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A 2D FLOOD INUNDATION MODEL BASED ON AN IMPLICIT PARALLELIZABLE SCHEME

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Summary. The European Directive 2007/60, introducing important modifications on flood risk evaluation and management, calls for extensive application in Europe of inundation models. Responding to the need of finding accurate but fast approaches, the present work describes a new implicit 2D parabolic flood inundation approach, based on an integrated finite difference scheme. The resulting non-linear system of equations is linearised using the Linear Theory approach to improve convergence, while the resulting linear system is then solved by means of the Jacobi iterative approach, which can be massively parallelized. The resulting model was tested on a number of numerical cases, offering a good compromise between computational speed and accurate reproduction of the flood event.

1 INTRODUCTION

The Flood Directive 2007/60 of European Union[(1)] introduces important modifications on flood risk evaluation and management. The new directive prescribes the definition of flood hazard maps and flood risk maps, along with the development of flood risk management plans. According to the directive, "assessments, maps and plans should be based on appropriate 'best practice' and 'best available technologies' not entailing excessive costs in the field of flood risk management". Therefore, to comply the directive, methods for modeling flood inundation should be reliable and capable of generating the required hydraulic information in an appropriate level of detail, but also practicable in terms of computational time and costs as well as input data requirement, since application over large inundation areas will be inevitably needed. In order to meet all these requirements, the use of 2D hydraulic models which describe flow processes using various degrees of approximation, either in flow equations or in computation scheme, seems recommendable. The reduced complexity allows these models to be faster and less demanding in terms of computational burden and data requirement with respect to more complex models, moreover different works showed that the use of simplified models does not necessarily implies a loss of accuracy and reliability in results. Consequently, the present work describes a flood inundation model which should offer a good compromise between computational speed and accurate reproduction of the flood event.

2 THE PROPOSED MODEL

2.1 The equations and the computational scheme

To date, several computational schemes for 2D hydraulic modeling have been proposed in literature. Flood inundation models are generally based on the finite differences, finite elements or finite volumes methods. More recently the Runge-Kutta Discontinuous Galerkin method has been successfully applied to 2D/3D problems in the case of hypercritical, trans-critical and convection dominated diffusive problems (4). In the present paper an hydraulic model based on the Integrated Finite Difference approach, similar to the finite volume method, is proposed for representing diffusion dominated phenomena such as flood plain inundation events.

The model herein proposed is based on the shallow water equations written in diffusive form. In the literature, the ability of diffusive approximations to correctly simulate flooding phenomena is still under discussion; however, to date models based on diffusive equations have been tested successfully against measurements from actual flood events, and they often performed as well as models based on complete equations (2) (3). As such, it is hypothesized that uncertainties over the data set (especially topography and roughness), dominate and thus influence model results to a greater extent than errors and approximations due to simplified mathematical description (3) (5). Lastly, models based on diffusive equations provided good results when tested against both analytical and results from physical or alternative numerical solutions (5) (6) (7).

Dropping the convective and the local acceleration terms from the original Saint Venant equations, the following equations can be derived for a generic cell i connected with n_i adjacent cells, after integration in space of the point mass balance equations over the entire cell domain:

$$\begin{cases} \dfrac{\partial H}{\partial x} = -\dfrac{n_k^2 \left|Q_k\right| Q_k}{B_k^2 y^{10/3}} \\ \dfrac{\partial V_i}{\partial t} = \displaystyle\sum_{k=1}^{n_i} Q_k + q_i \end{cases} \tag{1}$$

In Eq. (1), Q is the discharge passing through the contact face k between cells i and j; $\partial H / \partial x$ is the water surface slope between the cell centres, B and n are the surface width and the Manning roughness coefficient of the contact face k; $H = y + z$ is the water elevation in the cell centre, which is defined as the sum of the terrain elevation z plus the water stage y; V is the volume stored in the cell and q is the discharge entering or leaving the cell from outside the study area (assumed positive when flowing inwards).

The system of partial differential equations of Eq. (1) can then be integrated using the integrated finite difference (IFD) scheme. The study area is schematized through polygonal elements (non necessarily regular), the cells, connected along the contact faces. The head losses among the cell centres are estimated by integrating in space the momentum equations, on the assumption of a linear variation of the water stage. Thus, the system of Eq. (1) may be written as:

$$\begin{cases} \dfrac{3}{7}\dfrac{n_{ij}^2\left|Q_{ij,t}\right|Q_{ij,t}}{B_{ij}^2}\dfrac{\left(y_{i,t}^{\;-7/3}-y_{j,t}^{\;-7/3}\right)}{\left(y_{j,t}-y_{i,t}\right)}-\dfrac{H_{i,t}-H_{j,t}}{\Delta X_{ij}}=0 \\ \Omega_i\dfrac{y_{i,t}-y_{i,t-\Delta t}}{\Delta t}-\theta\left(\sum_{k=1}^{ni}Q_{ij,t}+q_{i,t}\right)-\left(1-\theta\right)\left(\sum_{k=1}^{ni}Q_{ij,t-\Delta t}+q_{i,t-\Delta t}\right)=0 \end{cases} \tag{2}$$

where Ω is the cell surface area and θ is the time discretisation coefficient.
Such schematization allows to solve the inundation model in analogy with a pipe network, where the centres of the cells correspond to the junction nodes and the flow through the connecting contact faces corresponds to the flow in the pipes. Accordingly the equations may be written using a matrix formulation, which was introduced to derive the Global Gradient Algorithm (GGA) [(8)], which has become the de facto standard in water distribution network steady state analysis and was recently modified to account for unsteady flow [(9)]: Eq. (2) can be re-written in matrix form as:

$$\begin{bmatrix} A_{11} & \vdots & A_{12} \\ \cdots & \cdots & \cdots \\ A_{21} & \vdots & A_{22} \end{bmatrix}\begin{bmatrix} Q \\ \cdots \\ H \end{bmatrix}=\begin{bmatrix} -A_{10}H_0 \\ \cdots \\ -q^* \end{bmatrix} \tag{3}$$

The elements in system (3) are herein described:

$Q_t=\left\lfloor Q_{1,t},Q_{2,t},\cdots,Q_{np,t}\right\rfloor$ is the discharges vector [1, n_p],

$H_t=\left\lfloor H_{1,t},H_{2,t},\cdots,H_{np,t}\right\rfloor$ is the vector of unknown water stages [1, n_n],

$H_t=\left\lfloor H_{nn+1,t},H_{nn+2,t},\cdots,H_{ntot,t}\right\rfloor$ is the vector of known water stages [1, n_{tot}],

$q_t*=\left\lfloor q*_{1,t},q*_{2,t},\cdots,q*_{np,t}\right\rfloor$ is the vector of known terms [1, n_n],

where (n_p) is the number of links, (n_n) is the number of nodes with unknown water stage, (n_p) is the total number of nodes, (n_{tot} –n_n) is the number of nodes with unknown water stage.

A_{11} is a diagonal matrix with dimension [n_p, n_p] where the elements are defined as:

$$A_{11}\left(k,k\right)=\frac{3}{7}\frac{n_{ij}^2\left|Q_{ij,t}\right|}{B_{ij}^2}\frac{\left(y_{i,t}^{\;-7/3}-y_{j,t}^{\;-7/3}\right)}{\left(y_{j,t}-y_{i,t}\right)} \tag{4}$$

A_{22} is a diagonal matrix with dimension [n_n, n_n] where the elements are defined as:

$$A_{22}\left(i,i\right)=-\frac{\Omega_i}{\theta\Delta t} \tag{5}$$

The generic element of the vector $q*_t$ is defined as:

$$q_t*\left(i\right)=q_{i,t}+\frac{\Omega_i}{\theta\Delta t}H_{i,t-\Delta t}+\frac{\left(1-\theta\right)}{\theta}\left(\sum_{k=1}^{nn}Q_{ij,t-\Delta t}+q_{i,t-\Delta t}\right) \tag{6}$$

The topology of the network (nodes and links) is described by matrix $\overline{A}_{12}$, defined as:

$$\overline{A}_{12}(i,k) = \begin{cases} -1 \text{ if flow in the link k leaves node i} \\ \;\;0 \text{ if link k doesn't include node i} \\ +1 \text{ if flow in the link k enters node i} \end{cases} \tag{7}$$

In order to guarantee the existence of a unique solution, the water elevation must be known in at least one cell. This implies the partition of $\overline{A}_{12}$ in two sub-matrices:

$$\overline{A}_{12} = \left[A_{12} \vdots A_{10} \right] \tag{8}$$

where $A_{12} = A_{21}^{T}$ describes the links between the unknown water head nodes, while $A_{10} = A_{01}^{T}$ describes the links with the known head nodes.

The system of Eq. (3) is linearised by using the Linear Theory (LT) approach [(10)]. In preliminary tests, the authors found that, for the accuracy requirements in typical flood inundation events (around 10^{-3} m),the LT approach requires more or less the same number of iterations of the Newton Raphson approach, with the advantage of a resulting symmetrical system matrix to be handled. In order to apply LT to the solution of the non linear system, the following recursive scheme is used:

$$\begin{cases} H^{\tau+1} = \left[A_{21} \left(A_{11}^{\tau} \right)^{-1} A_{12} - A_{22} \right]^{-1} \left[q^* - A_{21} \left(A_{11}^{\tau} \right)^{-1} A_{10} H_0 \right] \\ Q^{\tau} = \dfrac{1}{2} \left[Q^{\tau} - \left(A_{11}^{\tau} \right)^{-1} \left(A_{12} H^{\tau+1} - A_{10} H_0 \right) \right] \end{cases} \tag{9}$$

As can be noticed from Eqs. (9), the iterative solution requires the solution of a system of linear equations, which matrix $\left[A_{21} \left(A_{11}^{\tau} \right)^{-1} A_{12} - A_{22} \right]$ is a sparse symmetrical matrix with dominant diagonal. These are conditions that allow for the use of the recursive Jacobi approach, instead of more direct frontal Gauss-Seidel or matrix factorization approaches.

2.3 Advantages of the proposed approach

Hydraulic models integrated in time using implicit schemes are not only more accurate, but also more stable than the explicit ones, thus allowing for longer simulation time steps. However, implicit schemes need a considerably greater computational burden, and this may prevent their use in large areas, for which simplified explicit and/or locally integrated schemes are currently used. The proposed solution scheme may compensate for this drawback in terms of computational time.

The system of Eqs. (9) is sparse and symmetric therefore, available techniques for sparse matrices can be effectively employed. Unfortunately, these techniques, which are extremely efficient by accounting for sparsity and symmetry, cannot be efficiently parallelized, thus losing the advantages provided by the most recent parallel computing. Fortunately, the main diagonal of the system matrix is strictly dominant and the Jacobi approach, which is highly parallelizable, can be effectively implemented and used.

3 NUMERICAL CASES

After some preliminary tests, the IFD-GGA model has been applied to a number a numerical cases. Two of these cases are herein described. The aim is to evaluate the model performances in terms of accuracy and computational time, and to test the convergence of the solution method in various flow conditions.

It must be noted that the evaluation of reduced complexity models in numerical cases has been rarely applied in literature(6); indeed, the majority of research works describe model applications in real world cases, but very often this does not permit to carry out an in-depth analysis of applied models. For example, in a real flood event the calibration procedure plays a major role and may compensate for model limitations if the quantity and quality of available data are limited. On the contrary, numerical cases are based on simplified and well controlled conditions, reducing the number of variables to be considered and the uncertainty on their influence on model performance.

3.1 Case 1: channel

The first presented case deals with 1D flow over a regular channel with a mild slope. The geometry, resumed in Table 1, has been chosen to test the ability of IFD-GGA model to simulate a diffusive wave, since significant lamination effect is expected.

Bed slope	10^{-4}	Section shape	rectangular
Channel width	250 m	Manning Roughness	0.05 $m^{-1/3}s$
Channel length	50 km		

Table 1 : Geometry of case 1

The hydrograph starts from an initial condition of uniform flow along all the channel, with a discharge of 10 m^3s^{-1}; at *t*=0, the discharge is linearly increased from upstream, going from 10 to 100m^3s^{-1} in 5 hours; the discharge values remains constant for the next 30 hours and then decrease linearly in 5 hours from 100 to 10 m^3s^{-1}. Please note that the geometry and the flow conditions were chosen in order to have a very large channel, so that the form of momentum equation used in IFD-GGA model (eq. 1) may be considered a good approximation. The downstream boundary condition is the uniform flow. The produced result were compared with the well known model HEC-RAS(11).

Three computation grids have been considered: one with 400x1 cells of 125x250m size, a second with 200x1 cells of 250x250m and another with 100x1 cells of 500x250m. This means that each grid maintains a channel width of 250m, while the longitudinal grid resolution varies. For each grid, simulations with different time steps have been performed (see Table 2).

The first relevant observed result is that the accuracy of solution does not vary significantly until a certain time step value is reached; beyond this value the solution provided by the model shows oscillations and is no longer comparable with HEC-RAS solution, although the solution scheme still converges. Also, spatial resolution influences the maximum value of employable time step but this does not affect the accuracy of solution (see Table 2).

Please note that the short time steps are compensated by the Jacobi solution method, which is very fast; to give a reference value, the simulation with the 250x250m grid and a time step of 5s over a period of 72 hours took approximately 30s on a computer with a 1.86 Ghz processor and 2Gb RAM.

Figure1 shows the results in terms of flow profile for the 250x250m grid and a time step of 5s, compared with HEC-RAS results. The profiles computed by the two models are very regular and almost coincident, both in general shape and position of wave front at the different time intervals. The wave simulated by the IFD-GGA model is slightly faster, however the RMSE against HEC-RAS results is always small (the maximum value, computed for time step t = 21h30m, is around 2 cm). The downstream discharges are also very similar. Most important, the IFD-GGA model reproduces with a very good accuracy the wave attenuation, even if the computation scheme is approximated with respect of the complete scheme based on full De Saint Venant Equations used by HEC-RAS

$\Delta X - \Delta t$	1s	2s	5s	10s	15s	30s	60s
125m	Y	Y	N				
250m	Y	Y	Y	Y	Y	N	
500m	Y	Y	Y	Y	Y	Y	Y

Table 2 : results for case 1. “N” indicates that the convergence of solution scheme is not achieved, while “Y” indicates that the simulation was successfully completed.

3.2 Case 2: horizontal plane

The wave routing on a flat slope is a case in which hydraulic models may be more subjected to instability, particularly when flow velocity and water surface slope are also reduced. It is worth noting that, according to recent works [12], in such conditions explicit diffusive models show significant stability problems, especially in presence of high resolution grids (cell size below 10m) and deep water stages. Therefore their major advantage, that is, a very reduced computational time, may be compensated by implicit models if larger time steps may be used. In order to test the proposed IFD- GGA model in these conditions, a series of simulations on an horizontal plane were performed. The computation grid consists of 20x20 cells with a 10x10m size; the simulation starts from an initial condition of null water stage, without incoming discharge; at t=0, a discharge varying linearly from 0 to 50m^3s^{-1} in 1 hour enters from one corner; then the discharge value remains constant for the next 2 hours and decrease linearly in 1 hour from 50 to 0 m^3s^{-1}. The total simulation time is 5 hours. The area is drained by a weir located in the opposite corner with respect to incoming discharge. As for case 1, the model results depend on time resolution if the time step is larger than a certain value (1s in this case), however with such values a limited number of iterations for each time step is needed (around 4-5 iterations for the non-linear scheme and 2-3 for Jacobi method), therefore computational time is still acceptable (≈4min on a computer with a 1.86 Ghz processor and 2Gb RAM). The graphics in Figure 2 show two computed water stages with a time step of 1s.

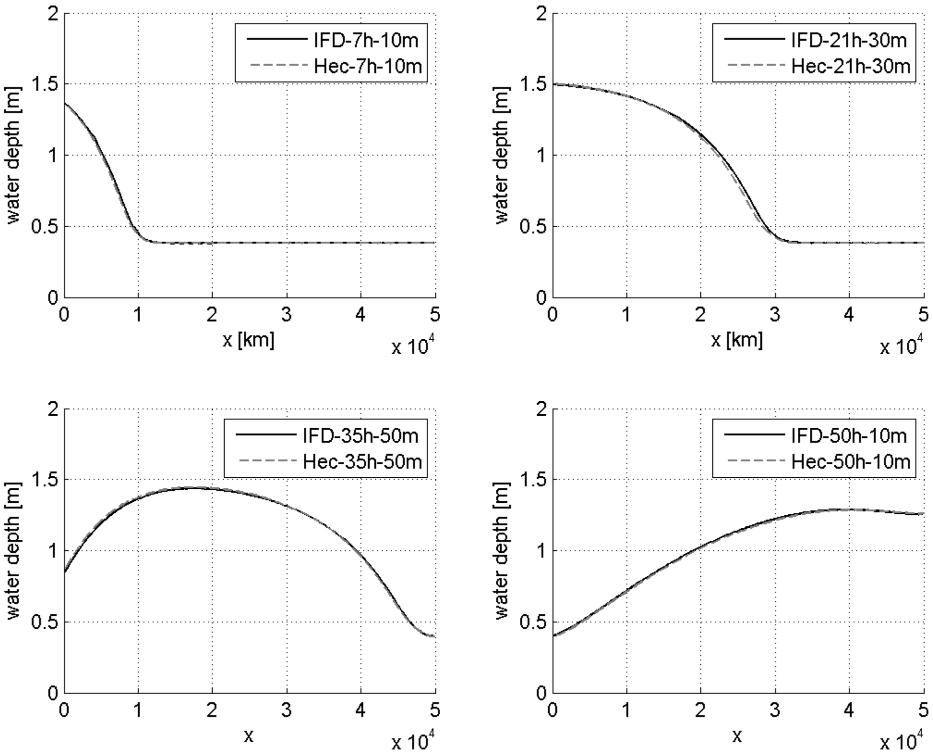

Figure 1. Case 1: comparison between flow profiles computed by IFD-GGA and HEC-RAS models.

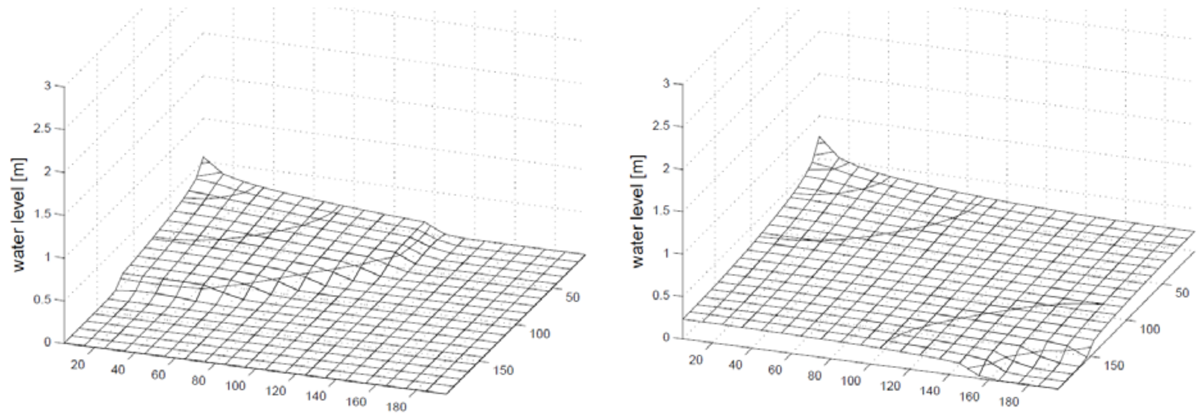

Figure 2. Case 2: water stages computed by IFD-GGA model after 10 min. (left) and 30 min. (right) from simulation start. Discharge enters from the upper left corner, while the outlet is located in the lower right corner.

4 CONCLUSIONS

The present work describes an implicit 2D parabolic flood inundation approach, based on an integrated finite difference scheme. The resulting non-linear system of equations is linearised using the Linear Theory approach to improve convergence, while the resulting linear system is then solved by means of the Jacobi iterative approach, which can be massively parallelized. The resulting model was tested on a number of numerical cases, offering a good compromise between computational speed and accurate reproduction of the flood event. Although the reliability of using the Jacobi approach needs to be carefully verified over a wide variety of problems, the first results are quite interesting and full of future potentialities. Next work will focus on presenting the results of the IFD-LT model in real world applications, and on realizing a parallelized version of the model.

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