

USING AN INTEGRATED SURFACE-SUBSURFACE MODEL TO SIMULATE RUNOFF FROM HETEROGENEOUS HILLSLOPES

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Key words: subsurface hydrology, surface water hydrology, coupled systems, hydrograph separation, surface-subsurface interactions

Summary: Traditionally, hydrologic models have treated surface and groundwater flow separately. Recently, integrated models of surface-subsurface processes are becoming more common. We use the fully integrated model Parflow to evaluate the role of heterogeneity on hillslope runoff production and baseflow. Simulations were generated with idealized, high-resolution hillslopes, which have constant slope and are with or without baseflow. Heterogeneous, correlated random fields were used to create spatial variability in hydraulic conductivity. Ensembles of multiple realizations were used to determine an average surface water outflow for a given hillslope configuration. An advantage to this technique is that we may explicitly interrogate individual realizations to perform accurate hydrograph separation between overland and subsurface flow. This technique allows determination of the contribution of variance from overland and baseflow in addition to average runoff behavior. Outflow from slopes without baseflow show typical hydrograph patterns which agree with previous studies of Hortonian runoff generation. However, baseflow cases understandably show a different surface runoff pattern from overland flow cases, with very little sensitivity to the variance of hydraulic conductivity. For these simulations, the contributions from overland runoff are shown to be a somewhat small portion of the hydrograph.

INTRODUCTION

Traditionally, hydrologic models have considered groundwater and surface water separately. Recently, models that fully-integrate these two systems have been developed. Improvements in solvers, numerical methods and computing power have all contributed to the ability of these models to consider both systems in an integrated manner. Previous approaches also considered simple homogenous or layered systems. Numerous studies have discussed the limitations to this assumption, concluding that the subsurface is heterogeneous in nature, from micro to macro scale.³ Quantifying the impacts of subsurface heterogeneity is essential to adequate hydrograph prediction for any hydrologic system. Heterogeneity can play a role in dispersion in the subsurface and impact residence times and travel paths of subsurface flows. Heterogeneity can also play a role in the magnitude and contributions to a hydrograph, either Dunne or Hortonian flow².

To model this coupled system, two main equations are solved: the kinematic wave equation and Richards' Equation. These two equations are integrated, to allow generation of runoff to occur at any point where the top boundary cell is saturated. These two equations are used

together in a coupled model called ParFlow⁵ (for complete details see Kollet and Maxwell, 2006)⁶.

This work builds upon a previous study of runoff generation from a heterogeneous hillslope under Hortonian conditions⁴. The impacts of heterogeneity on runoff generation were represented using a fully-coupled model and a geostatistical, stochastic approach for Hortonian, or purely overland, flow only. Cases were developed with varying rainfall-to-permeability ratios for multiple variances of the hydraulic conductivity field. They found that outflow was very sensitive to the ratio of the geometric mean of the hydraulic conductivity (K_g), the rainfall rate (Q_{rain}) and the variance of the hydraulic conductivity field. They also found that effective parameters were likely not to exist for lower values of K_g / Q_{rain} .

Using a similar approach, this current work builds on the study of Maxwell and Kollet (2008)⁴, to investigate the role of subsurface heterogeneity in baseflow and subsurface stormflow. A fully three-dimensional hillslope with random, Gaussian heterogeneity hydraulic conductivity fields, is used to simulate runoff under Hortonian (non baseflow) and Dunne (baseflow) type conditions. First, a comparison is made with Maxwell and Kollet (2008)⁷ for Hortonian cases(non-baseflow case), and then these simulations are expanded to cases including feedback from the water table (baseflow case).

The integrated surface-subsurface model, ParFlow, was used to conduct these simulations. This work seeks to answer the following research question(s):

- 1) What are the differences between a hydrograph with baseflow and non-baseflow in a heterogeneous hillslope?
- 2) What portion of the hydrograph is directly related to the subsurface flow in a watershed?
- 3) How does heterogeneity in the subsurface effect the runoff generation for baseflow?
- 4) What are the roles of heterogeneity on averaged baseflow and subsurface stormflow behavior?

Here, we use ensembles (many realizations of random fields) of stochastic, heterogeneous simulations of baseflow and overland flow to address the questions of heterogeneity impacts on outflow. The simulations will also determine differences in baseflow and non-baseflow cases and aid in interrogating outflow results for hydrograph separation.

For each case, simulations were conducted for 50 realizations of hydraulic conductivity. Each equally-likely realization of hydraulic conductivity contains a different spatial pattern of saturated hydraulic conductivity, but honors a specified mean, variance and spatial correlation structure. A uniform, unit rainfall was specified and the subsequent surface runoff patterns are analyzed. Outflow results were then averaged over the ensemble of realizations.

NON-DIMENSIONALIZATION OF VARIABLES

A series of non-dimensional variables will be used to better generalize the simulation results and to make it easier to compare to other studies and field observations. The variables used here are the same as those developed in Maxwell and Kollet (2008)⁴. The primary variable that is non-dimensionalized is qr' which is the rain rate (Q_{rain}) normalized by the average

permeability of the domain, K_g . This parameter is used to develop multiple different cases and as a basis for analysis. A non-dimensional time, t' , is also used, which is the total time normalized by the time of rain application.

$$qr' = \frac{Q_{rain}}{K_g} \quad (5)$$

$$t' = \frac{t}{t_a} \quad (6)$$

The model dimensions were also non-dimensionalized as well as manning's coefficient. These can be seen below.

$$x' = \frac{x}{K_g t_a}, y' = \frac{y}{K_g t_a}, z' = \frac{z}{K_g t_a} \quad (7)$$

$$n' = n \left(\frac{t_a}{(K_g t_a)^{\frac{2}{3}}} \right) \quad (8)$$

MODEL PARAMETERS

ParFlow, was used to create a simple small-scale domain. The model represents a hillslope domain of $x'=y'=3000$ and $z'=30$, which was the same domain outlined in Maxwell and Kollet (2008)⁴. A three-dimensional, wedge shaped domain was used to allow for the implementation of water table of specified depth. A finer x-y discretization was used to more accurately represent baseflow processes. The grid size was $n_x=n_y=60$ and $n_z=300$, for a total size of 1.08 million compute cells. The saturated hydraulic conductivity was populated using a correlated, random heterogeneous field technique, the Turning Bands Algorithm, which is described by Tompson et al. (1989)¹. Constant vanGenuchten parameters were used for all cases where $n=2.0$ and $\alpha=6.0$. Slopes were set to $S_x = -0.005$ and $S_y=0.0$ with a constant manning coefficient of $n'=2.32 \times 10^{-6}$. The specified slopes allow for overland flow in the x direction only and for interrogation of water table rise in the x direction only. Shown below are the hillslope setup and one representation of the specified hillslope heterogeneity.

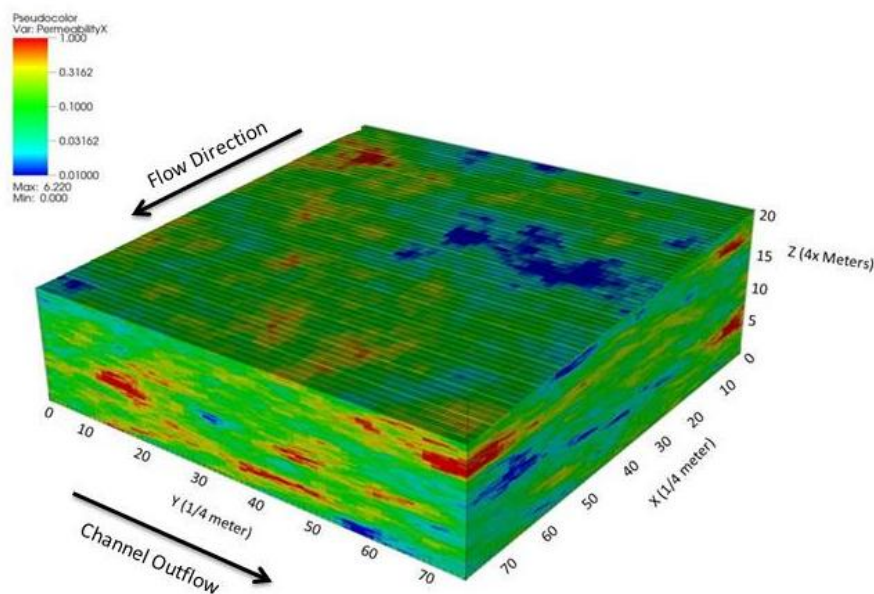


Figure 1 - Schematic of modeled hillslope. Heterogeneity is shown, where red is high permeability and blue is low permeability.

Rain was applied over the entire domain for a $\Delta t'=1$ and a recession period of $\Delta t'=1$ for all cases, with an additional recession period of $\Delta t'=35$ for baseflow cases. For the non-baseflow cases, the water table is initialized at the base of the domain to allow for only Hortonian runoff flow. However, for baseflow cases the water table was initialized at the slope base. This water depth was chosen to see quick responses in water table rise and the effects of Dunne flow combining with Hortonian flow.

For each case, 50 equally-likely realizations of hydraulic conductivity were simulated and a full transient flow simulation was conducted. Simulation results, such as the outflow hydrograph, were then averaged over all 50 realizations. This allows for the determination of an average outflow for the specified permeability and slope. The initial case that was modeled was for qr' equal to 1. Using this ratio, two different variances were used $\sigma^2=1.0$ and 0.25. The next case was with a qr' value of 0.5 and the same two variances. For the baseflow cases, two separate cases were run with a qr' ratio of 1, but with different permeability geometric mean, each of these was simulated 50 times. Overall, four different cases with 50 realizations each were simulated for both baseflow and non-baseflow cases for a total of 10 ensembles consisting of 500 total transient simulations.

RESULTS

Non-Baseflow Cases

The initial simulations were designed to only generate Hortonian overland flow. With the water table initialized far below the ground surface there was not sufficient time or duration of rainfall application to create baseflow. These simulation results were compared to previous studies completed by Maxwell and Kollet (2008)⁴, specifically the case with a variance of 1.0. The studies simulated here yielded the same results as those of Maxwell and Kollet (2008)⁴. Additional cases were simulated with a variance of 0.25. Figure 2a and 2b compares these cases with $qr'=1.0$ for the two variances. These figures show that the larger variance results in higher averaged outflow compared to the lower variance case. This difference is primarily due to differences in the lognormal distribution between variances. The smaller variance of 0.5 produces a narrower range of subsurface hydraulic conductivity values, resulting in more rain infiltrating and less runoff and outflow. Further simulation results show that when the rainfall rate is reduced to $qr'=0.5$, surface runoff decreases for both variances as seen in Figure 2b. Less surface runoff was generated because the lower rainfall rate results in fewer cells that have hydraulic conductivity values low-enough to impede infiltration, creating less ponding and generating lower runoff. Thus the lower qr' value results in more infiltration. We also see in Figures 2a and 2b that the larger variance produces more outflow, while the smaller variance produces less.

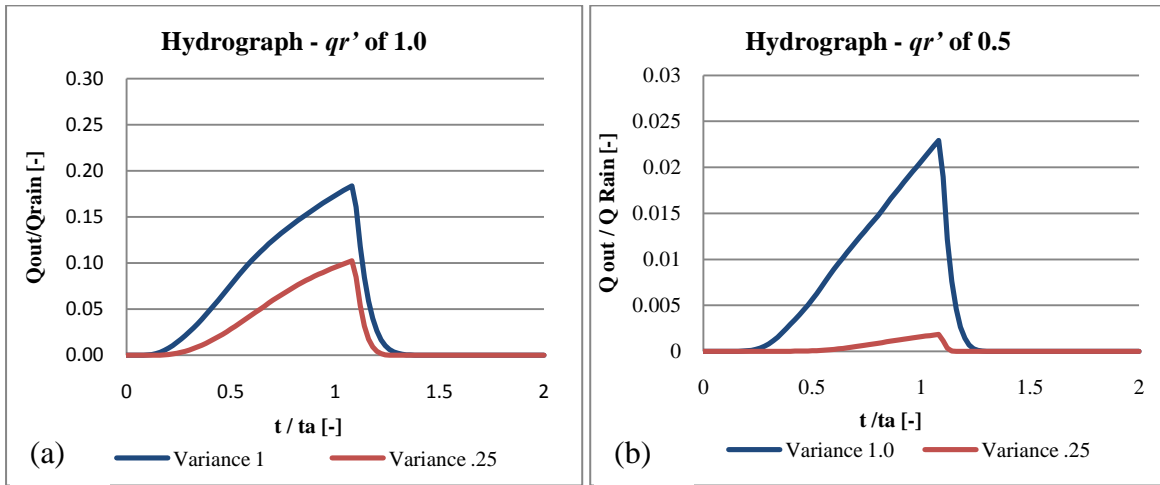


Figure 2a and 2b - Plot of outflow as a function of time comparing different variances of the hydraulic conductivity for two qr' ratios for the overland flow cases.

Baseflow Cases

Cases were also simulated with the same parameter values as the previous cases, but with a different water table initialization to create baseflow. In these cases the water table was set to be at a level equal to the slope outlet to allow interactions with the land surface. Figures 3a and 3b show results of these baseflow simulation cases for two qr' ratios, each with two different variances. This figure shows that the recession limb of the outflow hydrograph is sustained for many hours after a rainstorm, because of seepage out of the subsurface. A baseflow case with qr' ratio of 1.0 is shown in Figure 3a and a case with a qr' ratio of 0.5 is shown in Figure 3b. These cases are dramatically different than cases without a water table (Figures 2a and 2b). This difference results from the water table rise and interaction with the surface, generating Dunne outflow. The end result is a dynamic system that shows a combination of Dunne flow and Hortonian runoff, and longer attenuation of surface runoff.

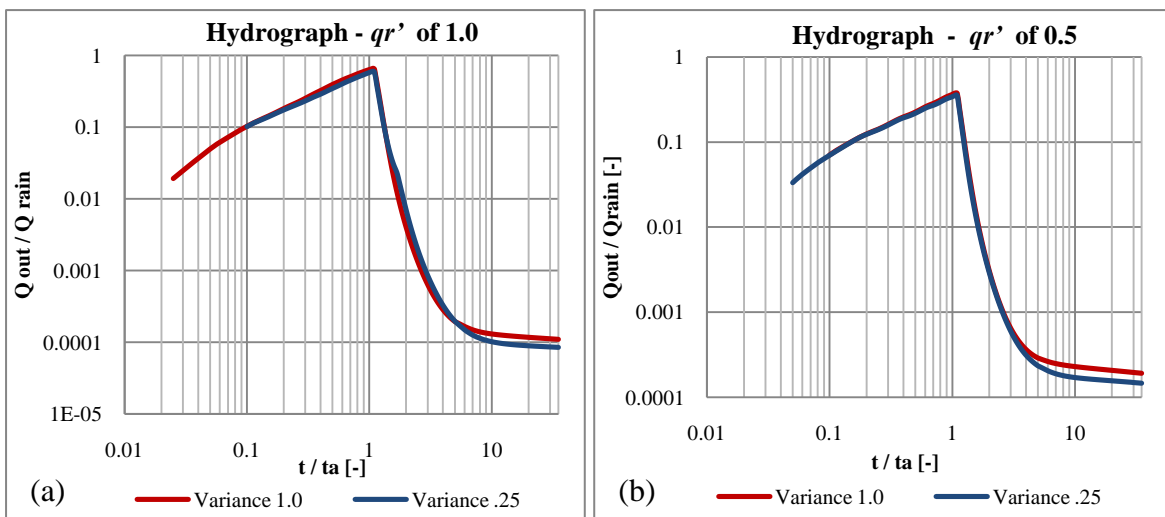


Figure 3a and 3b - Plot of outflow as a function of time comparing different variances of the hydraulic conductivity for two qr' ratios for the baseflow cases. Note the log scale in these plots.

Figures 3a and 3b also show very little hydrograph dependence on the variance of hydraulic conductivity. This is an intriguing result, indicating that the overall average hydraulic

conductivity controls hillslope runoff for the baseflow cases, not the small-scale variations. To further explore this concept, additional cases were simulated that held qr' at a constant ratio of 1.0 but varied the value used for the geometric mean of the saturated hydraulic conductivity. The results of this simulation are shown in Figure 4. This figure shows differences in runoff as a function of the averaged permeability and indicates that the effective permeability, not the variance, control the shape of the hydrograph. This is contrary to previous studies where the variance in heterogeneity affected the total outflow.

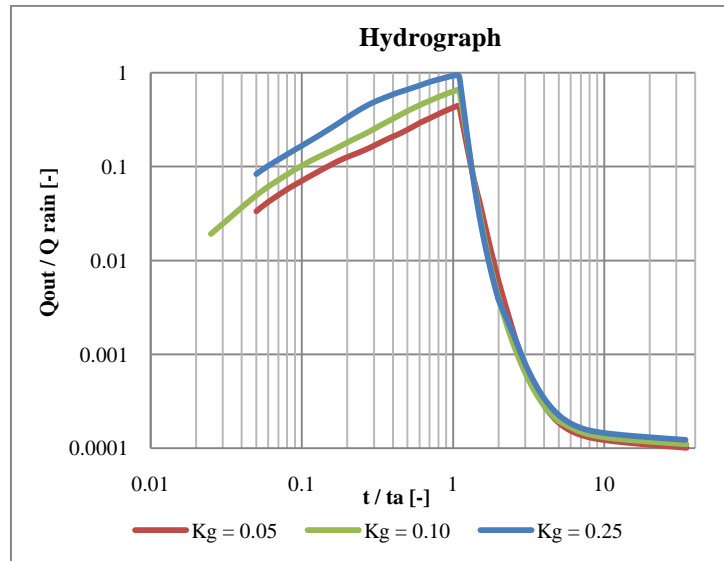


Figure 4 – Plot of outflow as a function of time for the baseflow cases comparing different geometric means of the hydraulic conductivity. Note the log scale in these plots.

HYDROGRAPH SEPARATION

To explore differences in the contribution of Dunne and Hortonian processes to the total outflow, an analytical hydrograph separation method was developed. An algorithm was used to determine water table rise and intersection with the ground surface (Figure 5). In this approach, a fully saturated column of cells was assumed to be Dunne flow while an undersaturated column of cells was assumed to be Hortonian flow. Each realization of each of the baseflow cases was analyzed using this approach to determine differences in Dunne and Hortonian contributions to total outflow. The baseflow cases of a qr' ratio of 1.0 are shown in Figures 6a and 6b. Cases with a qr' ratio of 0.5 are shown in Figures 7a and 7b.

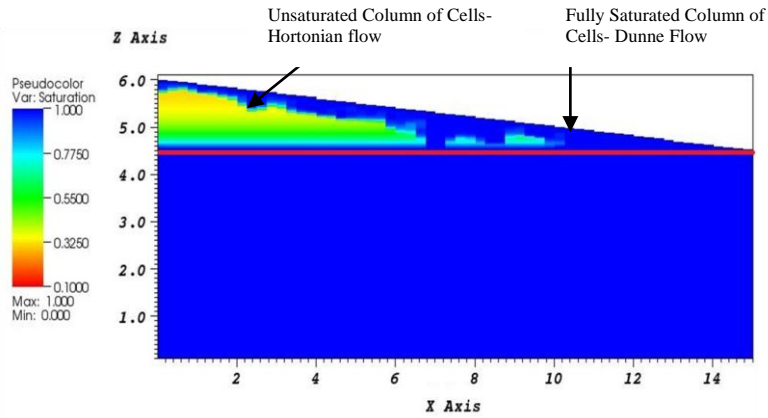


Figure 5 - Plot of saturation which demonstrates columns of unsaturated cells and fully saturated cells which are used in a hydrograph separation algorithm. Note the red line as the original water table.

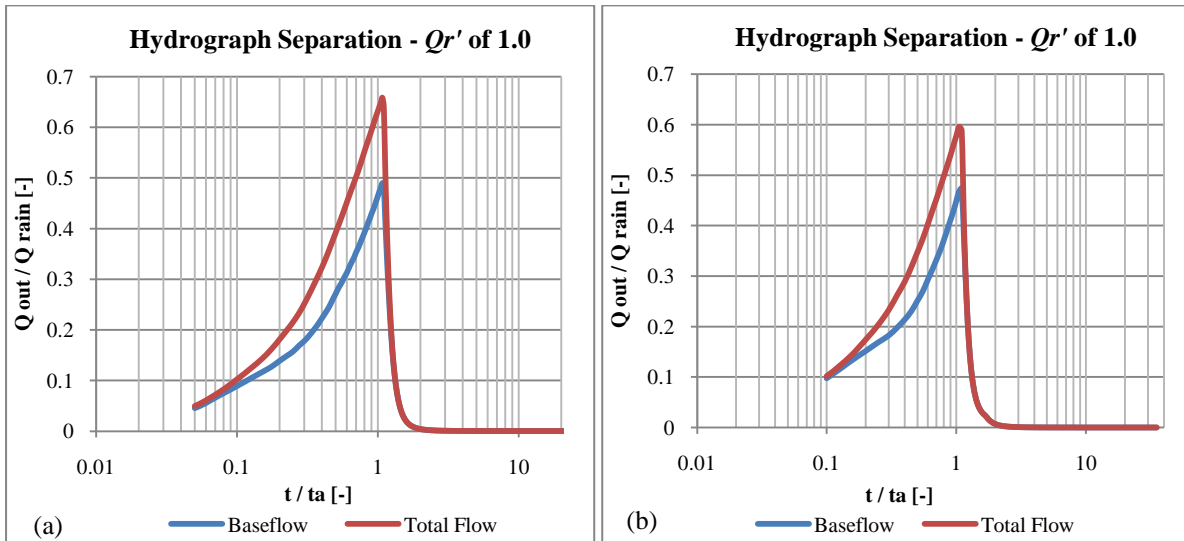


Figure 6a and 6b – Plot of outflow as a function of time for total flow (red) and overland flow only (blue) for $qr'=1.0$. Panel (a) is for a variance of 1.0 and (b) is for a variance of 0.25. Note the semi-log scale in this plot.

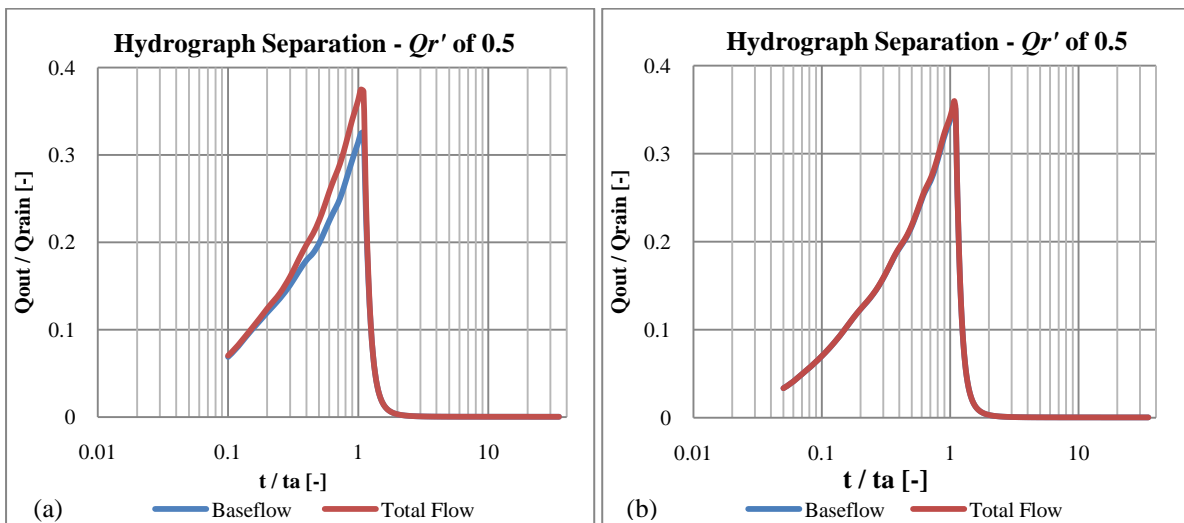


Figure 7a and 7b - Plot of outflow as a function of time for total flow (red) and overland flow only (blue) for $qr'=0.5$. Panel (a) is for a variance of 1.0 and (b) is for a variance of 0.25. Note the semi-log scale in this plot.

These figures show that flow is predominantly subsurface dominated. We see overland flow contributes to approximately 30% of the peak flow for $qr'=1.0$. Figure 7 indicates that surface flow contributes very little to total outflow for the qr' ratio of 0.5. The fact that subsurface flow dominates outflow in these systems contributes to the averaged behavior seen in Figure 3 and the differences seen for different Kg values in Figure 4.

CONCLUSIONS

This study used an integrated hydrologic model, ParFlow, with heterogeneous, random fields to represent hydraulic conductivity. Cases were simulated to study the differences in outflow for solely Hortonian (overland) flow and a mixture of Hortonian and Dunne flow. This work reached the following conclusions:

1. The hydrographs of baseflow cases (Figure 2a and Figure 2b) were found to be different from those of non-baseflow cases (Figure 3a and Figure 3b). In non-baseflow cases (Hortonian) runoff is controlled by the degree of heterogeneity in the subsurface whereas baseflow cases show a dependence only on the geometric mean of permeability. This study shows that for these specific baseflow cases changes in the strength of heterogeneity do not appear to affect total runoff which is contrary to previous studies that did not include baseflow.
2. In watersheds and hillslopes where runoff is primarily base flow; this work indicates that an average or effective, hydraulic conductivity may be derived for the subsurface properties (Figure 4) that will capture the average hydrograph.
3. The hydrograph separation presented here, indicates that for large variances in hydraulic conductivity, there is a greater amount of overland flow. Also that when there is a lower rainfall to Kg or qr' ratio, infiltration and Dunne flow increase. (Figure 6-Figure 7)

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