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IDENTIFICATION OF A COMBINED HARDENING AND APPLICATION TO A "ONE-STEP" INVERSE APPROACH

F. Sabourin*, P. Simon*

* Laboratoire de Mécanique des Contacts et des Solides (LaMCoS)
Institut National des Sciences Appliquées de Lyon
20 Avenue Einstein, 69621 Villeurbanne, France
e-mail: francis.sabourin@insa-lyon.fr, web page: http://lamcos.insa-lyon.fr

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1 INTRODUCTION

In a deep drawing process, the areas of the stamped part which have been bent and unbent have a great influence on the further shape after spring-back. Amongst other parameters, the cyclic plasticity properties of the material must be taken into account and, therefore, have to be identified. To impose a compression strain to a thin sheet metal, a simple way consists in imposing at each end of a metallic strip a cyclic rotation angle. For that, a bending-unbending apparatus has been developed and provides the corresponding moment-curvature diagrams.

In addition, three tensile tests carried out with specimens - respectively cut in the rolling direction, the transverse one and at 45° - account for the initial sheet metals anisotropy.

The tensile test in the rolling direction and the bending-unbending experiment results are then used to identify the chosen hardening model which is derived from *Chaboche-Ziegler*'s [1] formulation and called "combined" or "mixed" since it associates the properties of both an isotropic hardening (uniform expansion of the yield strength surface) and a non-linear kinematic hardening (translation of the center of this surface in stresses space).

Thanks to data coming from the tensile test, the bending-unbending process is numerically simulated as though the hardening were isotropic. Then, the difference between experimental and numerical moments give the initial values of the required parameters. These parameters allow to simulate the process again - but, this time, with the combined hardening - thus giving the "theoretical" moment-curvature results.

In order to improve the fitting of these theoretical results – by minimising their difference with the corresponding experimental values –, an optimisation procedure is finally used. To decrease the computation time, the plasticity incremental aspect is replaced by a secant modulus concept based on successive steps of radial loading.

This formulation can be implemented in the inverse approach proposed by *Batoz et al* [2] for sheet forming simulation thanks to an additional procedure. Firstly, it detects the elements located on a "curvature path", i.e. bent then unbent over die fillets, and the successive extreme curvatures. Then, starting with stress and strain states previously computed, this procedure imposes the extreme curvatures found before in two steps only while keeping an integrated form of the plasticity law commonly implemented in One Step software.

2 COMBINED HARDENING FORMULATION

2.1 Initial anisotropy

This kind of anisotropy due to rolling is taken into account with dimensionless material parameters f, g, h, n called *Hill'*s coefficients while r_0, r_{45} and r_{90} are plastic strain ratios; for a tensile specimen according to an angle θ from rolling direction, r_θ is the ratio of a plastic strain increment in the width direction (θ+π/2) to the corresponding plastic strain increment in the thickness direction ("z"). The particular values f = g = h = 1/2 and n = 3/2, lead to *Von Mises*' yield function for an isotropic hardening.

$$h=\frac{r_0}{1+r_0};\quad g=1\text{-}h;\quad f=h\frac{1}{r_{90}};\quad n=\frac{(r_0+r_{90})(2r_{45}+1)}{2r_{90}(1+r_0)};\quad [\mathbf{M}]=\begin{bmatrix} g+h & -h & \\ -h & f+h & \\ & & 2n \end{bmatrix} \tag{1}$$

Considering a plane stress assumption where "x" and "y", respectively, are "rolling" and "cross" directions, the equivalent stress is such as:

$$\sigma_{eq}^2=\langle\boldsymbol{\sigma}-\boldsymbol{\alpha}\rangle[\mathbf{M}]\{\boldsymbol{\sigma}-\boldsymbol{\alpha}\};\quad \langle\boldsymbol{\sigma}\rangle=\{\boldsymbol{\sigma}\}^T=\langle\sigma_{xx}\ \ \sigma_{yy}\ \ \sigma_{xy}\rangle;\quad \langle\boldsymbol{\alpha}\rangle=\{\boldsymbol{\alpha}\}^T=\langle\alpha_{xx}\ \ \alpha_{yy}\ \ \alpha_{xy}\rangle \tag{2}$$

Also called "back stresses", $\{\boldsymbol{\alpha}\}$ components specify the center location of the yield surface. Therefore, they remain equal to zero for an isotropic hardening.

2.2 Non-linear combined (or "mixed") hardening

The chosen formulation is the one due to *Chaboche* and *Ziegler* defined by five parameters (Q, b, C, γ, and σ_0) where σ_0 is the yield stress at zero plastic strain and $\bar{\sigma}$ might be called "comparison stress" since, if plastic flow occurs, $\bar{\sigma}=\sigma_{eq}$. With the equivalent plastic strain $\bar{\varepsilon}^p$, C and γ manage the evolution of the kinematic components of this model such that:

$$\{d\boldsymbol{\alpha}\}=C\frac{d\bar{\varepsilon}^p}{\bar{\sigma}}\{\boldsymbol{\sigma}-\boldsymbol{\alpha}\}-\gamma\{\boldsymbol{\alpha}\}d\bar{\varepsilon}^p;\qquad \bar{\sigma}=\sigma_0+Q(1-e^{-b\bar{\varepsilon}^p}) \tag{3}$$

Consistency and normality rule give the gradient vector $\{\mathbf{a}\}$ and the plastic flow increment:

$$\{d\boldsymbol{\varepsilon}^p\}=\{\mathbf{a}\}d\bar{\varepsilon}^p;\qquad \langle d\boldsymbol{\varepsilon}^p\rangle=\{d\boldsymbol{\varepsilon}^p\}^T=\langle d\varepsilon_{xx}^p\ \ d\varepsilon_{yy}^p\ \ 2d\varepsilon_{xy}^p\rangle;\qquad \{\mathbf{a}\}=\frac{1}{\bar{\sigma}}[\mathbf{M}]\{\boldsymbol{\sigma}-\boldsymbol{\alpha}\}; \tag{4}$$

If $[\mathbf{D}]$ is the (3x3) plane stress matrix of elastic constants, the incremental elastic-plastic strain- stress relation is:

$$\{d\boldsymbol{\sigma}\}=[\mathbf{D}]\{d\boldsymbol{\varepsilon}-d\boldsymbol{\varepsilon}^p\};\qquad d\bar{\varepsilon}^p=\frac{\langle\mathbf{a}\rangle[\mathbf{D}]\{d\boldsymbol{\varepsilon}\}}{H'+C\text{-}\gamma\langle\mathbf{a}\rangle\{\boldsymbol{\alpha}\}+\langle\mathbf{a}\rangle[\mathbf{D}]\{\mathbf{a}\}};\qquad H'=\frac{d\bar{\sigma}}{d\bar{\varepsilon}^p}=Qbe^{-b\bar{\varepsilon}^p} \tag{5}$$

3 INITIAL IDENTIFICATION

3.1 Theoretical basis

As already written in the introduction, the initial identification of the parameters C and γ is based on the difference ($\Delta\sigma$) during the unbending and opposite bending stages (see fig. 1, 2).

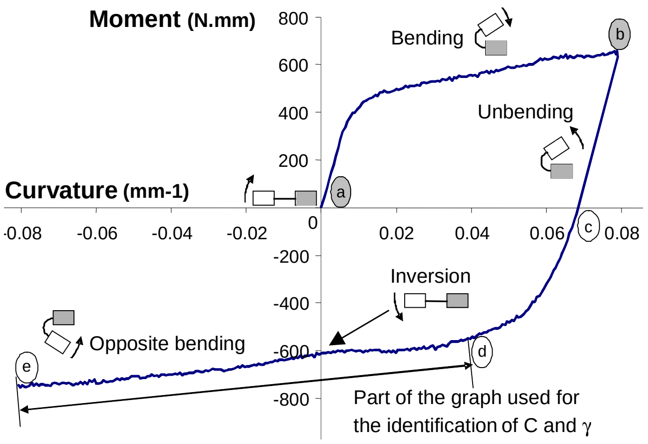

Figure 1: Bending-unbending diagram

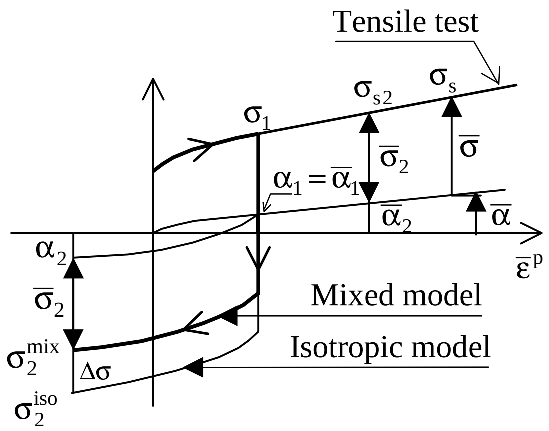

Figure 2: Tension-compression diagrams

Coming from the general previous equations, the useful 1D relations are:

$$\sigma_2^{mix} = \alpha_2 - \overline{\sigma}_2 ; \quad \overline{\sigma}_2 = \sigma_{s2} - \overline{\alpha}_2 ; \quad \sigma_2^{iso} = -\sigma_{s2} ; \quad \sigma_2^{mix} - \sigma_2^{iso} = \alpha_2 + \overline{\alpha}_2 \tag{6}$$

$$\alpha_1 = \overline{\alpha}_1 = \frac{C}{\gamma}(1 - e^{-\gamma\overline{\varepsilon}_1^p}) ; \quad \alpha_2 = -\frac{C}{\gamma} + (\alpha_1 + \frac{C}{\gamma})e^{-\gamma(\overline{\varepsilon}_2^p - \overline{\varepsilon}_1^p)} ; \quad \overline{\alpha}_2 = \frac{C}{\gamma}(1 - e^{-\gamma\overline{\varepsilon}_2^p}) \tag{7}$$

$$\text{Finally: } \sigma_2^{mix} - \sigma_2^{iso} = 2\alpha_1 e^{-\gamma(\overline{\varepsilon}_2^p - \overline{\varepsilon}_1^p)} \Rightarrow \ln(\sigma_2^{mix} - \sigma_2^{iso}) = -\gamma(\overline{\varepsilon}_2^p - \overline{\varepsilon}_1^p) + \ln(2\alpha_1) \tag{8}$$

3.2 Bending experiment

For a given curvature κ in the useful part of the bending test shown in fig. 1 – corresponding to a point "2" in fig. 2 –, and denoting by "w" and "t" the width and the thickness of the specimen, the necessary data for the identification are obtained as follow:

$$\sigma_2^{mix} = \frac{4M^{exp}(\kappa)}{wt^2} ; \quad \sigma_2^{iso} = \frac{4M^{iso}(\kappa)}{wt^2} ; \quad \overline{\varepsilon}_2^p \text{ such as } \sigma_s(\overline{\varepsilon}^p) = \sigma^{iso} ; \quad \overline{\varepsilon}_1^p \text{ known yet} \tag{9}$$

A linearization of equation (8) with several locations "2" – but the same location "1" at the end of the bending stage – gives γ and α_1 which, itself, gives the parameter C with eq. (7). Once the first set (C, γ) is known, other similar computations at each integration point through the thickness provide other values of C and γ and the ones giving the best 'precision' are kept.

4 IDENTIFICATION OPTIMISATION WITH A SECANT MODULUS CONCEPT

The results shown in figure 3 correspond to an incremental computation mainly based on equations 3, 4 and 5. For instance, to get the bending moment at the location "e" (see fig. 1), the computation starts at the location "a". To optimize the material parameters values, especially in the zone between "c" and "d" (fig. 1), it is more advisable to start from "b" and directly proceed up to a required curvature in the unbending part. The diagram 4 proves that a secant module concept makes it possible to achieve this goal.

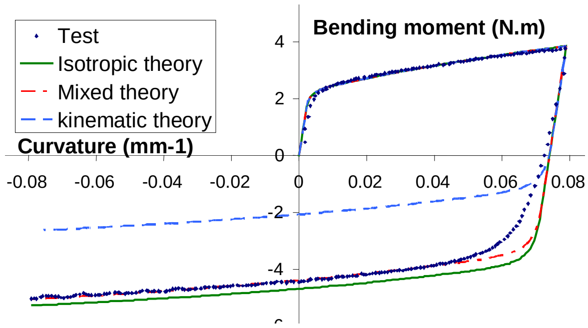

Figure 3: Incremental identification of an a aluminum

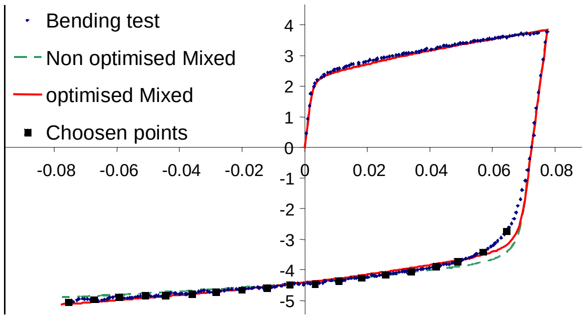

Figure 4: Further secant modulus calculations

The idea of successive secant moduli presented by *Sabourin et al* [3] for sheet forming One-step simulations has been used here in a more simple way since the strain and stress states are equal to zero at the beginning of the first bending stage.

Applied to the "U" deep drawing geometry of NumiSheet'93 benchmark, the pictures 5, 6 show the difference between two different hardening models and that an inverse approach, provided an additional procedure!, and an incremental computation give comparable results.

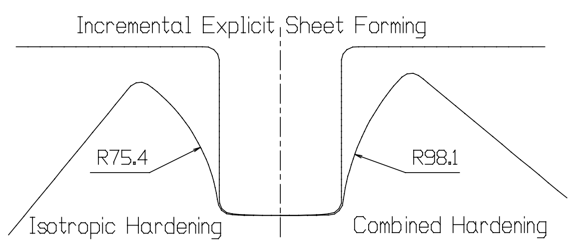

Figure 5: Incremental identification of an a aluminum

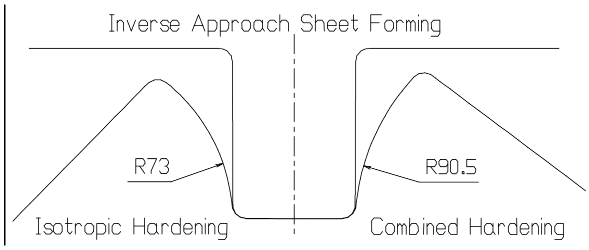

Figure 6: One-step simulations with secant moduli

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