\lambda \qquad {}^{t+\Delta t}\hat{S}_3 = -\left({}^{t+\Delta t}\hat{S}_1 + {}^{t+\Delta t}\hat{S}_2\right)$$

$${}^{t+\Delta t}\hat{S}_1 = \frac{p_1\,{}^{t+\Delta t}\hat{S}_1^E + p_2\,{}^{t+\Delta t}\hat{S}_2^E}{p_1^2 - p_2^2} \qquad {}^{t+\Delta t}\hat{S}_2 = \frac{p_2\,{}^{t+\Delta t}\hat{S}_1^E + p_1\,{}^{t+\Delta t}\hat{S}_2^E}{p_1^2 - p_2^2}$$

$${}^{t+\Delta t}\hat{S}_i = \frac{{}^{t+\Delta t}\hat{S}_i^E}{1 + \left(2G + \hat{C}\right)\Delta\lambda} \qquad i = 4,5,6 \qquad {}^{t+\Delta t}\hat{\sigma} = \sqrt{\frac{3}{2}}\left\|{}^{t+\Delta t}\hat{\mathbf{S}}\right\|$$

Check for yielding $\left|{}^{t+\Delta t}f_y\right| = \left|{}^{t+\Delta t}\hat{\sigma} - {}^{t+\Delta t}\hat{\sigma}_y\right| > \mathbf{TOL}$ - go to1.

Plastic strain and back stress

$${}^{t+\Delta t}\mathbf{e}^{P} = {}^{t}\mathbf{e}^{P} + \Delta\lambda\,{}^{t+\Delta t}\hat{\mathbf{S}} \qquad {}^{t+\Delta t}\boldsymbol{\alpha} = {}^{t}\boldsymbol{\alpha} + \hat{C}\Delta\lambda\,{}^{t+\Delta t}\hat{\mathbf{S}}$$

2. Mean elastic strain and mean stress

$${}^{t+\Delta t}e_m^E = c_2\left({}^{t+\Delta t}e_1 - {}^{t+\Delta t}e_1^P + {}^{t+\Delta t}e_2 - {}^{t+\Delta t}e_2^P\right) \qquad {}^{t+\Delta t}\sigma_m = c_m\,{}^{t+\Delta t}\mathbf{e}_m^E \qquad c_m = \frac{E}{1-2\nu}$$

Deviatoric stress and total stress

$${}^{t+\Delta t}\mathbf{S} = {}^{t}\boldsymbol{\alpha} + \left(1 + \hat{C}\Delta\lambda\right){}^{t+\Delta t}\hat{\mathbf{S}} \qquad {}^{t+\Delta t}\boldsymbol{\sigma} = {}^{t+\Delta t}\mathbf{S} + {}^{t+\Delta t}\boldsymbol{\sigma}_m$$

3 EXAMPLES

We consider the geometrical and material nonlinear analysis of spot welded girder with length L=900mm. The three different models are considered: a) without gap element, b) with gap element and c) glued contact surfaces (separation prevented). The welded spots are modelled by 20 beam elements with circular cross-section of 5mm diameter; 7437 improved shell elements of thickness 0.8mm used for sheets modelling; 660 gap finite elements were used for modelling of interface of sheets without initial opening. The gap normal stiffness is $Kn=10^9$N/mm. The von-Mises elasto-plastic isotropic material model is used. The material data for Ramberg-Osgood yield curve are:

$E=2x10^5$ N/mm^2 $\nu=0.3$

$\sigma_y = \sigma_{yv} + C_y \bar{e}_p^n \qquad \sigma_{yv} = 200 N/mm^2 \qquad C_y = 2000 N/mm^2 \qquad n = 1$

The girder is subjected to the incremental (300 steps) prescribed displacement of 120mm at the middle of span.

The Force-Displacement diagram is shown on Fig. 1. A good compliance of experimental and numerical model using [1] is shown on Fig. 2. The model with gap elements is superior related to model without gap because penetration is prevented as shown on Fig. 3.

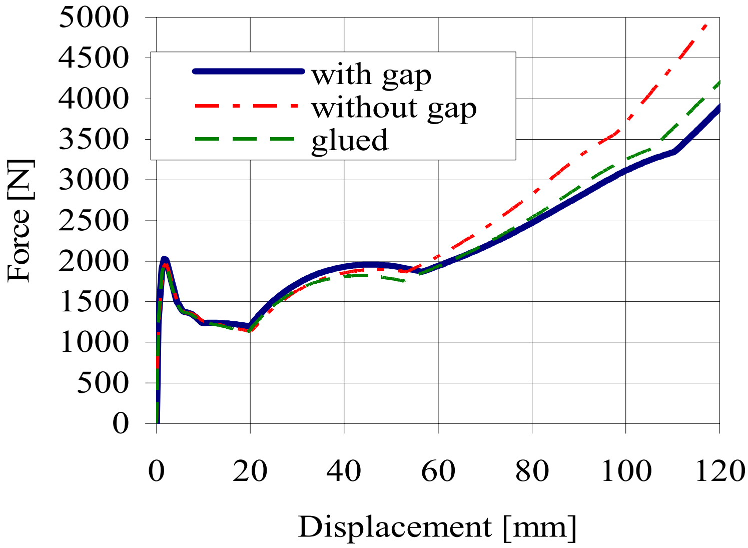

Figure 1. Force – Displacement relationship

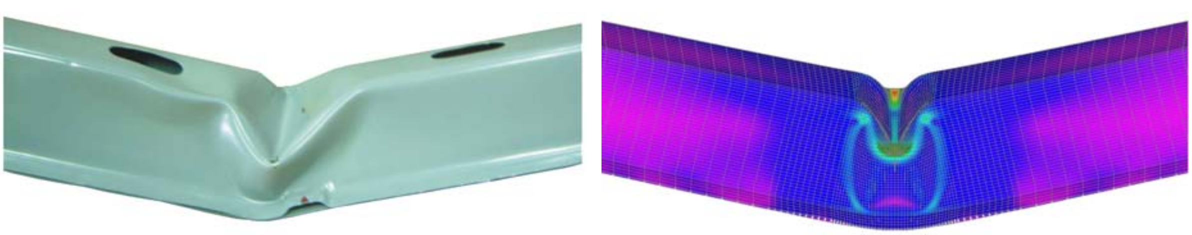

Figure 2. Experimental and numerical model

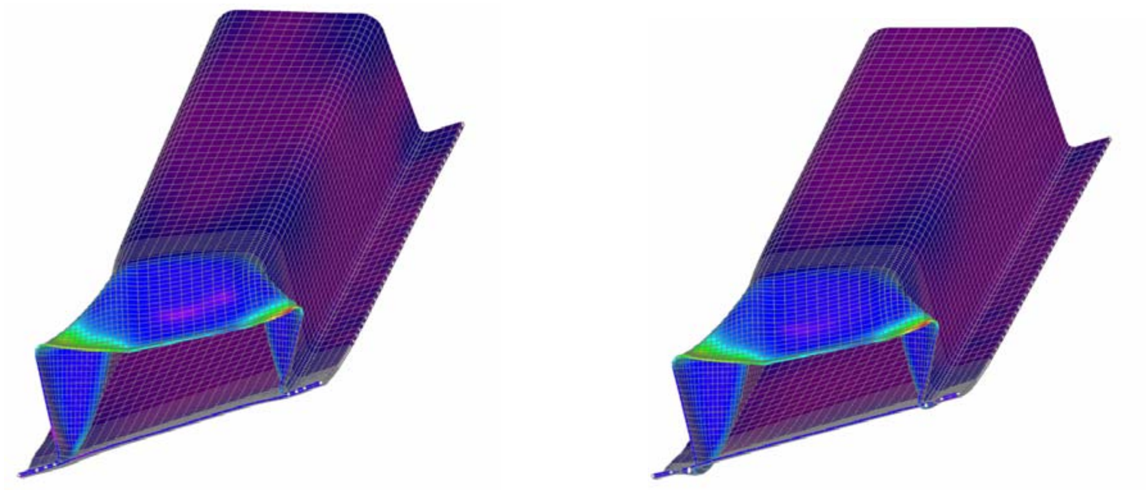

Figure 3. Deform configuration with and without gap

4 CONCLUSIONS

A nonlinear analysis was performed till the crash of structure, with gap and without it. The results of performed analyses show necessity of gap usage in modelling of interface of two spot welded sheets. The theoretical basis for elasto-plastic analysis for shell elements efficient in modeling of crash of real structures, Fig. 2. The proposed developed gap element showed an efficiency in analysis of real structure, i.e. thin walled structure of car body. The modelling of spot welded joint is an actual subject of research in nowadays, [4].

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