

IMPLICATIONS OF DIFFUSIVE WAVE CASCADING PLANE SIMULATIONS FOR THE STUDY OF SURFACE WATER - GROUNDWATER INTERACTION WITH A 3-D FULLY-INTEGRATED CATCHMENT MODEL

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Summary. The study of hydrological responses and interactions within coupled surface water - groundwater systems at catchment scales gave rise to the development of 3-D fully-integrated, physically-based models solving the diffusive wave equation coupled to the Richards' equation for variably-saturated flow. As their typical temporal and spatial resolution requirements are high, such models demand very long computation times. Looking for computationally efficient alternatives to these types of models (to which we will refer as the geometrically realistic model below) we investigated the usability of a simplified watershed model, the so-called equivalent diffusive wave cascading plane for parameter studies. The simplified model allows considerable savings in computation time while it keeps important model processes, certain geometric characteristics, and material properties of the catchment. Despite the geometric simplification the fully coupled system of surface water and subsurface water flow equations are solved within the diffusive wave cascading plane. We use the cascading plane to study the sensitivities of different catchment controlling factors (e.g. roughness coefficient, saturated hydraulic conductivity) and soil hydraulic functions (e.g. air-entry pressure, pore-size factor) on the hydrological response at the Lerma river basin a tributary to the Ebro River, Spain. The comparison with the calibrated geometrically realistic model indicates that the diffusive wave cascading plane has the potential to identify meaningful parameter ranges and sensitivity rankings that may be transferred from the simplified model to the realistic one at the benefit of reduced computation times.

1 Introduction

A common concern regarding fully-coupled transient modeling at catchment scales is its computational demand [1, 2, 3]. Fairly high temporal and spatial resolution is typically required for the accurate representation of the processes considered. Also, large uncertainties are usually associated with the definition of effective parameter values. The calibration and validation process demands very long computation times. Consequently, simplification would be desirable to speed-up parameter studies.

Looking for computationally-efficient alternatives to a geometrically realistic model (GRM) we investigated the usability of a simplified watershed model, the so-called equivalent diffusive wave cascading plane (DWCP) which represents an extension of the kinematic wave plane proposed by [4]. The DWCP of the catchment only preserves certain geometric characteristics: surface area, perimeter and the elevation distribution of the hypsometric curve (i.e. a hillslope representation of the actual catchment), while flow processes are still treated in a physical way.

The objective of this work is to show the potential of the DWCP to identify meaningful parameter ranges and sensitivity rankings that can be transferred to a GRM at the benefit of reduced computation times and faster calibration, based on the findings of the Lerma basin case study.

2 Case Study: Lerma Basin

The Lerma basin is located within the Ebro catchment (see Fig.1) and covers an area of 7.5 km². It is geologically characterized by quaternary deposits (glacis and alluvials) overlying tertiary materials (lutites and marlstones). The shallow aquifer (associated to the glacial deposits) is seasonally fed by precipitation and irrigation return flows and discharge into the Lerma creek.

Topographic information consist of a 25 m cell-sized digital terrain model (DTM). Daily rainfall, air temperature, wind velocity, relative humidity and radiation are obtained from a climatological station located within 5 kilometer from the Lerma Basin. Based on the hydroclimatological data, evapotranspiration is calculated with the Penman-Monteith method. Potential evapotranspiration (ET_0) is calculated based on grass as the crop reference [5, 6] and corrected by a crop factor (k_c) (i.e. related to land-use) to obtain the actual evapotranspiration (ET). Figure 2 shows daily precipitation (P) and daily observed discharge (Q) for the simulated period, i.e.the hydrological years 2006 and 2007

Soil properties are obtained from a soil characterization campaign including 10 points within the Lerma Basin. Soil samples were classified into different textures (i.e. clayey, clayey-loam and sandy-clayey-loam). Typical values of saturated hydraulic conductivities K_s for the textures identified within the Lerma basin, found in the literature [7, 8], range between 0.005 and 1.0 m/d. K_s values measured via pumping tests in the north of the Arba catchment are larger (between 1 and 7.7 m/d). For the definition of the Van Genuchten parameters α and n , only typical values found for texture classes are available [7]: α varies between 0.05 and 6.0 m^{-1} while values for n range between 1.01 and 2.0 [–], reflecting the large uncertainties associated to soil parameter values. Hence, investigating uncertainty of these paramameters has become the primary task for our case study.

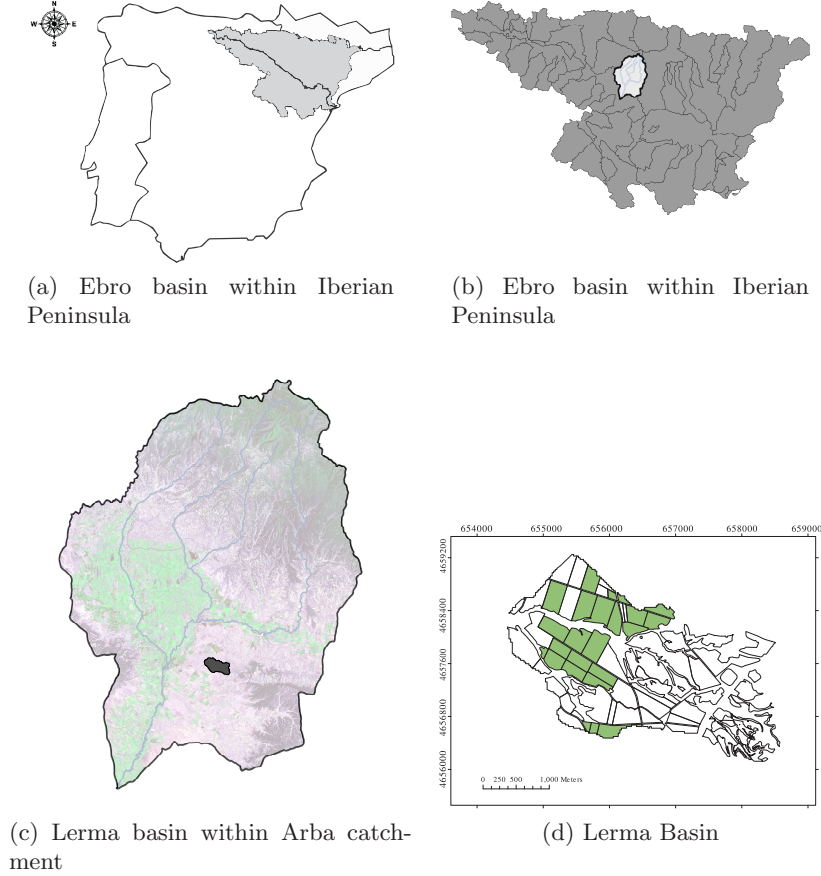


Figure 1: Location of study area in the Arba basin: (a) the Ebro basin in the Iberian peninsula; (b) the Arba catchment within the Ebro basin; (c) Lerma basin within the Arba catchment and (d) the Lerma basin

2.1 Diffusive wave cascading plane (DWCP)

Here, we propose the diffusive wave cascading plane (DWCP) which represents a simplification of the realistic model in order to speed-up the calibration process. The 3-D geometrically realistic model (GRM) is converted into a 2-D equivalent diffusive wave cascading plane. Surface area and perimeter of the GRM are preserved and the topographic structure is converted into a hillslope keeping the realistic model hypsometric curve (see Fig. 3).

Despite its geometric simplification we simulate the fully-coupled system of surface water and subsurface water flow with the DWCP applying the actual climatological and anthropogenic forcing (i.e. measured values of rainfall, evapotranspiration and irrigation).

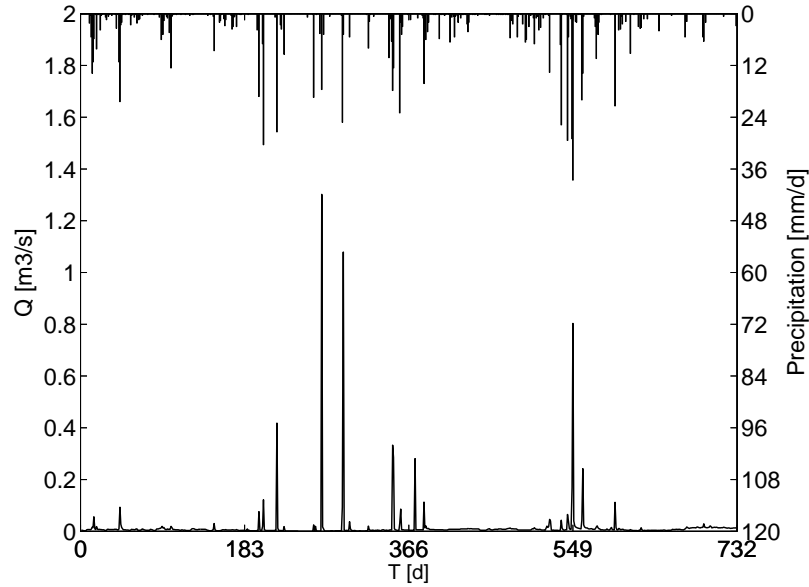


Figure 2: Daily precipitation(top) and daily discharges Q (bottom) at the basin outlet for the simulated years 2006-2007

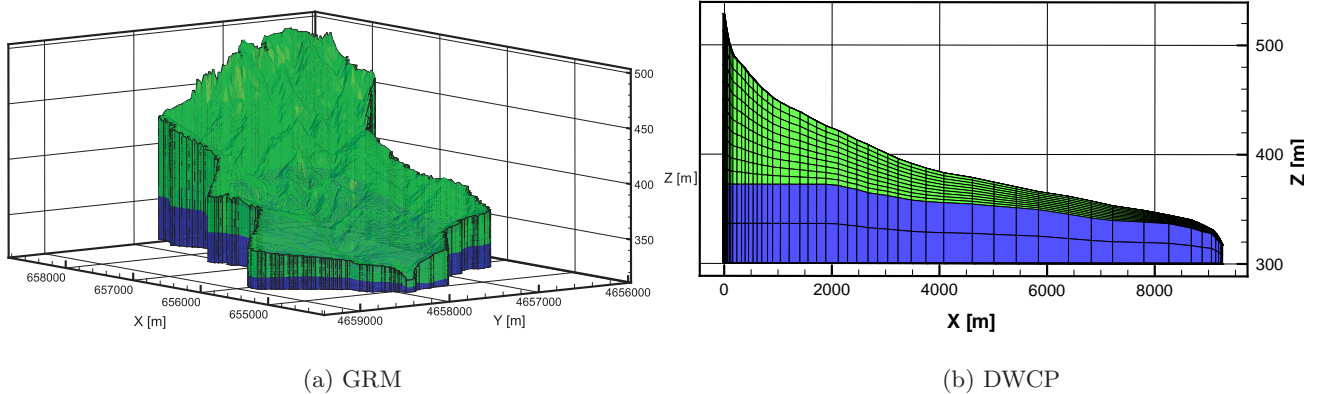


Figure 3: Geometrically Realistic Model (GRM) and Diffusive wave cascading plane (DWCP) for the Lerma basin (green: glaciais and blue: tertiary materials)

3 Simulations with the Diffusive Wave Cascading Plane(DWCP)

The starting point for model calibration is to identify those parameters that should become calibration parameters. Typically, these are both uncertain and sensitive. The determination of the relevant parameters using the realistic model is very time consuming due to the large

parameter space and to the long running times. Hence, we use our simplified cascading plane to define an appropriate set of calibration parameters.

In the first step a very simple set-up was used: available climatological variables (e.g. rainfall and evapotranspiration) and anthropogenic forces (e.g. irrigation) are applied to the cascading plane. Boundary conditions were set to no-flow except for the outlet, which is defined as critical depth. The choice of critical depth is supported by the fact that the gauging station at the basin outlet represents a weir, where critical depth is ensured. The subsurface was assumed to be homogeneous and the initial condition was generated by a dynamic water balance approach. The subsurface domain was incorporated into the DWCP by reproducing the hypsometric representation of the aquifer top and bottom surface. This results in a two layered subsurface structure with a shallow aquifer overlying an aquitard. The saturated hydraulic conductivity (K_s), the inverse air-entry pressure (α) and the pore distribution factor (n) are subject to investigation. The simplified model was used for the evaluation of sensitivities of soil hydraulic properties (i.e. K_s , α and n) and the roughness coefficient (k_x). Sensitivities indicate the level of influence of the hydraulic parameters on the hydrological response.

To quantify sensitivities we approximate the partial derivative of the simulated value S with respect to a parameter p_i for each observation (i.e. stream-discharge value) using the central-difference sensitivities scheme proposed by [9]:

$$\frac{\partial S}{\partial p_i} \Big|_{\vec{p}} \approx \frac{S(\vec{p} + \vec{e}_i \Delta p_i) - S(\vec{p} - \vec{e}_i \Delta p_i)}{2\Delta p_i} \quad (1)$$

where \vec{p} is the model parameter vector or evaluation point in parameter space, \vec{e}_i is the i th standard basis vector and Δp_i represents the change in value of parameter vector component p_i .

Generally, a full analysis of sensitivities is computationally demanding in nonlinear problems, because sensitivities may change significantly throughout the parameter space. To scan the parameter space we calculated the sensitivities at N locations in the parameter space, denoted by $\frac{\partial S}{\partial p_i} \Big|_{\vec{p}^{(j)}}$ where $j \in [1, N]$ and $\vec{p}^{(j)}$ denotes the j th evaluation point in parameter space. For the calculation of each $\frac{\partial S}{\partial p_i} \Big|_{\vec{p}^{(j)}}$, we simulated the two perturbed cases $S(\vec{p}^{(j)} + \vec{e}_i \Delta p_i)$ and $S(\vec{p}^{(j)} - \vec{e}_i \Delta p_i)$ and computed the sensitivities via Eq. 1. In order to make a perturbation about a small parameter value comparable to one about a larger parameter value, the sensitivities $\frac{\partial S}{\partial p_i} \Big|_{\vec{p}^{(j)}}$ are multiplied by the parameter value $\vec{p}^{(j)} \cdot \vec{e}_i = p_i^{(j)}$ to arrive at scaled sensitivities (as proposed by [10]). The composite scaled sensitivities (CSS_i^j) over a whole sequence of N_{obs} simulated discharge values S_k with $k = 1, \dots, N_{obs}$ are estimated as follows (after [10]):

$$CSS_i^j = \frac{1}{N_{obs}} \sum_{k=1}^{N_{obs}} \frac{\partial S_k}{\partial p_i} \Big|_{\vec{p}^{(j)}} p_i^{(j)} \quad (2)$$

Table 1 presents the range of model parameter values and the perturbation Δp_i used for each parameter p_i .

From the sensitivity analysis, we performed a parameter ranking (Fig. 4). We used the averaged-CSS values (average is calculated with all of the values j for each parameter i) to measure the importance of each parameter on the hydrological response. Larger CSS values

Parameter	Min. Value	Max. Value	dp_i
$K_s[m/d]$	0.004	8.64	0.01 [m/d]
$\alpha[m^{-1}]$	0.001	6.0	0.005 [m^{-1}]
$n[-]$	1.0	3.0	0.05 [-]
$k_x[s/m^{1/3}]$	0.03	0.05	0.001 [$s/m^{1/3}$]

Table 1: Information used to calculate sensitivities for k_s , α , n and k_x

indicate more meaningful parameters or parameters for which the observations provide more information. The roughness coefficient k_x appears to have a negligible sensitivity compared to K_s , α and n .

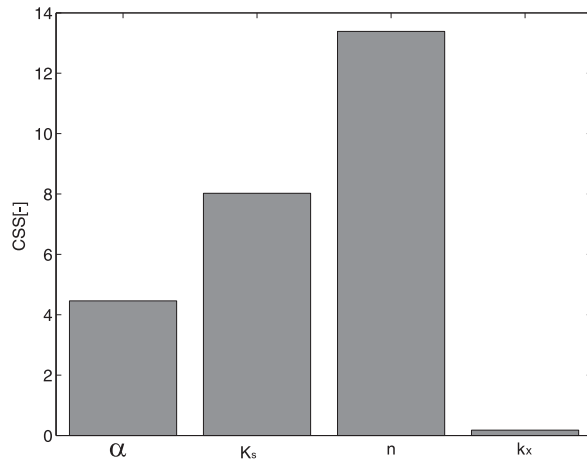


Figure 4: Composite scaled sensitivities calculated from the diffusive wave cascading plane simulations

4 Results and discussion

To test the suitability and transferability of the diffusive cascading plane parameter sensitivities, we compare it with a sensitivity analysis using the geometrically realistic model of Lerma using identical climatological forcing, i.e. the same rainfall and evapotranspiration. Additionally, a second cascading plane (DWCP2) is evaluated for which we did not keep the geometrical characteristics (i.e. we increased the values in 10%). For both cases, we calculated the composite scaled sensitivities about the same parameter values (i.e. p_i^j) used for the DWCP.

CSSs for the original DWCP and the GRM are very similar (see Figure 5). Both show a lower sensitivity for α compared to n and K_s . The change in CSS observed at different values of α follows the same pattern for the GRM and DWCP. Clayey and sandy textures associated to very small and very high values, respectively, have in general a higher sensitivity than those calculated for sandy-clay and sandy-clay-loam (i.e. $\alpha=2.0 - 5.0$).

CSSs for n show a pronounced peak (i.e peak-height=170) around $n = 1.28$. The peak-width (measured at the height of 60% of the peak) is 0.20. Around the peak, i.e. $n \in [1.08 - 1.48]$,

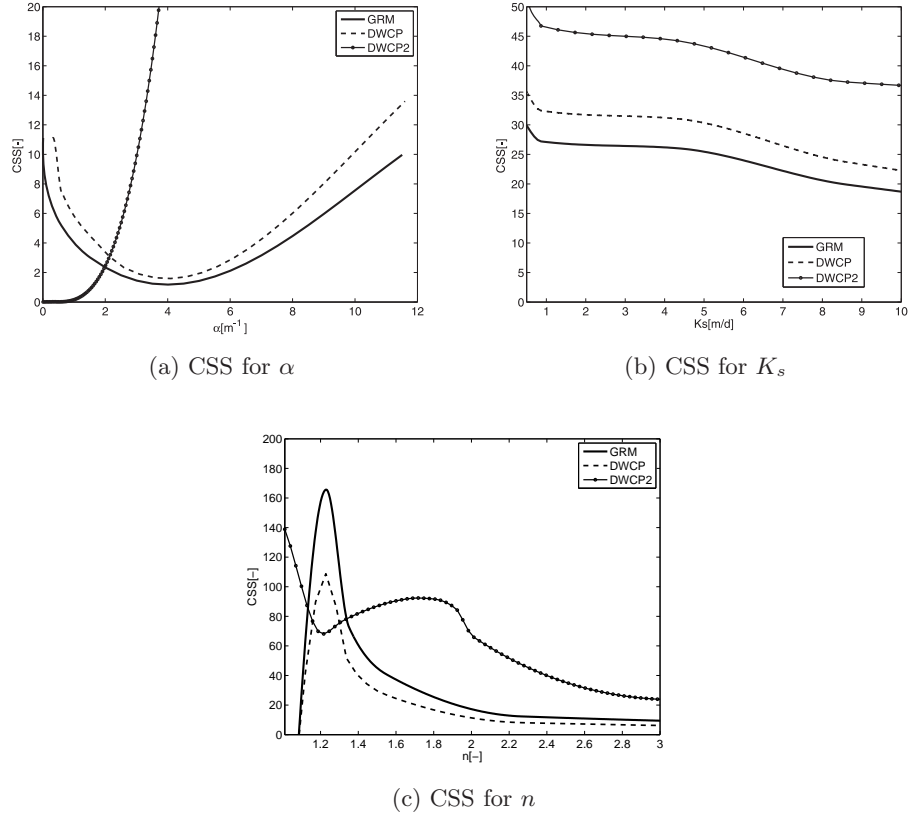


Figure 5: Composite scaled sensitivities for DWCP, DWCP2 and GRM

CSS change drastically. Most of the values of n reported in the literature (e.g. [7]) fall within this range. For $n > 2.2$, CSSs become much smaller (i.e. CSS=20) and for further increase in n its CSS remains constant.

CSSs of K_s are higher than those calculated for α for the whole simulated range. The trend for CSS of K_s is a slight decrease with increasing values of K_s . Overall the DWCP and GRM show a very similar behavior in terms of their CSSs.

For the mis-scaled DWCP2 case, a quite different CSS-structure can be observed. For α , an exponential growth of CSS is observed within the simulation range, while for n we can see a sharp decrease of CSS until $n = 1.25$. After that a gradual increase of CSS is observed, which extends to $n = 2.0$. Above $n = 2.0$ the CSSs decrease again. These results indicates that it is important to keep perimeter and area equal to the GRM study. On the other hand, it shows that the sensitivity structure of the GRM can not be reproduced by an arbitrary DWCP.

Within the course of the calibration of the Lerma basin GRM to match discharge series at the basin outlet, we systematically used parameter ranges for K_s , α and n which were identified as sensitive within the DWCP study. Calibration results show a good agreement between observed and simulated series (Figure 6).

The analysis of the calibrated GRM indicates that the DWCP has the potential to identify

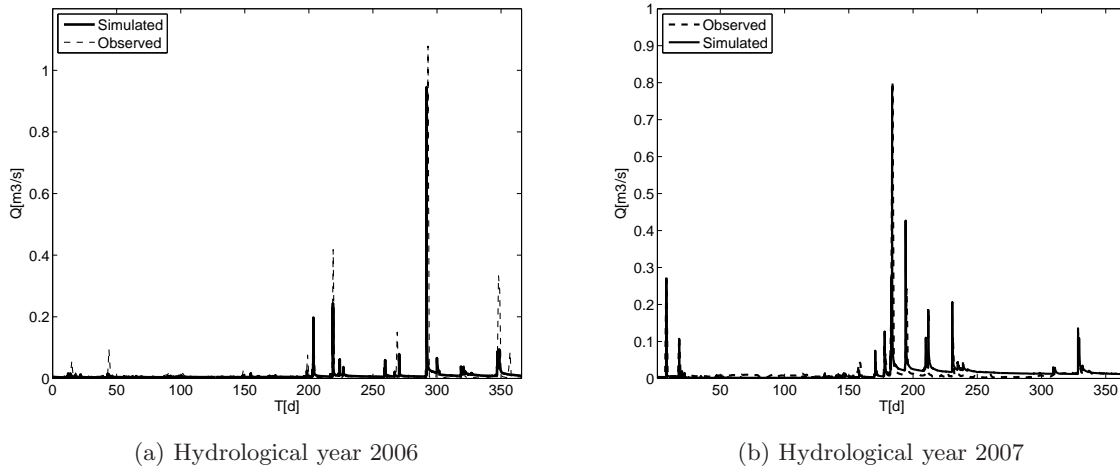


Figure 6: Simulation results with the GRM for the hydrological years 2006 and 2007

meaningful parameter ranges and sensitivity rankings that may be transferred from the DWCP to the GRM at the benefit of reduced computation times.

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