

## Compositions and fuzzy compositions in decision-making models

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In decision-making models, the compositions (Aitchison, 1986) are employed in various forms. They can represent the normalized weights of criteria in multiple-criteria decision-making models, or the probabilities of states of the world in the models of decision making under risk. The normalized weights express the relative information about the importance of criteria, while the probabilities reflect the expected incidence of the states of the world. The compositions are also the evaluations of objects according to their contributions to a common goal.

This paper addresses the situation when the compositions are set expertly. This is the typical way of obtaining the normalized weights of criteria. Various methods of setting the weights were introduced in order to properly reflect the preferences of the decision maker. Direct methods where the decision maker sets the weights directly can be distinguished from indirect ones, e.g., the method of pair-wise comparison, or Saaty's AHP.

The weights of criteria represent the parameters to the models of multiple-criteria evaluation or decision making. Statistical methods are applied to these models to aggregate the information about criteria preferences within a group of decision makers. However, in such a case, the final weights are usually obtained by the standard arithmetic mean, i.e., the data obtained from decision makers are not treated as compositional data. On the other hand, the weights from the Saaty matrix of pair-wise comparison, where the  $i$ -th row expresses the importance of particular criteria with respect to the  $i$ -th criterion, are calculated as the geometric mean of rows, which corresponds well to the concept of expected value of compositional data.

In principle, there is no difference between compositional data obtained by measuring and compositional data that are set expertly (the measuring instrument here is a human brain). But the latter method is burdened with uncertainty. Therefore, the data values should be modeled by the tools of the fuzzy sets theory. The special structure of fuzzy numbers was proposed for modeling fuzzy compositions that represent the weights of criteria – the structure of normalized fuzzy weights (Pavlačka & Talašová, 2006). In the paper, we show how to compute the fuzzy weighted average of fuzzy numbers with normalized fuzzy weights (the fact that normalized fuzzy weights represent a fuzzy composition must be taken into consideration by the calculation). We also describe the special ways of setting the normalized fuzzy weights including those based on the verbal description of preferences among criteria.

For modeling fuzzy data defined on the simplex, a more general mathematical object was proposed – a fuzzy vector of normalized weights (Pavlačka & Talašová, 2010). It is capable of expressing the uncertain weights even under further interactions among the weights, besides the fact that they are defined on the simplex.

These methods can also be applied for describing subjective probabilities of states of the world in the models of decision-making under risk.

## References

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